Def/Lemma: Let (M, ω) be a Liouville MFLD and $\partial_\infty M \subset M$, $\partial M \neq \phi$, WITH Liouville 1-form λ, and respective Liouville V.F Z on $M \setminus A$. We say that (M, ω) is a Liouville sector if one of the followwing conditions hold,

1) $\exists\ I : \partial M \to \mathbb{R}$, S.T $dI_p(C) > 0\ \forall p \in \partial M$ and $dI(Z) = I$ on $\partial M \setminus A$

2) $\underline{\partial(\partial_\infty M)}$ is convex and there is a diffeomorphism $\partial M \cong \mathbb{R} \times F$, F MFLD, S.T it sends the leaves of Charakteristic foliation to leaves of the form $\mathbb{R} \times \{p\}$, $p \in F$.

Proof: 1) $\Rightarrow$ 2) : I as above, $d\underline{I(Z) \doteq I}$ $\partial M \setminus A$, $\iff (\phi^t)^* I = e^t I$.

Use Collar Nghd, $U = [0,1[\times \partial(\partial_\infty M) \underset{\text{OPEN}}{\subseteq} \partial_\infty M$.

$$I : U \longrightarrow \mathbb{R}$$
$$(s, p) \in [0,1[\times \partial(\partial_\infty M) \longmapsto I(p)$$

$$U = \partial_\infty M$$

$$I : \Phi(\mathbb{R}_{\geq 0} \times U) \longrightarrow \mathbb{R}$$

$$\Phi(t, p) \longmapsto e^t \cdot I(p)$$

$\Rightarrow \exists\ Y \in CV(\partial_\infty M, \xi)\ .\ g \in C^\infty(\partial_\infty M)$ s.t

$$X_I = Y + gZ$$

$$\underset{\in}{c}\ T(\partial M | A)$$

$$0 < dI(c) = \omega(c, X_I) = \omega(c, Y) + g\underbrace{\omega(c, Z)}_{=0}$$

$$= \omega(c, Y)$$

$\Rightarrow \partial(\partial_\infty M)$ CONVEX!

Claim $\partial M \cong \mathbb{R} \times \underbrace{I^{-1}(0)}_{= F}$. Let $V \in \Gamma(c)$ s.t $dI(V) = 1$.

Claim: V is complete.

Step2: $d\phi^t(V) = \underline{\lambda^t} V \circ \phi^t$

Proof $d\phi^t(V) \circ \mathcal{L}\partial M$, $\underline{1\text{-dim}} \Rightarrow d\phi^t(V) = \overset{\swarrow}{g} \cdot V \circ \phi^t$

$$g = dI\,(gV) = dI\,(d\phi^t(V)) = d(\lambda^t I)(V) = \lambda^t$$

Step3: ψ^s flow V

$$\phi^t \circ \psi^s = \psi^{s\lambda^t} \circ \phi^t$$
$$\psi^s \circ \phi^t = \phi^t \circ \psi^{s/\lambda^t}$$

Proof: Uniqueness of ODE's and step2.

Step4: Using compactness $(A \cap \partial M) \cup \partial(\partial_\infty M)$, $\exists\, \varepsilon > 0$ s.t. $\forall p \in A_0$ $\forall s \in]-\varepsilon, \varepsilon[$

$\psi^s(p)$ is well-defined

Step 3: ψ^s flow V

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Step 4: Using compactness $(\overset{= A_0}{A \cap \partial M}) \cup \partial(\partial_\infty M)$, $\exists\, \varepsilon > 0$ s.t. $\forall p \in A_0$

$\psi^s(p)$ is well-defined $\forall s \in]-\varepsilon, \varepsilon[$

Let $p \in \partial M \setminus A$ $\exists\, q \in \partial(\partial_\infty M)$ $t \geq 0$ s.t. $p = \phi^t(q)$.

$$\psi^s(p) = \psi^s \circ \phi^t(q) = \phi^t \circ \psi^{s/\ell^t}(q),$$

$s/\ell^t \in]-\varepsilon, \varepsilon[\iff s \in]-\varepsilon \ell^t, \varepsilon \ell^t[\underset{t \geq 0}{\supseteq}]-\varepsilon, \varepsilon[$

$\Rightarrow$

Finally: $\Phi_0 : \mathbb{R} \times I^{-1}(0) \longrightarrow \partial M$, $\Phi_1 : \partial M \longrightarrow \mathbb{R} \times I^{-1}(0)$

$(s, p) \longmapsto \psi^s(p)$, $p \longmapsto (I(p), \psi^{-I(p)}(p))$

$dI(V) = 1 \iff (\psi^s)^* I = I + s$. $\square$

$\partial M \subseteq \overset{\text{OPEN}}{V} \subset M$

M MFLD, COMP, $X \in \mathfrak{X}(V)$, X is inward point ∂M

$\partial M \neq \emptyset$

$\Downarrow$

$\tilde{X} \in SV(T^*M, Z) \Rightarrow \tilde{X} \notin T(S^*M)$

$\Rightarrow$

$\tilde{X}$ HAMILTONIAN $\iff I \in HF(T^*M, Z)$

$\in SV(T^*M, Z) \longleftrightarrow \delta I(Z) = I$