First Name:
Name:
Legi-Nr.:

D–ERDW, D-HEST, D-USYS

Mathematics I, Summer 2018

PD Dr. Lorenz Halbeisen

Modus:

- Time: **90 minutes**.
- Permitted written aids: 10 A4-size sheets (= 20 pages) of your own summary, either hand-written or typed. Dictionaries. No pocket calculators.

Further Remarks:

- Put your student identity card on the table.
- **Switch off your mobile phone** and put it into your bag.
- In exercises 1-4 all answers in your work require proof. Exceptions: It is okay to use statements shown in class (like fundamental solutions to PDE's or integrals like $\int_{-\pi}^{\pi} \cos^2(x)dx = \pi$) without proving them.
- Start **every problem on a new sheet of paper** and write your name and the number of your student identity card on every sheet of paper.
- **Do not use pencils, erasable pens, red or green ink.**
- Leave enough empty space on the margins.
- Please **write neatly**! If something is not legible, it will be ignored.
- In exercises 5-10 exactly one answer is correct. Transfer your answers to the answer sheet in time. Only the answers on your answer sheet will be scored. Each multiple choice question gives 3 points when solved correctly and 0 points otherwise.
- We do not expect you to solve every problem to get the maximum grade 6.
- You can solve the problems in any order you want.

All the best!

1. (9 points) Compute the following integral:

$$\int_0^{\pi} \sin\left(\frac{x}{2}\right) \cdot \sin(x)\, dx.$$

2. (9 points) Compute the solution of the following initial value problem:

$$y = 6y' - 9y'', \quad y(0) = \sqrt{3}, \quad y''(0) = \sqrt{3}.$$

3. (3+6 points) The matrix A is given by

$$A = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}.$$

a) Compute $\det(A)$.

b) Compute A^{-1}.

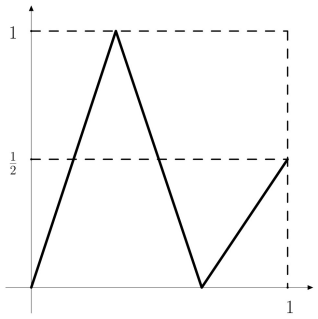
4. (9 points) Compute the solution of the following 2×2 system of differential equations:

$$\begin{cases} 2x - y = x', \\ 2y = y', \end{cases}$$

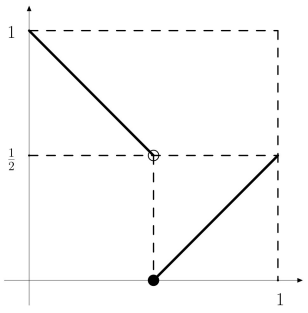
with $x(0) = 1$ and $y(0) = 1$.

5. (3 points) The following figures show four functions from $[0, 1]$ to $[0, 1]$. Which of them is surjective but not injective?

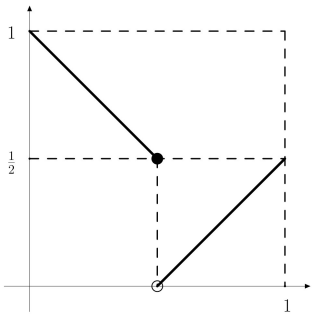
☐ (a)



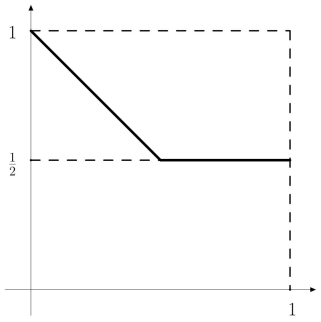
☐ (b)



☐ (c)



☐ (d)



6. (3 points) The limit

$$\lim_{x \to 0} \frac{\ln(1 + x^2)}{\ln(1 + x) \cdot \ln(1 + x)}$$

is

☐ (a) 0

☐ (b) not defined

☐ (c) e

☐ (d) 1

7. (3 points) Which of the following functions is a solution of the differential equation

$$y' = (y - 2)(y + 3)?$$

☐ (a) $\dfrac{1 - 2e^{5x}}{-e^{5x}}$

☐ (b) $\dfrac{2e^{5x}}{1 + e^{5x}}$

☐ (c) $\dfrac{2 - 2e^{5x}}{-e^{5x}}$

☐ (d) $\dfrac{2 - 3e^{5x}}{1 + e^{5x}}$

8. (3 points) The function

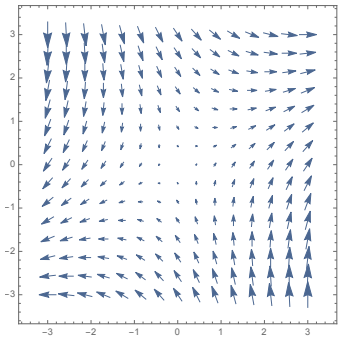
$$\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

is a solution of the following system of differential equations:

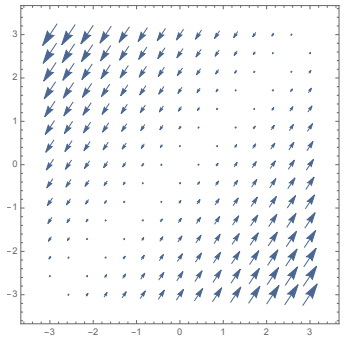
$$\begin{aligned} x_1(t) &= 2x_1(t) + 2x_2(t)\,, \\ x_2(t) &= 3x_1(t) - 3x_2(t)\,. \end{aligned}$$

Which is the phase portrait of the system of differential equations?

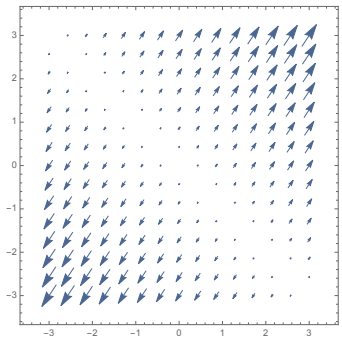
☐ (a)



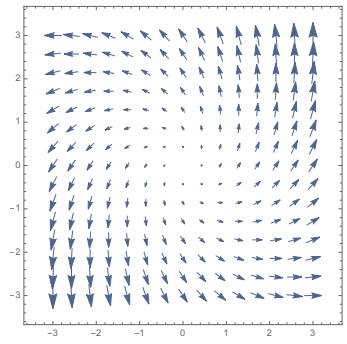
☐ (c)



☐ (b)



☐ (d)



9. (3 points) Consider the following sets of vectors in $\mathbb{R}^3$:

$$A := \left\{ \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\} \text{ and } B := \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

Which of these two sets are bases of $\mathbb{R}^3$?

- ☐ (a) both
- ☐ (b) A
- ☐ (c) B
- ☐ (d) none of them

10. (3 points) The expression

$$\left(\frac{2\sqrt{3} + 2i}{2e^{i\frac{\pi}{3}}} \right)^{24}$$

is equal to

- ☐ (a) $3^{12} - 1$
- ☐ (b) 2^{24}
- ☐ (c) 1
- ☐ (d) 3
