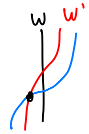
LECTURE 9

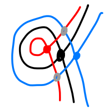
Dictionary of basic 4d-moves.

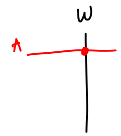
Ch 11: ◊ framed immersions. normal bundle of an immersion trivial. $\mathbb{Z}$ many diffeos to $\Sigma\times\mathbb{R}^2$. called framing. if $\partial\Sigma\neq\emptyset$ then get framing on $\partial\Sigma$ so $\partial\Sigma\times\mathbb{D}^2 \hookrightarrow \nu(\partial\Sigma)$.
if $\Sigma=$ Wh.disc W then ∂W has Whitney framing. Difference of these two framings is in $\mathbb{Z}=\pi_1 SO(2)$.

◊ Whitney moves along a framed embedded Wh.disc. If not framed:



If framed immersed:



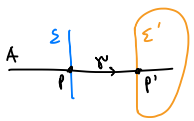


gives rise to four new self-intersections

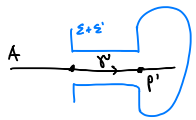
◊ finger moves

Ch 15:

◊ tubing

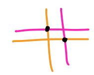


remove $\nu\sigma\cap\Sigma$ and $\nu\sigma\cap\Sigma'$ and glue the rest of $\partial(\nu\sigma)$ (annulus)

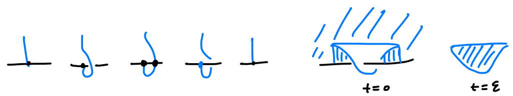


NOTE: If $\Sigma=\Sigma'$ assume $\operatorname{sgn} p = -\operatorname{sgn} p'$ in order to have the result orientable.

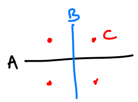
NOTE: We often tube into a parallel copy of Σ' instead (use a nonvan. normal v.f. to push Σ' off itself)
if Σ' framed, then get no inters with it.
if Σ' embedded, then get no new d.p.
if Σ' immersed, then get inters. with it (and among parallel push-offs):

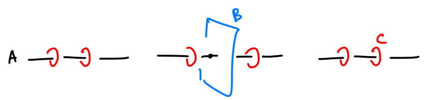


◊ boundary twisting

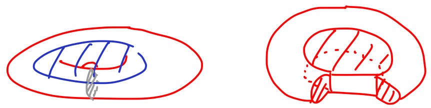


◊ Clifford torus





◊ symmetric surgery (contraction)



e.g. all finite groups
all abelian groups

key Thm [Freedman 1982] – s-cobordism Theorem in dim 5 –
If $(W, \partial_0 W, \partial_1 W)$ is an h-cobordism with $\dim W = 5$
and trivial Whitehead torsion $Wh(W, \partial_0 W) \in Wh(\pi_1 W)$ and $\pi_1 W$ is a good group,
then W is topologically trivial,
i.e. there is a homeomorphism $(W, \partial_0 W, \partial_1 W) \cong (\partial_0 W \times [0,1], \partial_0 W \times \{0\}, \partial_0 W \times \{1\})$

We will reduce to the following theorem.

key Thm [Freedman 1982] – Disc Embedding Thm –
If M is a smooth connected 4-manifold with $\partial M = \emptyset$ and $\pi_1 M$ a good group,
and $W_m : (D^2, \partial D^2) \looparrowright (M, \partial M)$ is a framed immersed collection with emb. boundary
which has a framed immersed collection $\{G_m\}$ of algebraic duals,
then
there exists a locally flat embedded collection $\{\overline{W}_m\}$
with the same framed boundary as $\{W_m\}$
not needed in the current proof: and with a framed immersed collection $\{\overline{G}_m\}$ of geometric duals
with $\overline{G}_m \underset{htpic}{\simeq} G_m$.

Proof of this is hard and we do not present it. See the DET book!