Lectures on Symplectic Geometry

— Errata for the 2006 text on author's website*—

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page 15, line 2
By Weinstein's lagrangian creed **[108]**, everything is a lagrangian manifold! not [105]

page 27, lines 6 and 5 from the bottom
is the graph of a diffeomorphism $\varphi : T^*X \to T^*X$, then φ is the symplectomorphism generated by f. In this case, $\varphi(x,\xi) = (y,\eta)$ if and only if... not f

page 28, lines 2 and 1 from the bottom
For both steps, it might be useful to recall that, given a function $\boldsymbol{h} : X \to \mathbb{R}$ and a tangent vector $v \in T_xX$, we have $d\boldsymbol{h}_x(v) = \frac{d}{du}\left[\boldsymbol{h}(\exp(x,v)(u))\right]_{u=0}$. not φ

page 32, lines 2-6

$$\begin{cases} \dfrac{\partial f}{\partial x} = -\dfrac{\chi(x)-\chi(y)}{|\chi(x)-\chi(y)|}\cdot\dfrac{d\chi}{ds}(x) = \cos\theta = v \\[2ex] \dfrac{\partial f}{\partial y} = -\dfrac{\chi(y)-\chi(x)}{|\chi(x)-\chi(y)|}\cdot\dfrac{d\chi}{ds}(y) = -\cos\nu = -w\ . \end{cases}$$

replace the two systems of equations by the single system above

page 32, line 12
$-|x_1 - x_2| - \ldots - |x_{N-1} - x_N| - |x_N - x_1|$ sign change

*The errata for the 2008 Springer text is at
https://people.math.ethz.ch/∼acannas/Papers/lsg_errata.pdf
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page 36, lines 9 and 13

$$\mathcal{L}_{v_t}\omega := \frac{d}{ds}\psi^*_{s,t}\omega\Big|_{s=t}$$

where $\psi_{s,t}$ is the flow of v_t, i.e., $s \mapsto \psi_{s,t_0}(p)$ is the unique maximal integral curve of v_t with value p at time $s = t_0$:

$$\tfrac{d}{ds}\psi_{s,t_0}(p)\big|_{s=t} = v_t\left(\psi_{t,t_0}(p)\right) \quad \text{and} \quad \psi_{t_0,t_0}(p) = p\,.$$

This is related to the previous ρ by $\rho_s = \psi_{s,0}$. If M is compact, the flow is globally defined and we have $\psi_{s,t} = \psi_{s,t_0}\circ\psi_{t_0,t}$, thus $\psi_{s,t}^{-1} = \psi_{t,s}$, and hence $\psi_{s,t} = \rho_s \circ \rho_t^{-1}$. We can use this last expression to write $\mathcal{L}_{v_t}\omega$ alternatively in terms of ρ. A good reference is the textbook by John Lee, *Introduction to Smooth Manifolds*.
the definition of Lie derivative by a time-dependent vector field v_t was wrong

page 38, Theorem 6.6
(i.e., a unique $q \in X$ minimizing $|q - \boldsymbol{p}|$) not $|q - x|$

page 39, Proposition 6.8
$\mu \in \boldsymbol{\Omega^{\ell-1}}(\mathcal{U})$ instead of $\Omega^{d-1}(\mathcal{U})$

page 40, line 5 from bottom
remove "(reviewed in the next section)"

page 46, line 9 from bottom
Hypothesis: X is a **half**-dimensional submanifold with... instead of n-dimensional

page 49, line 2 of Theorem 8.6
X a **compact** submanifold of dimension $k \geq n$,... assume compactness of X

page 55, last line i.e. footnote 9
... if $\mathbf{Id}-df_p : T_pM \to T_pM$ is nonsingular. not just df_p

page 79, line 6 of §14.2
$(\Lambda^\ell T^{1,0}) \otimes (\Lambda^m T^{0,1})$ so $\otimes$ instead of $\wedge$ in the definition of $\Lambda^{\ell,m}$

page 90, line 6
It follows immediately from the previous definition **and from Theorem 15.4** that
add clause

page 113, line 16 from bottom
$V(q) := \boldsymbol{-W_\gamma}$ sign change

page 121, last condition in Proposition 20.2
(d) $F(p) \to +\infty$ *as* $p \to \infty$ *in* V. omit "F is proper, that is,"

page 126, line 3 of part 7.
tautological 1-form twice replace "canonical" by "tautological"

page 141, line 3
whenever there is an **abelian** symmetry group the word "abelian" was missing

page 145, line 2 of part (d)

By the **inverse** function theorem instead of "implicit"

page 162, exercise 4, line 3
$\mu(z) = \frac{\mathbf{1}}{\mathbf{2i}} z z^*$ sign change, the formula in the hint is good

page 163, lines 5 and 10
$\mu(A) = \frac{\mathbf{1}}{\mathbf{2i}} AA^* - \frac{\mathrm{Id}}{2i}$ and $\mu(A) = \frac{\mathbf{1}}{\mathbf{2i}}[A, A^*]$ sign changes, following that in the previous page

page 180 and subsequent pages
the number π and the projection map π should be distinguished by different symbols

page 193, lines 3-4 of footnote
Z a **compact** manifold of dimension $k \geq n$ with... assume compactness of Z