Recap

Undirected graphical models

- graph G:
  set of vertices/nodes $V = \{1, \ldots, p\}$
  set of edges $E \subseteq V \times V$
- random variables $X = X^{(1)}, \ldots, X^{(p)}$ with distribution P
  identify nodes in V with components of X

graphical model: (G, P)

pairwise Markov property:
P satisfies the pairwise Markov property (w.r.t. G) if

$$(j, k) \notin E \Longrightarrow X^{(j)} \perp X^{(k)} | X^{(V \setminus \{j,k\})}$$

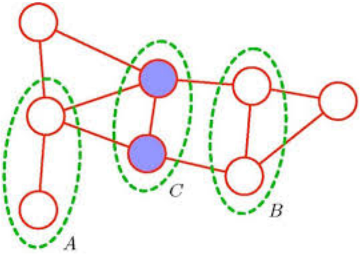
Global Markov property

(stronger property than pairwise Markov prop):

consider disjoint subsets $A, B, C \subseteq V$

P satisfies the global Markov property (w.r.t. G) if

$$A \text{ and } B \text{ are separated by } C \implies X^{(A)} \perp X^{(B)} | \underbrace{X^{(C)}}_{\text{only condition on subset } C}$$



global Markov property $\Longrightarrow$ pairwise Markov property

if P has a positive density w.r.t Lebesgue measure:
global Markov property $\Longleftrightarrow$ pairwise Markov property

the Markov properties imply some conditional independencies from graphical separation

for example with pairwise Markov property:

$$(j,k) \notin E \Longrightarrow X^{(j)} \perp X^{(k)} | X^{(V \setminus \{j,k\})}$$

reverse relation:
generally not true $\leadsto$ in general, we cannot interpret edges

in some special cases: prime example is P Gaussian

$$(j,k) \in E \iff X^{(j)} \not\perp X^{(k)} | X^{(V \setminus \{j,k\})}$$

but for A and B not separated by C: in general not true (even not for Gaussian P) that

$$X^{(A)} \not\perp X^{(B)} | X^{(C)}$$

... due to possible strange cancellations of “edge weights”