GGM

$(j,k) \in E$

want to check whether $X^{(j)} \perp\!\!\!\perp X^{(k)} \mid X^{V \setminus \{j,k\}}$

$p \sim \mathcal{N}(0, \Sigma)$

density $p_{\Sigma^{-1}=K}(x) \propto C \exp(x^T \underbrace{\Sigma^{-1}}_{=:K} x/2)$ concentration matrix

$p_j(x^{(j)}) = \int p(x^{(j)}, u)\, p_{\setminus j}(u)\, du$

$p_k(x^{(k)}) = \ldots$

$\rightarrow$ $p(x^{(j)} \mid x^{(V \setminus \{j,k\})}) = \ldots$

$p(x^{(k)} \mid x^{(V \setminus \{j,k\})}) = \ldots$

$p(x^{(j)} \mid x^{(V \setminus \{j,k\})}) \cdot p(x^{(k)} \mid x^{(V \setminus \{j,k\})}) \overset{?}{=} p(x^{(j)}, x^{(k)} \mid x^{(V \setminus \{j,k\})})$

true if $(\Sigma^{-1})_{j,k} = 0$

GLasso $\rightsquigarrow$ ~~obtain~~ we get an estimate

typically sparse $\hat{K} = \widehat{(\Sigma^{-1})}$

$\hat{K} \succ 0$ pos. definite

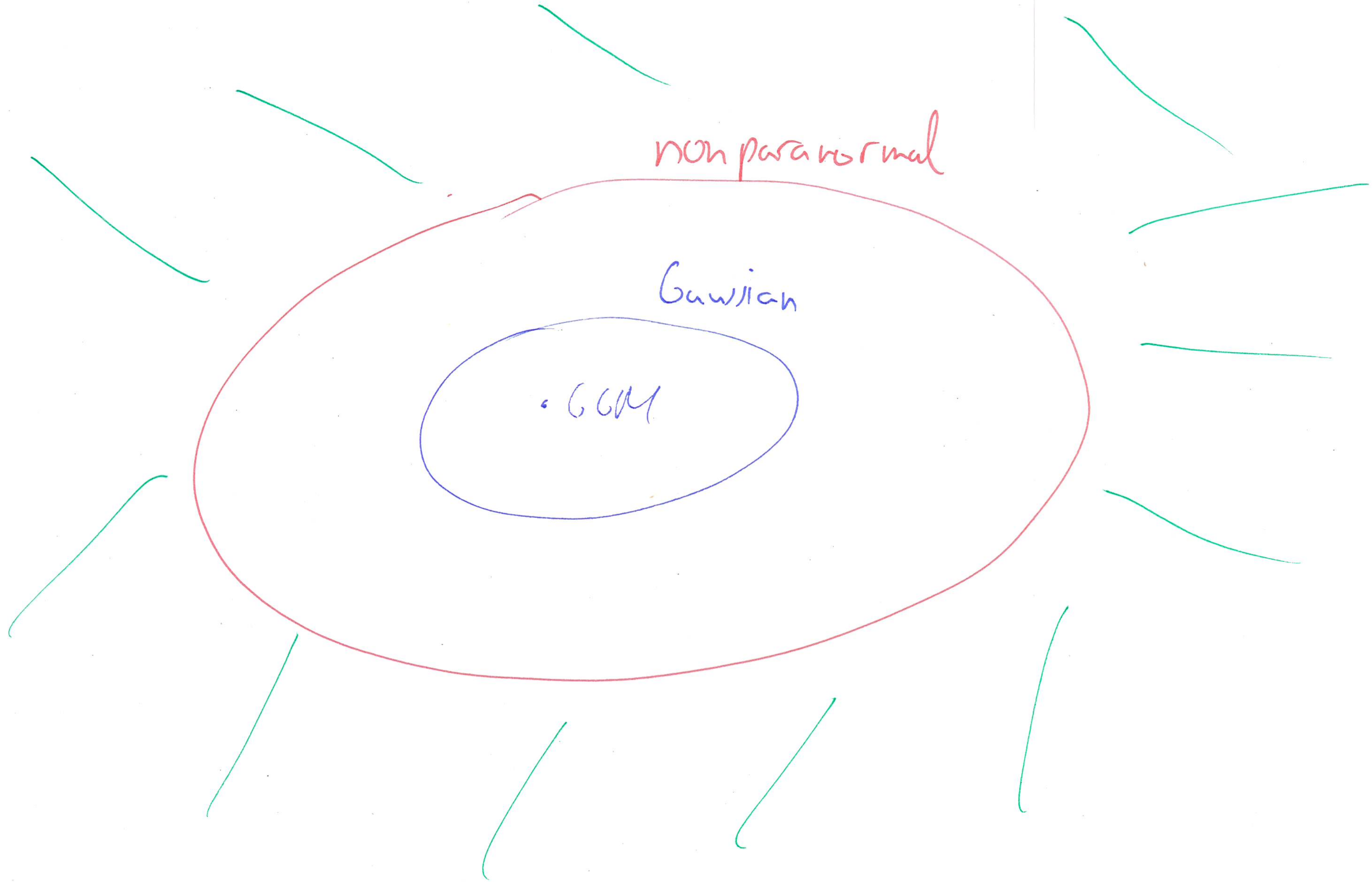
$\rightsquigarrow \hat{K}^{-1} = \hat{\Sigma}$

typically not sparse

$$f_j(\cdot) = \Phi^{-1}(F_j(\cdot))$$

$$F_j(u) = \mathbb{P}[X^{(j)} \leq u]$$

$$f_j(X^{(j)}) = \Phi^{-1}(\underbrace{F_j(X^{(j)})}_{\sim \mathrm{Unif}([0,1])}) \sim \mathcal{N}(0,1)$$


nonparanormal
Gaussian
GGM

pairwise MP:

$$(j,k) \notin E \implies X^{(j)} \perp X^{(k)} \mid X^{(V \setminus \{j,k\})}$$

$$\cancel{\Longleftarrow}$$

$$\Longleftarrow \text{ for a GGM}$$

$$\boxed{(j,k) \in E \iff X^{(j)} \not\perp X^{(k)} \mid X^{(V \setminus \{j,k\})}}$$

$$\iff (\Sigma^{-1})_{j,k} \neq 0$$

$$\beta^* = \underset{u}{\operatorname{argmin}}\ \mathbb{E}[(Y - Xu)^2]$$

$$= \underset{p \times p}{\mathrm{Cov}(X)^{-1}}\ \underset{p \times 1}{\mathrm{Cov}(X, Y)}$$

$$= \mathrm{Cov}(X)^{-1}\,\mathrm{Cov}(X, X\beta^0 + H\delta + \eta) \qquad \eta \perp\!\!\!\perp X$$

$$= \mathrm{Cov}(X)^{-1}\left(\mathrm{Cov}(X)\cdot\beta^0 + \mathrm{Cov}(X, H)\cdot\delta\right)$$

$$= \beta^0 + \underbrace{\mathrm{Cov}(X)^{-1}\,\mathrm{Cov}(X, H)\cdot\delta}_{=:\ \text{bias } b}$$

$$\neq 0 \text{ if } \begin{cases} \delta \neq 0 & \text{and} \\ \Gamma \neq 0 \end{cases}$$