# LECTURE 12.
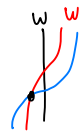
## Dictionary of basic 4d-moves.
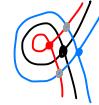
**Ch 11:**

◇ framed immersions.  normal bundle of an immersion trivial. $Z$ many diffeos to $\Sigma \times \mathbb{R}^2$. called framing. If $\partial\Sigma \neq \emptyset$ then get framing on $\partial\Sigma$ so $\partial\Sigma \times \mathbb{D}^2 \hookrightarrow \nu(\partial\Sigma)$.
If $\Sigma = $ Wh.disu $W$ then $\partial W$ has Whitney framing. Difference of these two framings is in $\mathbb{Z} = \pi_1 SO(2)$.

◇ Whitney moves    along a framed embedded Wh.disu.  If not framed:     If framed immersed:

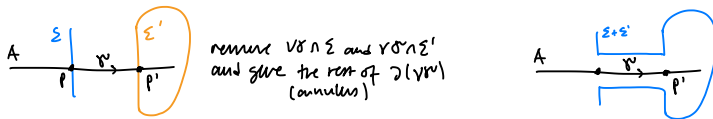gives rise to four new self-intersections

◇ finger moves

**Ch 15:**

◇ tubing            remove $\nu\sigma \cap \Sigma$ and $\nu\sigma \cap \Sigma'$ and glue the rest of $\partial(\nu\nu')$ (annulus)

NOTE: If $\Sigma = \Sigma'$ assume $sgn p = -sgn p'$. in order to have the result orientable.

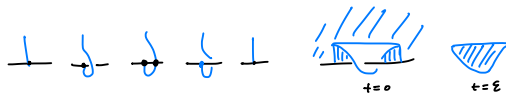NOTE: We often tube into a parallel copy of $\Sigma'$ instead (use a nonvan. normal v.f. to push $\Sigma'$ off itself)
    If $\Sigma'$ framed, then get no inters. with it.
    if $\Sigma'$ embedded, then get no new d.p.
    if $\Sigma'$ immersed, then get inters. with it (and among parallel push-offs):

◇ boundary twisting            $t=0$    $t=\varepsilon$

◇ Clifford torus
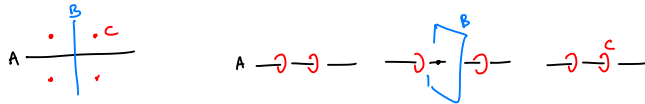
◇ symmetric surgery (contraction)