BOUNDED COHOMOLOGY AND REPRESENTATION VARIETIES OF LATTICES IN $PSU(1,n)$

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Abstract. We present a general technique to obtain rigidity results, based on homological methods in bounded cohomology developed in [3]. We illustrate this method by showing that lattices in $PSU(1,n)$, $n \geq 2$, admit no non-trivial deformations into $PSU(1,m)$, thus extending to the non-uniform case deformation rigidity theorems à la Goldman-Millson ([8], [5], [11]).

1. The results

Let X_i, $i = 1,2$ be symmetric spaces, G_i the associated group of orientation preserving isometries and $\Gamma < G_1$ a torsion free lattice. For certain k's we associate to every homomorphism $\pi : \Gamma \to G_2$ a linear map

$$I_\pi^{(k)} : \Omega^k(X_2)^{G_2} \to \Omega^k(X_1)^{G_1}$$

between the spaces of G_i-invariant differential k-forms on X_i, whose evaluation on "characteristic classes" is constant on connected components of the representation variety $\mathrm{Rep}(\Gamma, G_2)$ of Γ into G_2, and which generalizes invariants introduced by Goldman [6] and Toledo [11]. More precisely, let $\pi^* : \Omega^*(X_2)^{G_2} \to H^*_{DR}(\Gamma\backslash X_1)$ be the morphism with target in the de Rham cohomology of $\Gamma\backslash X_1$, induced by any choice $F_\pi : X_1 \to X_2$ of a smooth Γ-equivariant map; we assume on k that the L^2-cohomology $H^k_{(2)}(\Gamma\backslash X_1)$ injects into $H^k_{DR}(\Gamma\backslash X_1)$, and that the image of π^* is contained in $H^k_{(2)}(\Gamma\backslash X_1)$. Then we obtain $I_\pi^{(k)}$ by composing π^k with the orthogonal projection of $H^k_{(2)}(\Gamma\backslash X_1)$ onto $\Omega^k(X_1)^{G_1}$.

We turn now to a specific situation. Let $\Gamma < PSU(1,n)$ be a torsion free lattice in the group of orientation preserving isometries of complex hyperbolic n-space $\mathbb{H}^n_{\mathbb{C}}$, $\pi : \Gamma \to PSU(1,m)$ a homomorphism, $M = \Gamma\backslash\mathbb{H}^n_{\mathbb{C}}$ and ω_ℓ the Kähler form on $\mathbb{H}^\ell_{\mathbb{C}}$. Observe that if $n = 1$ and M is non-compact, then $H^2_{DR}(M) = 0$ and in particular the class $\pi^*(\omega_\ell)$

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vanishes, so that from now on, we shall assume that

$$n \geq 2 \qquad \text{or} \qquad M \text{ is compact .}$$

In this case $H^2_{(2)}(M)$ injects into $H^2_{DR}(M)$ ([13, Theorem 6.9]) and we identify it with its image. We shall then show that $\pi^*(\omega_m)$ lies in $H^2_{(2)}(M)$. If $< \cdot, \cdot >$ denotes the natural scalar product on $H^2_{(2)}(M)$ and ω_M is the Kähler class on M, we have

$$I^{(2)}_\pi(\omega_m) = \imath^{(2)}_\pi \omega_n \quad ,$$

with

$$\imath^{(2)}_\pi = \frac{< \pi^*(\omega_n), \omega_M >}{< \omega_M, \omega_M >} \quad .$$

Theorem 1.1. *Under the above assumption we have*

$$\left|\imath^{(2)}_\pi\right| \leq 1 \quad . \tag{1.1}$$

Moreover, equality holds if and only if:

- *$n \geq 2$ and there is an isometric embedding $F : \mathbb{H}^n_{\mathbb{C}} \to \mathbb{H}^m_{\mathbb{C}}$ which is Γ-equivariant;*
- *$n = 1$ and $\pi(\Gamma)$ leaves a complex line invariant .*

Since the cohomology class $\pi^*(\omega_m) \in H^2_{DR}(M)$ is a characteristic class of the principal $PSU(1,m)$ bundle over M associated to π, it is constant on connected components of $\mathrm{Rep}(\Gamma, PSU(1,m))$; in particular, equality holds in (1.1) if π lies in the component of the restriction to Γ of a "standard" representation of $PSU(1,n)$ into $PSU(1,m)$, because in this case $\pi^*(\omega_m) = \pm\omega_M$ and thus $|\imath^{(2)}_\pi| = 1$. Hence,

Corollary 1.2. *There are no non-trivial deformations of Γ in $PSU(1,m)$.*

For Γ cocompact the above corollary was obtained by Goldman and Millson in [8]. When M is compact, one has $I^{(2n)}_\pi(\omega^n_m) = \imath^{(2n)}_\pi \omega^n_n$, where

$$\imath^{(2n)}_\pi = \frac{1}{\mathrm{vol}(M)} \mathrm{eval}(\pi^*(\omega^n_m), [M])$$

with $[M]$ the fundamental class of M; in this case, Corlette (for $n \geq 2$, [5]) and Toledo (for $n = 1$, [11]) obtained analogous rigidity results concerning the invariant $\imath^{(2n)}_\pi$. Notice however that if M is not compact, then $H^{2n}_{DR}(M) = 0$ and in particular the invariant $\pi^*(\omega^n_m) \in H^{2n}_{DR}(M)$ vanishes.

For $n = 1$ and M non-compact the above corollary fails: indeed, Gusevskii and Parker have constructed lattices in $PSU(1,1)$ admitting quasi-Fuchsian deformations into $PSU(1,2)$, [9].

Sketch of the proof of Theorem 1.1: The theorem is a consequence of a formula involving the invariant $\imath_\pi^{(2)}$ and the Cartan angle $c_\ell : \mathbb{H}^\ell_{\mathbb{C}}(\infty)^3 \to [-1,1]$, which is a $PSU(1,\ell)$-invariant, alternating cocycle on the boundary $\mathbb{H}^\ell_{\mathbb{C}}(\infty)$ of $\mathbb{H}^\ell_{\mathbb{C}}$, measuring the extent to which triples of points fail to lie on the boundary of a complex geodesic ([7, §7.1]). Observe that if $\pi(\Gamma)$ is bounded, the left hand side of (1.1) is clearly zero, so that we may assume that $\pi(\Gamma)$ is unbounded and hence that there exists a Γ-equivariant measurable map $\varphi : \mathbb{H}^n_{\mathbb{C}}(\infty) \to \mathbb{H}^m_{\mathbb{C}}(\infty)$. Thus if μ denotes the invariant probability measure on $\Gamma\backslash SU(1,n)$, we have

$$\int_{\Gamma\backslash SU(1,n)} c_m(\varphi(gx_1),\varphi(gx_2),\varphi(gx_3))d\mu(g) = \imath_\pi^{(2)} \cdot c_n(x_1,x_2,x_3) \tag{1.2}$$

for almost every $(x_1,x_2,x_3) \in \mathbb{H}^n_{\mathbb{C}}(\infty)^3$. Applying (1.2) to triples of points for which c_n is maximal, we obtain the inequality in the theorem. As for the equality case, it is based on the fact that the configuration of points on which $|c_n|$ attains its maximal value 1 are exactly the triples of points lying on chains, that is on boundaries of complex geodesics. Thus, in the equality case, the boundary map φ sends almost every chain into a chain; a modification of a theorem due to E. Cartan [4] implies then that φ comes form an isometric embedding of $\mathbb{H}^n_{\mathbb{C}}$ into $\mathbb{H}^m_{\mathbb{C}}$. □

We shall now sketch a proof of the formula (1.2), which rests in an essential way on the homological characterization of bounded continuous cohomology and the relation of the latter to ordinary continuous cohomology.

2. The methods

2.1. **Preliminaries on bounded cohomology.** Let G be a locally compact group. The bounded continuous cohomology $H^*_{b,c}(G)$ of G (with trivial coefficients) is defined as the cohomology of the complex

$$0 \to L^\infty(G)^G \xrightarrow{d} L^\infty(G^2)^G \xrightarrow{d} \cdots \to L^\infty(G^n)^G \xrightarrow{d} \dots$$

where d is the standard coboundary operator. We recall here the main concepts pertaining to the theory of bounded cohomology and refer to [3], where this was developed, for details. In particular we shall not recall here the definition of continuous bounded cohomology with coefficients, although we shall use it in the proof of (2.3).

A continuous Banach G-module E is a Banach space on which G acts continuously by isometric automorphisms; G-morphisms are linear continuous G-equivariant maps between (continuous) Banach G-modules. We say that E is relatively injective (with respect to G) if for every injective admissible G-morphism $\imath : A \to B$ of continuous Banach G-modules A, B and every G-morphism $\alpha : A \to E$, there is a G-morphism $\beta : B \to E$ which extends α and such that $\|\beta\| \leq \|\alpha\|$

$$\begin{array}{ccc} A & \overset{\imath}{\hookrightarrow} & B \\ & \searrow & \\ & \alpha & \downarrow \beta \\ & & E \end{array}$$

A (relatively injective) admissible resolution E_* of $\mathbb{R}$ is then a sequence

$$0 \to \mathbb{R} \xrightarrow{d} E_0 \xrightarrow{d} E_1 \xrightarrow{d} \cdots \xrightarrow{d} E_n \xrightarrow{d} \dots$$

of (relatively injective) continuous Banach G-modules E_n equipped with G-morphisms d such that $d^2 = 0$ and continuous homotopy operators $h : E_n \to E_{n-1}$. The cohomology associated to any admissible resolution E_* is the cohomology of the corresponding non-augmented subcomplex of invariants

$$0 \to E_0^G \xrightarrow{d} E_1^G \xrightarrow{d} \cdots \xrightarrow{d} E_n^G \xrightarrow{d} \dots$$

If E_* and F_* are admissible resolutions of $\mathbb{R}$ and F_* is relatively injective, then the identity map $id : \mathbb{R} \to \mathbb{R}$ extends to a G-morphism of resolutions which is unique up to G-homotopy. If in addition also E_* is relatively injective, then any G-morphism of resolutions which extends the identity $id : \mathbb{R} \to \mathbb{R}$ induces a canonical isomorphism of the corresponding cohomology spaces (as topological vector spaces).

In what follows it will be essential to know that if (S, ν) is a regular amenable G-space, that is an amenable G-space such that the action of G on $L^1(S)$ is continuous, then the Banach G-module $L^\infty(S)$ is relatively injective (and in fact, this provides a characterization of amenable actions, although this will not be used here, [3]). It hence follows that the continuous bounded cohomology of G can be computed as the cohomology of the complex

$$0 \to L^\infty(S)^G \xrightarrow{d} L^\infty(S^2)^G \xrightarrow{d} \cdots \to L^\infty(S^n)^G \xrightarrow{d} \dots$$

or alternatively of the subcomplex of the G-invariant alternating bounded cocycles on S^n. As an example of such situation, and which will be of relevance to our case, recall that if Γ is a lattice in a semisimple Lie group G and $P < G$ is a minimal parabolic subgroup, then both Γ and G act amenably on G/P.

Moreover, if we have a continuous homomorphism $\pi : G_1 \to G_2$ between locally compact groups, and if (S_i, ν_i) is a regular amenable G_i-space, we can consider $L^\infty(S_2^*)$ as a a G_1-admissible resolution via

π and $L^\infty(S_1^*)$ as a G_1-admissible resolution by injective objects. We then obtain a G_1-morphism of complexes

$$\pi_b^* : L^\infty(S_2^*) \to L^\infty(S_1^*)$$

extending the identity $id : \mathbb{R} \to \mathbb{R}$, which is unique up to G_1-homotopy.

Recall moreover that there is a natural comparison map

$$H_{b,c}^*(G_i) \to H_c^*(G_i)$$

to the continuous cohomology of G_i, such that, if π^* is the map induced in cohomology by π, the diagram

$$\begin{array}{ccc} H_{b,c}^*(G_1) & \to & H_c^*(G_1) \\ \downarrow \pi_b^* & & \downarrow \pi^* \\ H_{b,c}^*(G_2) & \to & H_c^*(G_2) \end{array} \tag{2.1}$$

commutes.

2.2. **The diagram.** For $i = 1, 2$, let X_i be a symmetric space of non-compact type and let $G_i = \mathrm{Isom}(X_i)^0$. If $\Gamma < G_1$ is a lattice and if $\pi : \Gamma \to G_2$ is a homomorphism, let $F : X_1 \to X_2$ be a smooth Γ-equivariant map. The proof of the formula (1.2) is based on the commutativity of the following diagram,

(2.2)

$$\begin{array}{ccccc} H^*(\mathcal{B}((G_2/P_2)^*)^{G_2}) & \to & H_{b,c}^*(G_2) & \to & \Omega^*(X_2)^{G_2} \\ & \searrow \Phi^* & \downarrow \pi_b^* & & \downarrow F^* \\ & & H_b^*(\Gamma) & \to \ H_{(2)}^*(\Gamma\backslash X_1) \ \to & H_{DR}^*(\Gamma\backslash X_1) \\ & & \downarrow t_b & \downarrow t & \\ & & H_{b,c}^*(G_1) & \to \ \Omega^*(X_1)^{G_1} & \end{array}$$

which we now proceed to establish after giving the appropriate missing definitions.

2.2.1. *The triangle diagram.*
(i) The complex $\mathcal{B}(Y^*)$*:* If Y is a compact metric space with a continuous action of a locally compact group H which is regular with respect to some probability measure on Y, let $\mathcal{B}(Y^n)$ denote the space of all bounded Borel functions on Y^n, with the supremum norm. This is a Banach space on which H acts isometrically. Let $\mathcal{B}_0(Y^n)$ be the space of H-continuous vectors. One verifies that the complex

$$0 \to \mathbb{R} \xrightarrow{d} \mathcal{B}_0(Y) \xrightarrow{d} \cdots \xrightarrow{d} \mathcal{B}_0(Y^n) \xrightarrow{d} \dots$$

is an admissible H-resolution (see Section 2.1). *(ii) The triangle:* For $i = 1, 2$, let H_i be locally compact groups and let (Y_i, ν_i) be standard regular amenable H_i-spaces. Assume that Y_2 is a compact metric space and that the action of H_2 on Y_2 is continuous. First, by considering any Borel function on Y_2^n as an L^∞-function, we get an H_2-morphism of H_2-complexes

$$f^* : \mathcal{B}(Y_2^*) \to L^\infty(Y_2^*)$$

extending the identity $id : \mathbb{R} \to \mathbb{R}$. Moreover, we observed already in Section 2.1 that there is an H_1-morphism of complexes

$$\pi_b^* : L^\infty(Y_2^*) \to L^\infty(Y_1^*)$$

extending the identity $id : \mathbb{R} \to \mathbb{R}$, which is unique up to H_1-homotopy.

Finally, since H_1 acts amenably on (Y_1, ν_1), there is an H_1-equivariant measurable map $\varphi : Y_1 \to \mathcal{M}(Y_2)$ (where $\mathcal{M}(Y_2)$ denotes the space of probability measures on Y_2), which gives rise to a continuous H_1-morphism of complexes

$$\Phi^* : \mathcal{B}(Y_2^*) \to L^\infty(Y_1^*)$$

defined in the following way. If $c : Y_2^n \to \mathbb{R}$ is a bounded Borel function, define

$$\Phi^n(c)(y_1, \dots, y_n) = (\varphi(y_1) \otimes \cdots \otimes \varphi(y_n))(c).$$

Observe that since every Borel function is the pointwise limit of a sequence of continuous functions, the evaluation map of probability measures on bounded Borel functions is Borel measurable in the weak topology.

Clearly Φ^* extends $id : \mathbb{R} \to \mathbb{R}$ and is therefore unique up to H_1-homotopy.

From what we gathered so far, we have the diagram

$$\begin{array}{ccc} \mathcal{B}(Y_2^*) & \xrightarrow{f^*} & L^\infty(Y_2^*) \\ & \searrow_{\Phi^*} & \downarrow \pi_b^* \\ & & L^\infty(Y_1^*) \end{array} .$$

Since Φ^* and $\pi_b^* \circ f^*$ are both continuous H_1-morphisms of complexes $\mathcal{B}(Y_2^*) \to L^\infty(Y_1^*)$ extending the identity $id : \mathbb{R} \to \mathbb{R}$, and they are hence unique up to H_1-homotopy, they induce the same maps in cohomology. Thus the diagram

$$\begin{array}{ccc} H^*((\mathcal{B}(Y_2^*)^{H_2}) & \to & H_{b,c}^*(H_2) \\ & \searrow_{\Phi^*} & \downarrow \pi_b^* \\ & & H_{b,c}^*(H_1) \end{array}$$

commutes and implies the commutativity of the triangle diagram in (2.2) if we take $Y_2 = G_2/P_2$, $H_2 = G_2$ and $H_1 = \Gamma$ (so that $H_{b,c}^*(\Gamma) = H_b^*(\Gamma)$).

2.2.2. *The upper square.* We have the commutative diagram (see (2.1) in Section 2.1)

$$\begin{array}{ccc} H^*_{b,c}(G_2) & \to & H^*_c(G_2) \\ \downarrow \pi^*_b & & \downarrow \pi^* \\ H^*_b(\Gamma) & \to & H^*(\Gamma) \end{array}$$

where the horizontal arrows are the natural comparison maps. Using the theorem of van Est [12], we identify $H^*_c(G_2)$ with the space $\Omega^*(X_2)^{G_2}$ of G_2-invariant differential forms on X_2; identifying $H^*(\Gamma)$ with the de Rham cohomology $H^*_{DR}(\Gamma\backslash X_1)$, the map π^* is then given by $F^*:\Omega^*(X_2)^{G_2}\to H^*_{DR}(\Gamma\backslash X_1)$, that is it is induced by the pullback via F of differential forms. We consider now the diagram

(2.3)

$$\begin{array}{ccccc} H^*_b(\Gamma) & & H^*_{(2)}(\Gamma\backslash X_1) & \to & H^*_{DR}(\Gamma\backslash X_1) \\ \downarrow & & \downarrow & & \downarrow \\ H^*_{b,c}(G_1, L^\infty(\Gamma\backslash G_1)) & \to & H^*_c(G_1, L^2(\Gamma\backslash G_1)) & \to & H^*_c(G_1, L^2_{loc}(\Gamma\backslash G_1)) \end{array}$$

where all the horizontal arrows are natural comparison maps and the vertical arrows are isomorphisms defined as follows. If $f\in L^\infty((G_1/P_1)^n)^\Gamma$, define $\imath f(g_1,\dots,g_n)(h)=f(hg_1,\dots,hg_n)$, for $h\in G_1$. Then $\imath f$ is in the space $L^\infty_{w*}(G_1,L^\infty(\Gamma\backslash G_1))^{G_1}$ of G_1-equivariant essentially bounded weakly-$*$ measurable functions on G_1 with values in $L^\infty(\Gamma\backslash G_1)$ and the induced map in cohomology

$$H^*_b(\Gamma)\xrightarrow{\imath^*}H^*_{b,c}(G_1,L^\infty(\Gamma\backslash G_1))$$

is an isomorphism, where we used again the characterization of bounded cohomology recalled in Section 2.1. The isomorphism between $H^*_{(2)}(\Gamma\backslash X_1)$ and $H^*_c(G_1,L^2(\Gamma\backslash G_1))$ is due to Borel (see [2, Theorem 3]) and the last arrow is, modulo the identification of $H^*(\Gamma)$ with $H^*_{DR}(\Gamma\backslash X_1)$, the Eckman-Shapiro isomorphism in continuous cohomology (see [1]). We use this diagram to define the factorization

$$H^*_b(\Gamma)\to H^*_{(2)}(\Gamma\backslash X_1)\to H^*_{DR}(\Gamma\backslash X_1)$$

of the natural comparison map

$$H^*_b(\Gamma)\to H^*_{DR}(\Gamma\backslash X_1) \quad .$$

2.2.3. *The lower square.* Integration over $\Gamma\backslash G_1$ gives G_1-invariant projections $L^\infty(\Gamma\backslash G_1)\to\mathbb{C}$ and $L^2(\Gamma\backslash G_1)\to\mathbb{C}$ and hence a commutative diagram

$$\begin{array}{ccc} H^*_{b,c}(G_1,L^\infty(\Gamma\backslash G_1)) & \to & H^*_c(G_1,L^2(\Gamma\backslash G_1)) \\ \downarrow & & \downarrow \\ H^*_{b,c}(G_1) & \to & H^*_c(G_1). \end{array}$$

Via the isomorphisms used in Section 2.2.2 we obtain a commutative diagram

$$\begin{array}{ccc} H_b^*(\Gamma) & \to & H_{(2)}^*(\Gamma\backslash X_1) \\ \downarrow t_b & & \downarrow t \\ H_{b,c}^*(G_1) & \to & \Omega^*(X_1)^{G_1} \end{array}$$

whose maps t_b and t can be written explicitly as follows:
(i) For $f \in L^\infty((G_1/P_1)^n)^\Gamma$, we have

$$t_b f(x_1,\dots,x_n) = \int_{\Gamma\backslash G_1} f(gx_1,\dots,gx_n)d\mu(g).$$

(ii) For ω an L^2-differential form on $\Gamma\backslash X_1$,

$$t\omega = \int_{\Gamma\backslash G_1} g_*\omega.$$

In fact, t admits the following useful interpretation: identifying $\Omega^*(X_1)^{G_1}$ with a subspace of the L^2-harmonic forms on $\Gamma\backslash X_1$, itself naturally a subspace of $H_{(2)}(\Gamma\backslash X_1)$, the map t is the precisely the orthogonal projection of $H_{(2)}^*(\Gamma\backslash X_1)$ onto $\Omega^*(X_1)^{G_1}$.

2.3. **Proof of the formula (1.2).** Let $G = PSU(1,\ell)$, $X = \mathbb{H}_{\mathbb{C}}^\ell$ and let $S = \mathbb{H}_{\mathbb{C}}^\ell(\infty)$ be the boundary of hyperbolic ℓ-space. Then we have that $\Omega^2(X)^G = \mathbb{R}\cdot\omega_\ell$ and the comparison map $H_{b,c}^2(G) \to \Omega^2(X)^G$ is an isomorphism (see [3, Proposition 9]). Combining the fact that

$$H_{b,c}^2(G) = \{\alpha \in L_{alt}^\infty(S^3)^G : \, d\alpha = 0\}$$

with the fact that the Cartan angle

$$c_\ell : S^3 \to [-1,1]$$

is an alternating G-invariant Borel cocyle ([7, §7.1]), we get that if $[c_\ell]$ denotes the class of c_ℓ in $L^\infty(S^3)$, then $H_{b,c}^2(G) = \mathbb{R}\cdot[c_\ell]$. Since $\pi(\Gamma)$ is unbounded, we may assume that $\varphi : \mathbb{H}_{\mathbb{C}}^n(\infty) \to \mathcal{M}(\mathbb{H}_{\mathbb{C}}^m(\infty))$ takes values in the Dirac masses. Thus we shall use the commutativity of the triangle diagram Section 2.2.1 for the Γ-equivariant measurable map $\varphi : \mathbb{H}_{\mathbb{C}}^n(\infty) \to \mathbb{H}_{\mathbb{C}}^m(\infty)$, and deduce that the L^∞ function

$$(x_1,x_2,x_3) \mapsto c_\ell(\varphi(gx_1),\varphi(gx_2),\varphi(gx_3))$$

is a representative for $\pi_b^*([c_\ell]) \in H_b^2(\Gamma)$ which in turn, using the formula for t_b in Section 2.2.3, implies that the map

$$(x_1,x_2,x_3) \mapsto \int_{\Gamma\backslash PSU(1,n)} c_m(\varphi(gx_1),\varphi(gx_2),\varphi(gx_3))d\mu(g) \tag{2.4}$$

is a representative for $t_b\pi_b^*([c_m])$.

On the other hand, using the fact that $H^2_{(2)}(M)$ injects into $H^2_{DR}(M)$ and the commutativity of the two squares in the diagram (2.2) in Section 2.2 we have that

$$\begin{array}{rcl} t_b\pi_b^*([c_m]) & = & \lambda \cdot [c_n] \\ t\pi^*(\omega_m) & = & \lambda \cdot \omega_n . \end{array}$$

Finally, since t is the orthogonal projection of $H^2_{DR}(M)$ onto $\mathbb{R} \cdot \omega_M$, we obtain that

$$\lambda = \frac{< \pi^*(\omega_m), \omega_M >}{< \omega_M, \omega_M >}$$

which, together with (2.4), proves the formula (1.2).

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