Lectures on Symplectic Geometry

— Errata for the 2006 text on author's website*—

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Many thanks to all the readers who contributed useful comments and corrections on this text, in particular, Alessandro Fasse, Tommaso Goldhirsch, Yael Karshon, Reto Kaufmann, Rui Loja Fernandes, Lisa Ricci, Tianchen Tang, and Alan Weinstein, and Manuel Wiedmer.

page 15, line 2
By Weinstein's lagrangian creed **[108]**, everything is a lagrangian manifold! not [105]

page 27, lines 6 and 5 from the bottom
is the graph of a diffeomorphism $\varphi : T^*X \to T^*X$, then φ is the symplectomorphism generated by f. In this case, $\varphi(x,\xi) = (y,\eta)$ if and only if... not f

page 28, lines 2 and 1 from the bottom
For both steps, it might be useful to recall that, given a function $\boldsymbol{h} : X \to \mathbb{R}$ and a tangent vector $v \in T_xX$, we have $d\boldsymbol{h}_x(v) = \dfrac{d}{du}\left[\boldsymbol{h}(\exp(x,v)(u))\right]_{u=0}$. not φ

page 32, lines 2-6

$$\begin{cases} \dfrac{\partial f}{\partial x} = -\dfrac{\chi(x)-\chi(y)}{|\chi(x)-\chi(y)|}\cdot\dfrac{d\chi}{ds}(x) = \cos\theta = v \\ \dfrac{\partial f}{\partial y} = -\dfrac{\chi(y)-\chi(x)}{|\chi(x)-\chi(y)|}\cdot\dfrac{d\chi}{ds}(y) = -\cos\nu = -w\ . \end{cases}$$

replace the two systems of equations by the single system above

page 32, line 12
$-|x_1 - x_2| - \ldots - |x_{N-1} - x_N| - |x_N - x_1|$ sign change

*The errata for the 2008 Springer text is at
https://people.math.ethz.ch/∼acannas/Papers/lsg_errata.pdf

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page 36, lines 9 and 13

$$\mathcal{L}_{v_t}\omega := \frac{d}{ds}\psi_{s,t}^*\omega\Big|_{s=t}$$

where $\psi_{s,t}$ is the flow of v_t, i.e., $s \mapsto \psi_{s,t_0}(p)$ is the unique maximal integral curve of v_t with value p at time $s = t_0$:

$$\tfrac{d}{ds}\psi_{s,t_0}(p)\big|_{s=t} = v_t\left(\psi_{t,t_0}(p)\right) \qquad \text{and} \qquad \psi_{t_0,t_0}(p) = p\,.$$

This is related to the previous ρ by $\rho_s = \psi_{s,0}$. If M is compact, the flow is globally defined and we have $\psi_{s,t} = \psi_{s,t_0}\circ\psi_{t_0,t}$, thus $\psi_{s,t}^{-1} = \psi_{t,s}$, and hence $\psi_{s,t} = \rho_s\circ\rho_t^{-1}$. We can use this last expression to write $\mathcal{L}_{v_t}\omega$ alternatively in terms of ρ. A good reference is the textbook by John Lee, *Introduction to Smooth Manifolds*.
the definition of Lie derivative by a time-dependent vector field v_t was wrong

page 38, Theorem 6.6
(i.e., a unique $q \in X$ minimizing $|q - \boldsymbol{p}|$) not $|q - x|$

page 39, Proposition 6.8
$\mu \in \boldsymbol{\Omega^{\ell-1}}(\mathcal{U})$ instead of $\Omega^{d-1}(\mathcal{U})$

page 40, line 5 from bottom
remove "(reviewed in the next section)"

page 46, line 9 from bottom
Hypothesis: X is a **half**-dimensional submanifold with... instead of n-dimensional

page 55, last line i.e. footnote 9
... if $\mathbf{Id}-df_p : T_pM \to T_pM$ is nonsingular. not just df_p

page 79, line 6 of §14.2
$(\Lambda^\ell T^{1,0}) \otimes (\Lambda^m T^{0,1})$ so $\otimes$ instead of $\wedge$ in the definition of $\Lambda^{\ell,m}$

page 90, line 6
It follows immediately from the previous definition **and from Theorem 15.4** that add clause

page 113, line 16 from bottom
$V(q) := \boldsymbol{-W_\gamma}$ sign change

page 121, last condition in Proposition 20.2
(d) $F(p) \to +\infty$ *as* $p \to \infty$ *in* V. omit "F is proper, that is,"

page 126, line 3 of part 7.
tautological 1-form twice replace "canonical" by "tautological"

page 141, line 3
whenever there is an **abelian** symmetry group the word "abelian" was missing

page 162, exercise 4, line 3
$\mu(z) = \frac{\mathbf{1}}{\mathbf{2i}} zz^*$ sign change, the formula in the hint is good

page 163, lines 5 and 10
$\mu(A) = \frac{\mathbf{1}}{\mathbf{2i}} AA^* - \frac{\mathrm{Id}}{2i}$ and $\mu(A) = \frac{\mathbf{1}}{\mathbf{2i}}[A, A^*]$ sign changes, following that in the

previous page

page 180 and subsequent pages
the number π and the projection map π should be distinguished by different symbols