Recap

P-values based on multi sample splitting

need to avoid “double dipping” using the data twice for variable selection and using statistical inference (tests, confidence intervals) afterwards

$\rightsquigarrow$ sample splitting

multiple sample splitting is much more reliable and statistically better than splitting once

- is very generic, also for “any other” model class
- is powerful in terms of multiple testing correction
- it relies in theory on the screening property of the selector
  in practice: it is a quite competitive method!

Undirected graphical models

(Ch. 13 in Bühlmann and van de Geer (2011))

- graph G:
  set of vertices/nodes $V = \{1, \ldots, p\}$
  set of edges $E \subseteq V \times V$
- random variables $X = X^{(1)}, \ldots, X^{(p)}$ with distribution P
  identify nodes in V with components of X

graphical model: (G, P)

pairwise Markov property:
P satisfies the pairwise Markov property (w.r.t. G) if

$$(j, k) \notin E \Longrightarrow X^{(j)} \perp X^{(k)} | X^{(V \setminus \{j,k\})}$$

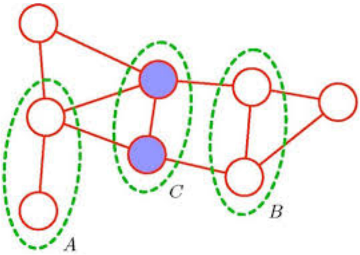
Global Markov property

(stronger property than pairwise Markov prop):

consider disjoint subsets $A, B, C \subseteq V$

P satisfies the global Markov property (w.r.t. G) if

A and B are separated by C $\Longrightarrow X^{(A)} \perp X^{(B)} | \underbrace{X^{(C)}}_{\text{only condition on subset } C}$



global Markov property $\Longrightarrow$ pairwise Markov property

Proof:
consider $(j, k) \notin E$

denote by $A = \{j\}, B = \{k\}, C = V \setminus \{j, k\}$;
since $(j, k) \notin E$, $A = \{j\}$ and $B = \{k\}$ are separated by C

by the global Markov property: $X^{(j)} \perp X^{(k)} | X^{(V \setminus \{j,k\})}$

$\square$

$\leadsto$ global Markov property is more "interesting"