Eigenfunctions

For $u(x,t)$, $0 \le x \le L$, $t \ge 0$.	For $u(x,t)$, $0 \le x \le L$, $t \ge 0$.
$\begin{cases} \boldsymbol{u_{tt} = c^2\, u_{xx}} & \text{1-diml wave eq.} \\ \boldsymbol{u(0,t) = u(L,t) = 0} & \text{hom. Dirichlet BC} \\ u(x,0) = u_0(x) \\ u_t(x,0) = v_0(x) \end{cases}$	$\begin{cases} \boldsymbol{u_t = c^2\, u_{xx}} & \text{1-diml heat eq.} \\ \boldsymbol{u(0,t) = u(L,t) = 0} & \text{hom. Dirichlet BC} \\ u(x,0) = f(x) \end{cases}$
$\sin\frac{n\pi x}{L} \cdot \cos(\lambda_n t)$ and $\sin\frac{n\pi x}{L} \cdot \sin(\lambda_n t)$, $n = 1,2,3,\ldots \quad \lambda_n = \frac{n\pi c}{L}$	$e^{-\lambda_n^2 t} \cdot \sin\frac{n\pi x}{L}$, $n = 1,2,3,\ldots \quad \lambda_n = \frac{n\pi c}{L}$

For $u(x,t)$, $0 \le x \le L$, $t \ge 0$.	For $u(x,t)$, $0 \le x \le L$, $t \ge 0$.
$\begin{cases} \boldsymbol{u_{tt} = c^2\, u_{xx}} & \text{1-diml wave eq.} \\ \boldsymbol{u(x+L,t) = u(x,t)} & \text{periodic BC} \\ u(x,0) = u_0(x) \\ u_t(x,0) = v_0(x) \end{cases}$	$\begin{cases} \boldsymbol{u_t = c^2\, u_{xx}} & \text{1-diml heat eq.} \\ \boldsymbol{u(x+L,t) = u(x,t)} & \text{periodic BC} \\ u(x,0) = u_0(x) \end{cases}$
$1,\ \cos\frac{2n\pi x}{L}\cos(\lambda_n t),\ \sin\frac{2n\pi x}{L}\cos(\lambda_n t),$ $t,\ \cos\frac{2n\pi x}{L}\sin(\lambda_n t),\ \sin\frac{2n\pi x}{L}\sin(\lambda_n t),$ $n = 1,2,3,\ldots \quad \lambda_n = \frac{2n\pi c}{L}$	$1,\ e^{-\lambda_n^2 t}\cos\frac{2n\,\pi x}{L}$ and $e^{-\lambda_n^2 t}\sin\frac{2n\,\pi x}{L}$, $n = 1,2,3,\ldots \quad \lambda_n = \frac{2n\pi c}{L}$

For $u(x,y)$, $0 \le x \le a$, $0 \le y \le b$.	For $u(x,t)$, $0 \le x \le L$, $t \ge 0$.
$\begin{cases} \boldsymbol{u_{xx} + u_{yy} = 0} & \text{2-diml Laplace eq.} \\ \boldsymbol{u(x,0) = u(0,y) = u(a,y) = 0} & \text{hom. part of BC} \\ u(x,b) = f(x) \end{cases}$	$\begin{cases} \boldsymbol{u_t = c^2\, u_{xx}} & \text{1-diml heat eq.} \\ \boldsymbol{u_x(0,t) = u_x(L,t) = 0} & \text{hom. Neumann} \\ u(x,0) = f(x) \end{cases}$
$\sin\frac{n\pi x}{a} \cdot \sinh\frac{n\pi y}{a}$, $\quad n = 1,2,3,\ldots$	1 and $e^{-\lambda_n^2 t} \cdot \cos\frac{n\,\pi x}{L}$, $n = 1,2,3,\ldots \quad \lambda_n = \frac{n\pi c}{L}$