# DS-GA 1014: Homework Problem Set 1

Optimization and Computational Linear Algebra for Data Science

(Fall 2018)

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Due on Friday September 7, 2018

**This homework problem set is due on September 7 on NYU Classes.**

**If you have questions about the homework feel free to contact Brett Bernstein (brett.bernstein@nyu.edu) or myself, or stop by our office hours.**

**Unless otherwise stated all answer must be mathematically justified.**

**Try not to look up the answers, you'll learn much more if you try to think about the problems without looking up the solutions. If you need hints, feel free to email me or Vlad.**

**You can work in groups but each student must write his/her own solution based on his/her own understanding of the problem. Please list, on your submission, the students you work with for the homework (this will not affect your grade).**

**Late submissions will be graded with a penalty of 10% per day late. Weekend days do not count, from Friday to Monday counts only 1 day.**

**If you need to impose extra conditions on a problem to make it easier (or consider specific cases of the question, like taking $n$ to be 2, e.g.), state explicitly that you have done so. Solutions where extra conditions were assume, or where only special cases where treated, will also be graded (probably scored as a partial answer).**

**Problems with a $(*)$ are extra credit, they will not (directly) contribute to your score of this homework. However, for every 4 (four) extra credit questions successfully answered you get a homework "bye": your lowest homework score (or one you did not hand in) gets replaced by a perfect score.**

**Problem 1.1** *Find the minimum value, a minimizer, the maximum value, and maximizer of the following functions (or show that there are no minimizers/maximizers or that the extremal value is* $\infty$*):*

1. $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = x^4 - 2x^2 + 4$
2. $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-3|}$.

**Problem 1.2** *Let* $v$ *and* $w$ *be two vectors in* $\mathbb{R}^2$*, show that either they are linearly dependent or they span the whole of* $\mathbb{R}^2$*.*

**Problem 1.3** *Show that any three vectors in* $\mathbb{R}^2$ *need to be linearly dependent.*

($*$) **Problem 1.4 (For Extra Credit)** *Let* $w = \alpha_1 v_1 + \cdots + \alpha_m v_m$ *where* $v_1, \ldots, v_m$ *in* $\mathbb{R}^n$ *and* $a_i > 0$ *for all* $i$*.*

*Prove that either (i)* $v_1, \cdots, v_m$ *are linearly independent or (ii) there exist coefficients* $b_i \geq 0$*, for* $i = 1, \ldots, m$ *such that* $w = b_1 v_1 + \cdots + b_m v_m$ *and at least one of the* $b_i$*'s is zero.*