DS-GA 1014: Homework Problem Set 2

Optimization and Computational Linear Algebra for Data Science

(Fall 2018)

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Due on Friday September 14, 2018

This homework problem set is due on September 7 on NYU Classes.

If you have questions about the homework feel free to contact Brett Bernstein (brett.bernstein@nyu.edu**) or myself, or stop by our office hours.**

Unless otherwise stated all answer must be mathematically justified.

Try not to look up the answers, you'll learn much more if you try to think about the problems without looking up the solutions. If you need hints, feel free to email me or Vlad.

You can work in groups but each student must write his/her own solution based on his/her own understanding of the problem. Please list, on your submission, the students you work with for the homework (this will not affect your grade).

Late submissions will be graded with a penalty of 10% **per day late. Weekend days do not count, from Friday to Monday counts only 1 day.**

If you need to impose extra conditions on a problem to make it easier (or consider specific cases of the question, like taking n **to be** 2**, e.g.), state explicitly that you have done so. Solutions where extra conditions were assume, or where only special cases where treated, will also be graded (probably scored as a partial answer).**

Problems with a $(*)$ **are extra credit, they will not (directly) contribute to your score of this homework. However, for every 4 (four) extra credit questions successfully answered you get a homework "bye": your lowest homework score (or one you did not hand in) gets replaced by a perfect score.**

Problem 2.1 *Let* $f : \mathbb{R}^2 \to \mathbb{R}^3$ *be a linear transformation such that*

$$f(1,2) = (1,2,3) \quad \text{and} \quad f(2,2) = (1,0,1).$$

1. *Give the set* $\{x \in \mathbb{R}^2 \mid f(x) = (1,4,5)\}$.
2. *Give the set* $\{x \in \mathbb{R}^2 \mid f(x) = (2,4,5)\}$.

Problem 2.2 *Given a linear transformation* $L : \mathbb{R}^m \to \mathbb{R}^n$*, show that* $\ker(L)$ *is a subspace of* $\mathbb{R}^m$.

Problem 2.3 *Consider two linear transformations* $L : \mathbb{R}^2 \to \mathbb{R}^3$ *(corresponding to a* 3×2 *matrix) and* $U : \mathbb{R}^3 \to \mathbb{R}^2$ *(corresponding to a* 2×3 *matrix). Consider the linear transformation* LU *given by* $LU(v) = L(U(v))$ *(which is from* $\mathbb{R}^3$ *to* $\mathbb{R}^3$*). Can it be that* $\ker(LU) = \{0\}$*? Justify your answer: If yes, give an example, if not prove that it cannot be the case.*

Problem 2.4 *(Essentially in Problem 45 on Section 3.5. of Strang's book) Let* V *and* W *be two subspaces in* $\mathbb{R}^n$*. Show that if*

$$\dim(V) + \dim(W) > n,$$

then there must exist a non-zero vector in the intersection of them, i.e.: $V \cap W \neq \{0\}$.

$(*)$ **Problem 2.5 (For Extra Credit)** *(Essentially in Problem 38 on Section 2.7. of Strang's book) We can think about a permutation of* n *elements as a linear transformation from* $\mathbb{R}^n$ *to* $\mathbb{R}^n$ *that permutes the basis elements* $e_1, \ldots, e_n$*. In that case it is an* $n \times n$ *matrix with only* 0*'s and* 1*'s as entries and such that every row and every column have exactly one entry being* 1.

Is it true that for any n *and any permutation* P *there exists an integer* $k \geq 1$ *such that*

$$P^k = I,$$

where I *is the identity matrix (and* P^k *means* P *multiplied with itself* k *times). Justify your answer.*