Recap

Undirected graphical models

- graph G:
  set of vertices/nodes $V = \{1, \ldots, p\}$
  set of edges $E \subseteq V \times V$
- random variables $X = X^{(1)}, \ldots, X^{(p)}$ with distribution P
  identify nodes in V with components of X

graphical model: (G, P)

pairwise Markov property:
P satisfies the pairwise Markov property (w.r.t. G) if

$$(j, k) \notin E \Longrightarrow X^{(j)} \perp X^{(k)} | X^{(V \setminus \{j,k\})}$$

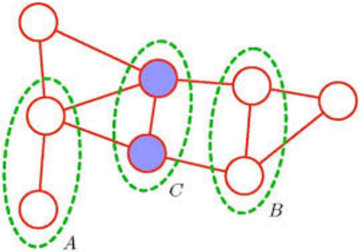
Global Markov property

(stronger property than pairwise Markov prop):

consider disjoint subsets $A, B, C \subseteq V$

P satisfies the global Markov property (w.r.t. G) if

A and B are separated by C $\Longrightarrow X^{(A)} \perp X^{(B)} | \underbrace{X^{(C)}}_{\text{only condition on subset } C}$



global Markov property $\Longrightarrow$ pairwise Markov property

if P has a positive density w.r.t Lebesgue measure:
global Markov property $\Longleftrightarrow$ pairwise Markov property

Gaussian Graphical Model (GGM): $P \sim \mathcal{N}_p(0, \Sigma)$:

$$(j, k) \in E \Longleftrightarrow (\Sigma^{-1})_{jk} \neq 0$$

Neighborhood selection: nodewise regression

$$X^{(j)} = \beta_k^{(j)} X^{(k)} + \sum_{r \neq j,k} \beta_r^{(j)} X^{(r)} + \varepsilon^{(j)},\ j = 1 \ldots, p$$

$$X^{(k)} = \beta_j^{(k)} X^{(j)} + \sum_{r \neq k,j} \beta_r^{(k)} X^{(r)} + \varepsilon^{(k)}$$

for GGM:

$$(j, k) \in E \Longleftrightarrow \beta_k^{(j)} \neq 0 \ \Longleftrightarrow \beta_j^{(k)} \neq 0$$

nodewise regression (Meinshausen & Bühlmann, 2006)

- run Lasso for every node variable $X^{(j)}$ versus all others $\{X^{(k)};\ k \neq j\}$ ($j = 1, \ldots, p$)
- estimated active set $\hat{S}^{(j)} = \{r;\ \hat{\beta}_r^{(j)} \neq 0\}$ ($j = 1, \ldots, p$)
- estimate edges in $\hat{E}$:

$$\text{or rule:} \qquad (j,k) \in \hat{E} \Longleftrightarrow j \in \hat{S}^{(k)} \text{ or } k \in \hat{S}^{(j)}$$

$$\text{and rule:} \qquad (j,k) \in \hat{E} \Longleftrightarrow j \in \hat{S}^{(k)} \text{ and } k \in \hat{S}^{(j)}$$

just run Lasso p times: it's fast!

`R`-packages `huge` and also in `glasso` (and set ‘approx = T’)