ERRATA FOR

FUNCTIONAL ANALYSIS

AMS, GRADUATE STUDIES IN MATHEMATICS **191,** 2018

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Abstract. These notes correct a few typos and errors in the book *Functional Analysis*, AMS, Graduate Studies in Mathematics **191**, 2018.

p 4, l -14: In equation (1.1.3) replace the inequality "$f \leq c$" by "$|f| \leq c$".

p 8, l 15: Replace "$B_\varepsilon(x') \subset U_{i'}$" by "$B_\varepsilon(x) \subset U_{i'}$".

p 8, l 18: Replace "$J_n \setminus J_{n-1} = \{i_n\}$" by "'$J_{n+1} \setminus J_n = \{i_n\}$".

p 18, l 17: Replace "$\|A(\delta\|x\|_X^{-1}x)\|_X \leq 1$" by "$\|A(\delta\|x\|_X^{-1}x)\|_Y \leq 1$".

p 23, l -12 to -9: The sequence x_{i_k} in the proof of Lemma 1.2.13 may not exist. To correct the argument, the sequence should be chosen as follows (thanks to Tahl Nowik for pointing out this error as well suggesting the correction).

"Choose $i_1 \in \mathbb{N}$ such that $\inf_{y\in Y} \|x_{i_1} - x_j + y\| < 2^{-1}$ for every integer $j \geq i_1$. Once $i_1, \ldots, i_k$ have been constructed, choose $i_{k+1} > i_k$ to be the smallest integer bigger than i_k such that $\inf_{y\in Y} \left\|x_{i_{k+1}} - x_j + y\right\| < 2^{-k-1}$ for every integer $j \geq i_{k+1}$."

p 27, l 14: Typo: The displayed line should read

$$\|x\|_p := \left(\sum_{i=1}^{\infty} |x_i|^p\right)^{1/p} \qquad \text{for } x = (x_i)_{i\in\mathbb{N}} \in \ell^p.$$

p 65, l 16: In Theorem 2.3.2 replace "normed vector space" by "real vector space".

p 72, l -12: The assertion of Corollary 2.3.17 continues to hold for every real normed vector space X. Completeness is not required. To see this, note that for every closed linear subspace $Y \subset X$ the projection

$$\pi : X \to X/Y$$

is an open mapping by definition of the quotient topology on X/Y. Namely, if U is an open subset of X, so is the set

$$\pi^{-1}(\pi(U)) = \bigcup_{x\in U} (x + Y) = \bigcup_{y\in Y} (U + y) \subset X,$$

and hence $\pi(U)$ is an open subset of X/Y. With this understood, the proof of Corollary 2.3.17 carries over verbatim to the case where X is a real normed vector space, without any appeal to the Open Mapping Theorem 2.2.1.

p 88, l -4: Replace "p_n" by "x_n" (twice).

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p 111, l 11: The set $\mathcal{F} \subset \{f : X \to \mathbb{R} \,|\, f \text{ is linear}\}$ is required to be nonempty.

p 124, l 17/18: Replace $(x_{n_{i,k}})_{i\in\mathbb{N}}$, respectively $(x_{n_{i,k+1}})_{i\in\mathbb{N}}$, by $(x^*_{n_{i,k}})_{i\in\mathbb{N}}$, respectively $(x^*_{n_{i,k+1}})_{i\in\mathbb{N}}$. (Three times.)

We also remark that the Banach–Alaoglu Theorem for separable Banach spaces (Theorem 3.2.1) can be proved without using any version of the axiom of choice, as was pointed out to us by Mikhail Katz. Here is how this works.

First observe that there is a map that assigns to every bounded sequence of real numbers a convergent subsequence. Namely, given a bounded sequence $(a_n)_{n\in\mathbb{N}}$ of real numbers, let $a := \limsup_{n\to\infty} a_n$ and define a sequence $(n_i)_{i\in\mathbb{N}} =: \mathcal{T}((a_n)_{n\in\mathbb{N}})$ of positive integers recursively by

$$n_1 := \min\{n \in \mathbb{N} \,|\, |a_n - a| < 1\}$$

and

$$n_{i+1} := \min\left\{n \in \mathbb{N} \,\middle|\, n > n_i,\, |a_n - a| < \tfrac{1}{i}\right\} \qquad \text{for } i \in \mathbb{N}.$$

Then $|a_{n_i} - a| < 1/i$ for all $i \in \mathbb{N}$ and so $\lim_{i\to\infty} a_{n_i} = a$.

Now define $(n_{i,1})_{i\in\mathbb{N}} := \mathcal{T}((\langle x^*_n, x_1\rangle)_{n\in\mathbb{N}})$ and, for $k \in \mathbb{N}$,

$$n_{i,k+1} := n_{j_i,k}, \qquad (j_i)_{i\in\mathbb{N}} := \mathcal{T}\left(\left(\langle x^*_{n_{j,k}}, x_{k+1}\rangle\right)_{j\in\mathbb{N}}\right).$$

Then the sequence $(\langle x^*_{n_{i,k}}, x_k\rangle)_{i\in\mathbb{N}}$ converges and the sequence $(x^*_{n_{i,k+1}})_{i\in\mathbb{N}}$ is a subsequence of $(x^*_{n_{i,k}})_{i\in\mathbb{N}}$ for every $k \in \mathbb{N}$.

We also mention that the general Banach–Alaoglu Theorem (Theorem 3.2.4) is equivalent to the Boolean Prime Ideal Theorem as well as to the Tychonoff Theorem for compact Hausdorff spaces. (See the paper `http://karagila.org/wp-content/uploads/2016/10/axiom-of-choice-in-analysis.pdf` by Asaf Karagila and the references therein.)

p 136, l -5/6: The claim contains two typos. Replace "$x^* \in X^*$" by "$x \in X$" and replace "$x \in S$" by "$x^* \in S$". Here is the correct formulation of the claim for a given nonzero element $x^{**} \in X^{**}$.

Claim. *For every finite set $S \subset X^*$ there is an element $x \in X$ such that*

$$\|x\| \le 2\,\|x^{**}\|, \qquad \langle x^*, x\rangle = \langle x^{**}, x^*\rangle \quad \textit{for all } x^* \in S.$$

p 137, l 19: Replace the phrase "the weak topologies of X and X^*" by the phrase "the weak topology of X and the weak* topology of X^{**}".

p 181, l 13: Replace "$F : X \to Y$" by "$F : Y \to X$".

p 310, l 6: Delete the reference to Corollary 2.4.2.

p 310, l 13: Replace "Step 1" by "Step 2".

p 310, l -12: Replace the equation "$\Lambda_i = \Lambda_{x_i^*, y^*}$" by "$\Lambda_i = \Lambda_{x_i^*, y_i^*}$".

p 367, l -4: The proof of strong continuity of the inverse semigroup uses the estimate

$$\sup_{0\le h\le T} \|S(h)^{-1}\| < \infty$$

for all $T > 0$, which follows from the identity

$$S(h)^{-1} = S(T)^{-1}S(T-h), \qquad 0 \le h \le T,$$

the Open Mapping Theorem, and part (i) of Lemma 7.1.8.

p 388, l -2: The number $-\delta$ in equation (7.4.4) should be replaced by $+\delta$, i.e. the spectrum of A is contained in the sector

$$C_\delta := \left\{\omega_0 + re^{\mathbf{i}\theta} \,|\, r \geq 0,\ \pi/2 + \delta \leq |\theta| \leq \pi\right\} \tag{7.4.4}$$

(see Figure 7.4.1).

p 412, l 10: The letter "f" in equation (7.5.10) should be capitalized; thus

$$Z := \{t \in I \,|\, F \text{ is not differentiable at } t\}\,. \tag{7.5.10}$$

Here $F : I = [0,1] \to X$ is a continuous function with values in a Banach space.

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