Recap

Stability Selection

random subsampling of half of the data:

$$I^{*1}, \ldots, I^{*B} \text{ independent}$$
$$I^{*b} \underbrace{\text{random subsample}}_{\text{without replacement}} \subset \{1, \ldots, n\}, |I^{*b}| = \lfloor n/2 \rfloor$$

feature selection algorithm $\hat{S}_\lambda \subseteq \{1, \ldots, p\}$

stability of selected single features:

$$\hat{\Pi}_j(\lambda) = \mathbb{P}^*[j \in \hat{S}_\lambda(I^*)] \approx B^{-1} \sum_{b=1}^{B} I(K \subseteq \hat{S}_\lambda(I^{*b}))$$

Why half-sampling?
I.e., subsampling without replacement with $|I^{*b}| = \lfloor n/2 \rfloor$?

Freedman (1977): sampling without replacement with subsample size $m = \lfloor n/2 \rfloor$ is closest to i.i.d. sampling n individuals with replacement (i.e., bootstrap resampling)

“closest” means w.r.t total variance distance

Connecting to false discoveries

$$\hat{S}_{\text{stable}} = \{j; \max_{\lambda \in \Lambda} \hat{\Pi}_j(\lambda) \geq \pi_{\text{thr}}\}$$

Choice of π_{thr}?

as a measure for type I error control (against false positives):

$$V = \text{ number of false positives} = |\hat{S}_{\text{stable}} \cap S_0^c|$$

where S_0 is the set of the true relevant features
$\hat{S}_\Lambda = \cup_{\lambda \in \Lambda} \hat{S}(\lambda)$
$q_\Lambda = \mathbb{E}[|\hat{S}_\Lambda(\underbrace{I}_{\text{random subsample}})|]$

Theorem 10.1
Assume:

- exchangeability condition:
  $\{1(j \in \hat{S}(\lambda)\}), j \in S_0^c\}$ is exchangeable for all $\lambda \in \Lambda$
- $\hat{S}$ is not worse than random guessing

$$\frac{\mathbb{E}|S_0 \cap \hat{S}_\Lambda|)}{\mathbb{E}(|S_0^c \cap \hat{S}_\Lambda|)} \geq \frac{|S_0|}{|S_0^c|}.$$

Then, for $\pi_{\text{thr}} \in (1/2, 1)$:

$$\mathbb{E}[V] \quad \leq \quad \frac{1}{2\pi_{\text{thr}} - 1} \frac{q_\Lambda^2}{p}.$$

strategy: specify upper bound for $\mathbb{E}[V] \leq v_0$ and solve for π_{thr}

Stability Selection is very generic!

- specify a feature selector which (typically) selects "moderately" too many features
- retain the stable features such that $\mathbb{E}[V] \leq v_0$