DS-GA 1014: Extended Syllabus Lecture 1

Optimization and Computational Linear Algebra for Data Science (Fall 2018)

Afonso S. Bandeira
bandeira@cims.nyu.edu
http://www.cims.nyu.edu/~bandeira

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These are not meant to be Lecture Notes. They are simply extended syllabi with the most important definitions and results from the lecture. As such, they lack the intuition and motivation and so they are not a good place to learn the material the first time, just to briefly review it. These extended syllabi will also have references.

There are many amazing books about linear algebra and virtually all of them will contain the material for this particular lecture, examples include the book suggested for the course [2]. Another place you can read about some of these is the Lecture Notes for DSGA1002 [1].

Please let me know if you find any typos

- Span: Given vectors $v_1, \ldots, v_m \in \mathbb{R}^n$ the span of $v_1, \ldots, v_m$ is the set of the vectors that can be written as linear combinations of these:

$$\begin{aligned} \mathrm{Span}(v_1, \ldots, v_m) &= \langle v_1, \ldots, v_m \rangle \\ &= \left\{ v \in \mathbb{R}^n : v = \alpha_1 v_1 + \cdots + \alpha_m v_m \text{ for some } \alpha_1, \ldots, \alpha_m \in \mathbb{R} \right\}. \end{aligned}$$

- Linear dependency: vectors $v_1, \ldots, v_m \in \mathbb{R}^n$ are linear dependent if there exist $\alpha_1, \ldots, \alpha_m \in \mathbb{R}$ not all zero such that $v \in \mathbb{R}^n : 0 = \alpha_1 v_1 + \cdots + \alpha_m v_m$. Otherwise, they are linearly independent.

- Basis: A basis for a vector space is a set of vectors that span the whole vector space (say $\mathbb{R}^n$) and that is linearly independent.

  The following results related to basis were proven in Lab 2:

  - If $v_1, \ldots, v_m$ spans a vector space V then any vectors $w_1, \ldots, w_p$ with $p > m$ must be linearly dependent.
  - Any two bases of a vector space have the same length.
  - Every subspace of $\mathbb{R}^n$ has a basis.

- Dimension: The dimension of a vector space V is the number of vectors in a basis of that vector space (the dimension of $\mathbb{R}^n$ is n), we call this $\dim(V)$.

- Subspace: Given a vector space V, a subspace S is a subset $S \subset V$ such that: (i) for all $v \in S$ and $\alpha \in \mathbb{R}$ we have $\alpha v \in S$, and (ii) for all $v_1, v_2 \in S$ we have $v_1 + v_2 \in S$.

References

[1] Carlos Fernandez-Granda, *Lecture Notes of DSGA1002*, 2015 version available at `http://www.cims.nyu.edu/~cfgranda/pages/DSGA1002_fall15/notes.html`, 2015

[2] Gilbert Strang, *Introduction to Linear Algebra*, Fifth Edition, 2016