Quantifying Regulatory Capital for Operational Risk: Utopia or Not?

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Contents

A. The New Accord (Basel II)

- **1988**: Basel Accord (Basel I): minimal capital requirements against credit risk, one standardised approach, Cooke ratio
- **1996**: Amendment to Basel I: market risk, internal models, netting
- **1999**: First Consultative Paper on the New Accord (Basel II)
- **to date**: CP3: Third Consultative Paper on the New Basel Capital Accord (`www.bis.org/bcbs/bcbscp3.htmcp3`)
- **2004**: Revision: (final) version
- **2006–2007**: full implementation of Basel II ([13])

Basel II: What is new?

- **Rationale** for the New Accord: More flexibility and risk sensitivity

- **Structure** of the New Accord: Three-pillar framework:

❶ Pillar 1: minimal capital requirements (risk measurement)

❷ Pillar 2: supervisory review of capital adequacy

❸ Pillar 3: public disclosure

- Two options for the measurement of credit risk:
  - Standard approach
  - Internal rating based approach (IRB)
- Pillar 1 sets out the minimum capital requirements (Cooke Ratio):

$$\frac{\text{total amount of capital}}{\text{risk-weighted assets}} \geq 8\%$$

- MRC (minimum regulatory capital) $\stackrel{\text{def}}{=}$ 8% of risk-weighted assets
- Explicit treatment of operational risk

Operational Risk:

The risk of losses resulting from inadequate or failed internal processes, people and systems, or external events

Remark: Business Risk is not included!

- **Notation**: C_{OP}: capital charge for operational risk
- **Target**: $C_{\mathsf{OP}} \approx 12\%$ of MRC (down from initial 20%)
- **Estimated total losses** in the US (2001): \$50b
- **Some examples**
  - 1977: Credit Suisse Chiasso-affair
  - 1995: Nick Leeson/Barings Bank, £1.3b
  - 2001: Enron (largest US bankruptcy so far)
  - 2002: Allied Irish, £450m

B. Risk measurement methods for OP risks

Pillar 1 regulatory minimal capital requirements for operational risk:
Three distinct approaches:

❶ Basic Indicator Approach

❷ Standardised Approach

❸ Advanced Measurement Approaches (AMA)

Basic Indicator Approach

- Capital charge:

$$\boxed{C_{\mathsf{OP}}^{\mathsf{BIA}} = \alpha \times GI}$$

- $C_{\mathsf{OP}}^{\mathsf{BIA}}$: capital charge under the Basic Indicator Approach

- GI: average annual gross income over the previous three years

- $\alpha = 15\%$ (set by the Committee based CISs)

Standardised Approach

- Similar to the BIA, but on the level of each business line:

$$\boxed{C_{\mathsf{OP}}^{\mathsf{SA}} = \sum_{i=1}^{8} \beta_i \times GI_i}$$

$\beta_i \in [12\%, 18\%]$, $i = 1, 2, \ldots, 8$

- 8 business lines:

Corporate finance	Payment & Settlement
Trading & sales	Agency Services
Retail banking	Asset management
Commercial banking	Retail brokerage

Advanced Measurement Approaches (AMA)

- Allows banks to use their internally generated risk estimates
- Preconditions: Bank must meet qualitative and quantitative standards before being allowed to use the AMA
- Risk mitigation via insurance possible
- **AMA1**: Internal measurement approach (dropped!)
- **AMA2**: Loss distribution approach

Internal Measurement Approach

- Capital charge (similar to Basel II model for Credit Risk):

$$\boxed{C_{\text{OP}}^{\text{IMA}} = \sum_{i=1}^{8} \sum_{k=1}^{7} \gamma_{ik}\, e_{ik}} \qquad \text{(dropped!)}$$

e_{ik}: expected loss for business line i, risk type k

γ_{ik}: scaling factor

- 7 loss types: Internal fraud
  External fraud
  Employment practices and workplace safety
  Clients, products & business practices
  Damage to physical assets
  Business disruption and system failures
  Execution, delivery & process management

C. Loss Distribution Approach

- For each business line/loss type cell (i, k) one models

$L_{i,k}^{T+1}$: OP risk loss for business line i, loss type k over the future (one year, say) period $[T, T+1]$

$$L_{i,k}^{T+1} = \sum_{\ell=1}^{N_{i,k}^{T+1}} X_{i,k}^{\ell} \quad \text{(next period's loss for cell } (i,k))$$

Note that $X_{i,k}^{\ell}$ is truncated from below

Remark: Look at the structure of the loss random variable L^{T+1}

$$
\begin{aligned}
L^{T+1} &= \sum_{i=1}^{8} \sum_{k=1}^{7} L_{i,k}^{T+1} \quad \text{(next period's total loss)} \\
&= \sum_{i=1}^{8} \sum_{k=1}^{7} \sum_{\ell=1}^{N_{i,k}^{T+1}} X_{i,k}^{\ell}
\end{aligned}
$$

A methodological pause 1

$$L = \sum_{k=1}^{N} X_k \quad \text{(compound rv)}$$

where (X_k) are the severities and N the frequency

Models for X_k:

- gamma, lognormal, Pareto (≥ 0, skew)

Models for N:

- binomial (individual model)
- Poisson(λ) (limit model)
- negative binomial (randomize λ as a gamma rv)

- Choice of a risk measure g $(\alpha \in (0,1)$ fixed)

$$C_{i,k}^{T+1,\mathrm{OR}} = g(L_{i,k}^{T+1}) = \begin{cases} F^{\leftarrow}_{L_{i,k}^{T+1}}(\alpha) = \mathrm{VaR}_\alpha(L_{i,k}^{T+1}) \\ \mathrm{ES}(L_{i,k}^{T+1}) = E\left(L_{i,k}^{T+1} \middle| L_{i,k}^{T+1} > \mathrm{VaR}_\alpha(L_{i,k}^{T+1})\right) \end{cases}$$

- VaR_α is not coherent (example)
- ES_α is coherent (modulo trivial change)

$$C^{T+1,\mathrm{OR}} = \sum_{i,k} g(L_{i,k}^{T+1}) \quad \text{(perfect correlation)}$$

- Why?
- Dependence effects (copulae)

VaR_α is in general not coherent:

- 100 iid loans: 2%-coupon, 100 face value, 1% default probability (period: 1 year):

$$X_i = \begin{cases} -2 & \text{with probability } 99\% \\ 100 & \text{with probability } 1\% \quad \text{(loss)} \end{cases}$$

- Two portfolios $L_1 = \sum_{i=1}^{100} X_i$, $L_2 = 100X_1$

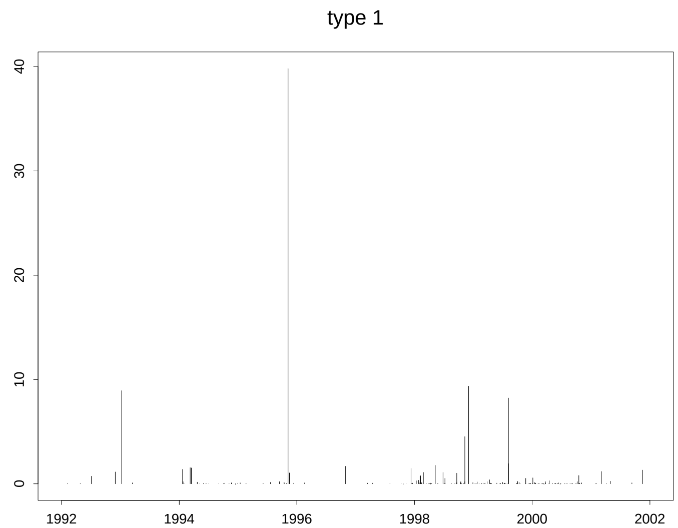
- $\underbrace{\mathrm{VaR}_{95\%}(L_1)}_{\mathrm{VaR}_{95\%}(\sum_{i=1}^{100} X_i)} > \underbrace{\mathrm{VaR}_{95\%}(100X_1)}_{\sum_{i=1}^{100} \mathrm{VaR}_{95\%}(X_i)}$ (!)

- Hence the well-diversified portfolio L_1 gets a higher (VaR-)risk charge than the very concentrated, "all eggs in one basket" portfolio L_2

- This cannot happen when $(X_1, \ldots, X_d)$ has a multivariate normal (or more generally, elliptical) distribution

- Link to Operational Risks: skewness

D. Some data


type 1
0
10
20
30
40
1992
1994
1996
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2000
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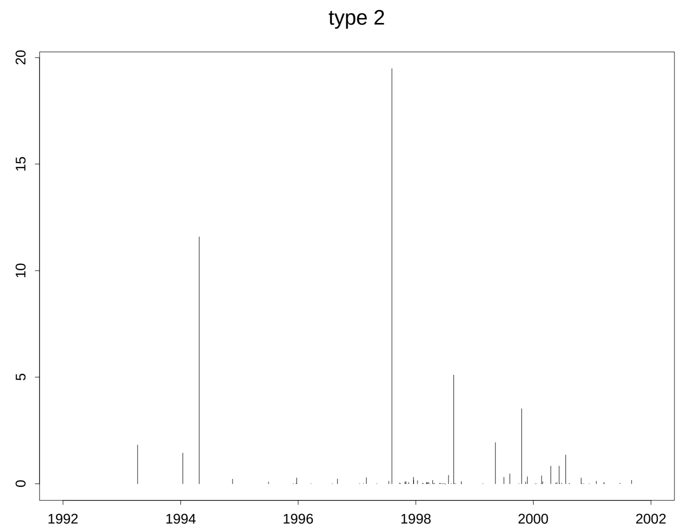
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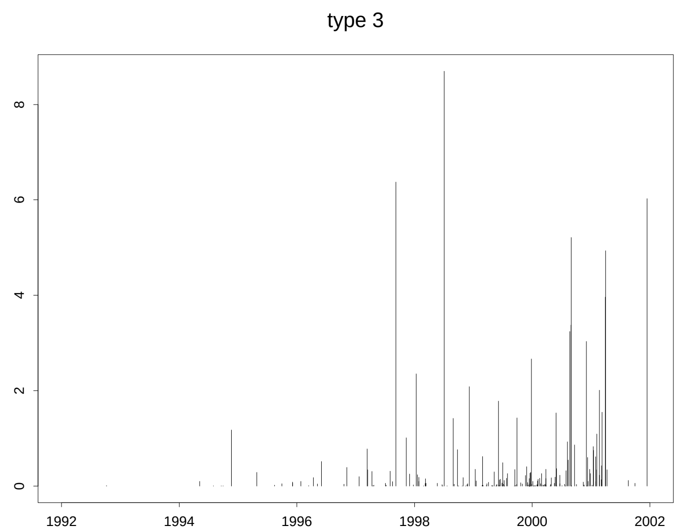
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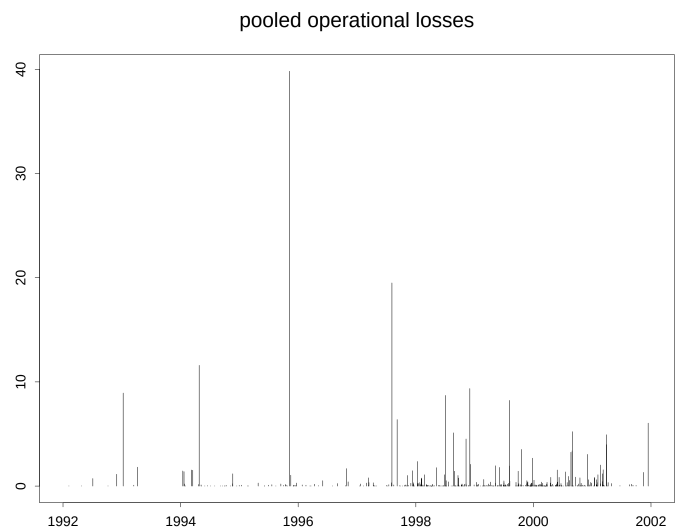
pooled operational losses
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1992
1994
1996
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2000
2002

- Stylized facts about OP risk losses:
  - Loss amounts show extremes
  - Loss occurence times are irregularly spaced in time (reporting bias, economic cycles, regulation, management interactions, structural changes, . . .)
  - Non-stationarity (frequency(!), severity(?))
- Large losses are of main concern
- Repetitive versus non-repetitive losses
- **Warning flag**: observations are not in line with standard modelling assumptions

A methodological pause 2

- severity models need to go beyond the classical models (binomial, homogeneous Poisson, negative binomial: $\longrightarrow$ stochastic processes)

- as stochastic processes:

  - Poisson(λt), $\lambda > 0$ deterministic (1)
  - Poisson($\lambda(t)$), $\lambda(t)$ deterministic
    non-homogeneous Poisson, via time change $\longrightarrow$ (1)
  - Poisson($\Lambda(t)$), $\Lambda(t)$ stochastic process
    - double stochastic (or Cox-) process
    - basic model for credit risk

- a desert-island model: (NB,LN) (cover of [4])

E. The Capital Charge Problem

- Estimate $g_\alpha(L^{T+1})$ for α large

  Basel II: $g_\alpha = \mathrm{VaR}_\alpha$, $\alpha = 99.97\%$ (reason)

- In-sample estimation of $\mathrm{VaR}_\alpha(L^{T+1})$ for α large is difficult, if not impossible (lack of data)

- Even for nice (repetitive) data one needs a structural model: Insurance Analytics ([11])

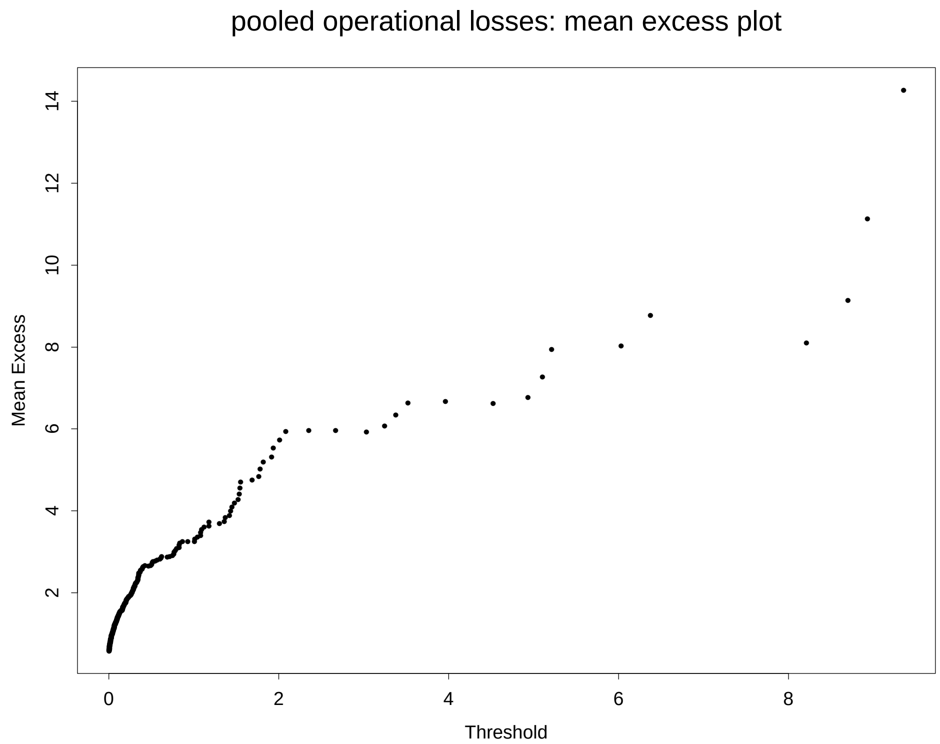
- Standard Actuarial Techniques

  * Analytic approximations (normal, translated gamma, Edgeworth, saddle-point, . . .)
    However: long-tailedness (Pareto, power tails)

$$P(X > x) \sim x^{-\alpha} L(x), \; x \text{ large}$$

  * Inversion methods (FFT)
  * Recursive methods (Euler-Panjer)
  * (Rare event) simulation
  * Expert system Ansatz
  * Extreme Value Theory (EVT): α large

- Back to the data



- $P(L > x) \sim x^{-\alpha}L(x),\ 1 < \alpha < 3$

- 20 – 80 rule

- one-claim-causes-ruin phenomenon ([1])

Summary

- $\alpha \simeq 1$ and heavy-tailed loss-sizes, hence extremes matter
  - Extreme Value Theory (EVT) ([8])
- adding risk measures over different risk classes, hence dependence matters
  - Copulae $(F_{\underline{X}}(\underline{x}) = C(F_1(x_1), \ldots, F_d(x_d)))$ ([9])
- complicated loss-frequencies, hence point processes matter
  - double-stochastic (or Cox) processes ([5])
- full model analytically not tractable, hence
  - rare event simulation ([3])

F. Accuracy of VaR-estimates

- **Assumptions**:
  - $L_1, \ldots, L_n$ iid $\sim F_L$
  - For some ξ, β and u large ($G_{\xi,\beta}$: GPD):

$$F_u(x) := \mathbb{P}[L - u \le x | L > u] \sim G_{\xi,\beta(u)}(x), \ u \text{ large}$$

  - Use that: $1 - F_L(x) = (1 - F_L(u))\,(1 - F_u(x-u))\,, \quad x > u$

- Tail- and quantile estimate:

$$\boxed{\begin{aligned} 1 - \hat{F}_L(x) &= \frac{N_u}{n}\Big(1 + \hat{\xi}\,\frac{x-u}{\hat{\beta}}\Big)^{-1/\hat{\xi}}, \quad x > u \\ \widehat{\mathrm{VaR}}_\alpha = \hat{q}_\alpha &= u - \frac{\hat{\beta}}{\hat{\xi}}\Big(1 - \Big(\frac{N_u}{n(1-\alpha)}\Big)^{\hat{\xi}}\Big) \end{aligned}} \tag{1}$$

- **Idea**: Comparison of estimated quantiles with the corresponding theoretical ones by means of a simulation study ([12], [6]).

- **Simulation procedure**:

  ❶ Choose F_L and fix $\alpha_0 < \alpha < 1$, N_u ($\#$ of data points above u)

  ❷ Calculate $u = q_{\alpha_0}$ and the true value of the quantile q_α

  ❸ Sample N_u independent points of F_L above u by the rejection method. Record the total number n of sampled points this requires

  ❹ Estimate ξ, β by fitting the GPD to the N_u exceedances over u by means of MLE

  ❺ Determine $\hat{q}_\alpha$ according to (1)

  ❻ Repeat N times the above to arrive at estimates of Bias$(\hat{q}_\alpha)$ and SE$(\hat{q}_\alpha)$

  ❼ Require bias and standard error to be small $\Rightarrow$ datasize

Example: Pareto distribution with $\alpha = 2$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	A minimum number of **100 exceedances** (corresponding to 333 observations) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of **200 exceedances** (corresponding to 667 observations) is required to ensure accuracy wrt bias and standard error.
$q = 0.9$	0.99	Full accuracy can be achieved with the minimum number **25** of **exceedances** (corresponding to 250 observations).
	0.999	A minimum number of **100 exceedances** (corresponding to 1000 observations) is required to ensure accuracy wrt bias and standard error.

Summary

- Minimum number of observations increases as the tails become thicker ([12], [6]).
- Large number of observations necessary to achieve targeted accuracy.
- **Remember**: The simulation study was done under idealistic assumptions (iid, exact Pareto). Operational risk losses, however, typically do NOT fulfil these assumptions.

G. Conclusions

- OP risk $\neq$ market risk, credit risk
- OP risk losses resemble non-life insurance losses
- Actuarial methods (including EVT) aiming to derive capital charges are for the moment of limited use due to
  - lack of data
  - inconsistency of the data with the modelling assumptions
- OP risk loss databases must grow
- Sharing/pooling internal operational risk data? Near losses?

- Choice of risk measure: ES better than VaR

- Heavy-tailed ruin estimation for general risk processes ([10]): an interesting mathematical problem related to time change

- Alternatives?

  - Insurance. Example: FIORI, Swiss Re (Financial Institution Operating Risk Insurance)
  - Securitization / Capital market products

- OP risk charges can not be based on statistical modelling alone

▶ Pillar 2 (overall OP risk management such as analysis of causes, prevention, . . .) more important than Pillar 1

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