

MATHEMATICS I EXAM

for students of Agricultural Science, Earth Sciences,
Environmental Sciences, and Food Science

Family name:

First name:

ETH ID Nr.:

Department:

Important:

- Please fill the header above and lay your ETH-card visible on the table.
- Please write neatly with a non erasable blue or black pen, in particular not with a pencil.
- This exam has 12 exercises and lasts for 90 minutes.

For exercises 1-4:

- Please write down all intermediate steps of your calculations and solutions.
- Write your name and ETH ID / Legi-Nr. on each additional sheet.
- The maximal score of each exercise part is given in the right margin.

For exercises 5-12:

- Mark your answers clearly.
- There is always only one correct answer.

Permitted aids:

- Written notes (40 A4-Pages),
- **no** calculator, **no** mobile phone, **no** laptop.

Good Luck!

1. Let

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} .$$

- (a) Determine the eigenvalues of A . 4 points
- (b) Is A invertible? Explain your answer. 2 points
- (c) *Choose* one eigenvalue of A and determine the eigenvectors belonging to this eigenvalue. 3 points

2. Consider the differential equation

$$y = 2y' - 2y'' .$$

- (a) Determine the general solution of this differential equation. 4 points
- (b) Determine the solution with the following initial conditions:

$$y(0) = 1 , \quad y'(0) = 1 .$$

4 points

3. Compute the following integral:

$$\int_0^\pi \cos(2x) \cdot \cos\left(\frac{x}{2}\right) dx .$$

8 points

4. Determine the solution of the following 2×2 system of differential equations

$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = -x, \end{cases}$$

with $x(0) = 0$ and $y(0) = 1$.

9 points

For exercises 5-12: Every correct answer gives 2 points. Wrong or multiple answers give 0 points. Please mark the correct answer.

5. What is the biggest value of the function

$$f(x) = 4 + 3x^2 - 2x^3$$

for x in the interval $[0, 2]$?

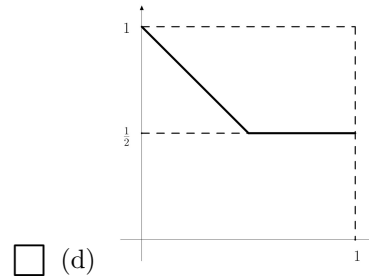
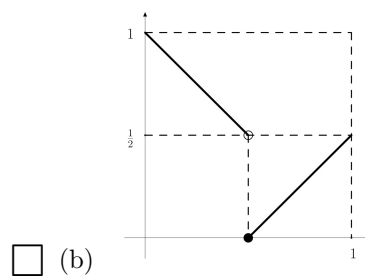
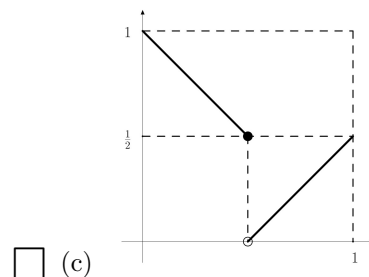
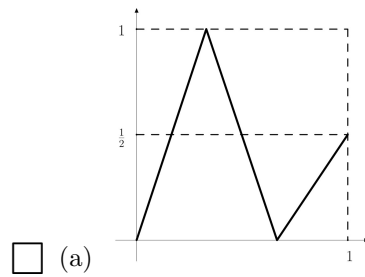
(a) 4.

(c) 8.

(b) 5.

(d) 9.

6. The following figures show four functions from the interval $[0, 1]$ to $[0, 1]$. Which of them is invertible?



7. The limit

$$\lim_{x \rightarrow 0} \frac{\sin(x^2)}{\sin^2(x)}$$

is

(a) 0.

(c) 1.

(b) not defined.

(d) $+\infty$.

8. The expression

$$\frac{-1 + 3i}{i + 1}$$

is equal to:

(a) $1 + 2i$.

(c) $2 + i$.

(b) $1 - 2i$.

(d) $-2 + i$.

9. Which function solves the following differential equation?

$$y' = y(y + 3)$$

(a) $y(x) = \frac{3}{e^{-3x} + 1}$.

(c) $y(x) = \frac{3}{e^{3x} + 1}$.

(b) $y(x) = \frac{3}{e^{3x} - 1}$.

(d) $y(x) = \frac{3}{e^{-3x} - 1}$.

10. What is the rank of A ?

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 2 & 2 & 4 \\ 2 & 0 & 2 & 4 & 2 \\ 3 & 2 & 5 & 6 & 7 \end{pmatrix}$$

(a) 2.

(c) 4.

(b) 3.

(d) 5.

11. Which pair of vectors can be linearly dependent for some values of θ ?

(a) $\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$.

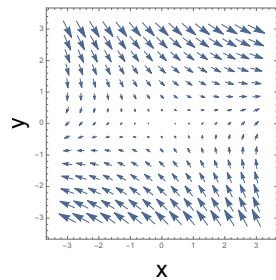
(c) $\begin{pmatrix} 1 \\ \sin \theta \end{pmatrix}, \begin{pmatrix} \sin \theta \\ 1 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 \\ \cos \theta \end{pmatrix}, \begin{pmatrix} -\cos \theta \\ 1 \end{pmatrix}$.

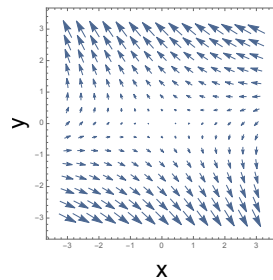
(d) $\begin{pmatrix} 1 \\ \cos \theta \end{pmatrix}, \begin{pmatrix} \sin \theta \\ 1 \end{pmatrix}$.

12. Which is the phase portrait of the following system of differential equations?

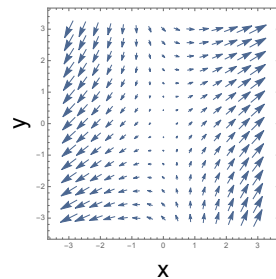
$$\begin{aligned}\dot{x}(t) &= x(t) + 3y(t), \\ \dot{y}(t) &= x(t) - 3y(t).\end{aligned}$$



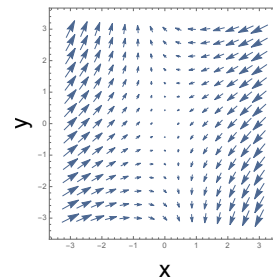
(a)



(c)



(b)



(d)