DS-GA 3001.03: Homework Problem Set 8

Optimization and Computational Linear Algebra for Data Science

(Fall 2016)

Afonso S. Bandeira
bandeira@cims.nyu.edu
http://www.cims.nyu.edu/~bandeira

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This homework problem set is due on November 22, before class at the homework dropbox in the CDS reception.

If you have any questions about the homework post them on the piazza page for the course, contact Vlad Kobzar at vkobzar@cims.nyu.edu or myself at bandeira@cims.nyu.edu, or stop by our office hours.

Try not to look up the answers, you'll learn much more if you try to think about the problems without looking up the solutions. If you need hints, feel free to email me or Vlad.

You can work in groups but each student must write his/her own solution based on his/her own understanding of the problem. Please list, on your submission, the students you work with for the homework (this will not affect your grade).

If you need to impose extra conditions on a problem to make it easier (or consider specific cases of the question, like taking n to be 2, e.g.), state explicitly that you have done so. Solutions where extra conditions were assume, or where only special cases where treated, will also be graded (probably scored as a partial answer).

Problems with a $(*)$ are extra credit, they will not (directly) contribute to your score of this homework. However, for every 4 (four) extra credit questions successfully answered you get a homework "bye": your lowest homework score (or one you did not hand in) gets replaced by a perfect score.

Problem 6.1 *Let* $f : \mathbb{R}^n \to \mathbb{R}$ *be an affine function*

$$f(x) = a^T x + b.$$

Show that f *is convex and concave (recall that* f *is concave if* $-f$ *is convex). Is* f *strictly convex?*

Problem 6.2 *Show that if* $f_1, \ldots, f_m$ *are convex functions and* $\alpha_1, \ldots, \alpha_m \geq 0$ *then*

$$f(x) = \sum_{k=1}^{m} \alpha_k f_k(x)$$

is a convex function.

Problem 6.3 *Show that subspaces are convex sets.*

Problem 6.4 *Show that if* f *is convex then a local minima of* x *needs to be a global minima. A local minima of* f *is a point* $x^\natural$ *for which there exists* $\epsilon > 0$ *such that for all* x *satisfying* $\left\|x - x^\natural\right\|_2 \leq \epsilon$ *we have* $f(x) \geq f(x^\natural)$. *(*Hint: *draw a picture).*

($*$) **Problem 6.5 (For Extra Credit)** *Prove or disprove: If* $f : \mathbb{R} \to \mathbb{R}$ *and* $g : \mathbb{R} \to \mathbb{R}$ *are two convex functions then* $h : \mathbb{R} \to \mathbb{R}$ *given by* $h(x) = f(g(x))$ *is also a convex function.*