Modeling exchange rate dependence dynamics at different time horizons

Alexandra Dias[a,*], Paul Embrechts[b,1]

[a]*Warwick Business School, Finance Group, University of Warwick, CV4 7AL Coventry, UK.*
[b]*Department of Mathematics, ETH Zurich, 8092 Zurich, Switzerland.*

Abstract

Despite an extensive body of research, the best way to model the dependence of exchange rates remains an open question. In this paper we present a new approach which employs a flexible time-varying copula model. It allows the conditional correlation between exchange rates to be both time-varying and modeled independently from the marginal distributions. We introduce a dynamic specification for the correlation using the Fisher transformation. Applied to Euro/US dollar and Japanese Yen/US dollar, our results reveal a significantly time-varying correlation, dependent on the past return realizations. We find that a time-varying copula with the proposed correlation specification gives better results than alternative dynamic benchmark models. The dynamic copula model outperforms at six different time horizons, ranging from hourly to daily, confirming the model specification.

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*Corresponding author. Tel.: +44(0) 247 657 4297; fax: +44(0) 247 652 3779.
Email address: alexandra.dias@wbs.ac.uk (Alexandra Dias)

1. Introduction

In economics and finance multivariate problems are often of interest in areas like risk management, asset pricing, portfolio allocation and forecasting. There, the specification of financial variables' multivariate distributions is crucial for the computation of quantities related to their inter-dependence. The prime example is the linear correlation, the perfect measure of dependence for multivariate normal variables and to some extent for elliptical variables. However there is strong evidence that the univariate distributions of many financial variables are non-normal and significantly fat-tailed. This empirical fact often rules out the use of the multivariate normal distribution. In principle, there is no reason for different marginal variables to have the same degree of fat-tailedness or even to have univariate distributions of the same type. Moreover, most financial data exhibit skewness. This also questions the use of elliptical distributions in other contexts.

Financial time-series are often modeled with GARCH type models. In the multivariate GARCH literature there exist several models, like CCC-GARCH, DVEC, matrix-diagonal GARCH, BEKK and principal components GARCH.[2]

In all these models the conditional multivariate distribution is Gaussian

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[2]For a survey on multivariate GARCH models see Bauwens et al. (2006).

or Student-t. An alternative approach to overcome that distributional constraint is to use copula-GARCH models; see Jondeau and Rockinger (2006), Fortin and Kuzmics (2002), Patton (2006a) and Patton (2006b).[3] Copula based models allow complete freedom to combine different conditional marginal distributions in a dependence structure.

By construction, the conditional variance of the univariate distributions in a GARCH model is time-varying. Additionally, in the copula-GARCH model the parameters of the dependence structure can also be time-varying. This is potentially useful as there is evidence that asset's dependence is time-varying, a fact that has raised considerable interest in the dynamic behavior of correlation between different risks as a function of time; see for instance Boyer et al. (1999), Engle (2009), Longin and Solnik (2001), Loretan and English (2000) and Loretan and Phillips (1994). Because of the fundamental importance of the notion of linear correlation in finance and insurance, such dynamics may have a non-trivial impact on the pricing and hedging of underlying instruments, or on the risk measurement of such positions. As a consequence, a systematic modeling of the dynamic behavior of the dependence structure underlying multivariate variables is of considerable importance. This can be achieved with time-varying copula-GARCH models.

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[3]Another possible alternative, which we opt not to pursue here, is to model regime changes. In the context of time series analysis, see for instance Hamilton (1990).

In the present article we present a time-varying copula-GARCH model. We propose a specification for the dynamics of the dependence parameter using the Fisher transformation in order to model the dynamic dependence between Euro and Japanese Yen versus U.S. Dollar exchange rates. With copula-GARCH models we allow for possibly different fat-tailed univariate distributions. We look for the most appropriate copula family for both time-invariant and time-varying copula models.

We compare the proposed model with two competing models: the dynamics proposed by Tse and Tsui (2002) and the BEKK model introduced by Engle and Kroner (1995). We evaluate the estimated dependence paths by comparing them with two benchmark non-parametric measures of dependence: the realized correlation and Kendall's tau. Finally we investigate six time horizons ranging from hourly to daily. Our goal is to examine the consistency of our findings across different time horizons. Theoretical properties of time aggregation of univariate GARCH models can for instance be found in Drost and Werker (1996) and Meddahi and Renault (2004).

The paper is organized as follows. In Section 2 we introduce the time-varying specification for copula-GARCH models and its estimation methodology. In Section 3 we describe the data and the computation of the bivariate returns on the Euro and Japanese Yen spot rates, both quoted against the U.S. Dollar. These are deseasonalized bivariate returns at six time horizons

from one hour to daily. The first step of the copula-GARCH modeling is performed in Section 4. This consists of univariate GARCH filtering and consequent specification tests. In Section 5 we estimate the time-varying dependence paths using several dynamic copula models, including the time-varying copula dynamics introduced by Tse and Tsui (2002) and the time-varying correlation implied by a BEKK model. The evaluation of the estimated time-varying dependence paths against realized correlation and Kendall's tau is given in Section 6. Our results are summarized in Section 7 which concludes the paper.

2. The model specification and estimation

We proceed to the specification of the dynamic copula model starting by introducing the general copula-GARCH type model.

2.1. The copula-GARCH type model

We denote the observed bivariate time series of the exchange rate returns by $(\mathbf{x}_t) = (x_{1t}, x_{2t})'$, $t = 1, \ldots, T$ and model these data with a copula-AR-GARCH[4] specification as follows. Each of the returns' univariate processes

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[4]We considered also copula-ARMA-GARCH models but the moving average dynamics is not significant for the FX data analyzed here.

satisfies

$$X_{it} = \mu_{it} + \epsilon_{it}, \qquad \mu_{it} = \mu_i + \sum_{l=1}^{r} \phi_{il} \left(X_{it-l} - \mu_i \right),$$

$$\epsilon_{it} = \sigma_{it} Z_{it}, \qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^{p} \alpha_{ik} \epsilon_{it-k}^2 + \sum_{j=1}^{q} \beta_{ij} \sigma_{it-j}^2, \qquad i = 1, 2,$$

where ϕ_{il} are autoregressive coefficients, the parameters α_{i0}, α_{ik} and β_{ij} are non-negative, and $\sum \alpha_{ik} + \sum \beta_{ij} < 1$. The innovations Z_{it} are independent of $(X_{is})_{s \leq t}$, have mean zero, unit variance, and bivariate distribution function F with continuous univariate marginal distribution functions F_1 and F_2. The bivariate innovations are characterized by a copula-based model

$$F(z_1, z_2; \boldsymbol{\theta}_t) = C(F_1(z_1), F_2(z_2); \boldsymbol{\theta}_t), \qquad\qquad (2)$$

where C is a copula function which, as long as the marginal distribution functions are continuous, we know to exist uniquely by Sklar's Theorem (Sklar (1959)). We suppose the copula C is parameterized by the vector $\boldsymbol{\theta}_t \in \mathbb{R}^q$ with $q \in \mathbb{N}$.

A dynamic copula model

The search for the most appropriate dependence parameter dynamics depends strongly on the interpretation that a specific (copula) dependence parameter may have. We shall see in Section 5 that the Gaussian and the t-copula yield the best models for the dependence structure of the Euro and

Japanese Yen returns. For that reason we focus here on the dynamics for these copulae parameters.

Let ρ_t be the correlation parameter of a Gaussian or t-copula at time t. Given the coefficients α, β and γ, set

$$\rho_t = h^{-1}(\alpha + \beta \operatorname{sign}(Z_{1t-1}Z_{2t-1})|Z_{1t-1}Z_{2t-1}|^{1/2} + \gamma h(\rho_{t-1})), \qquad (3)$$

where $h(\cdot)$ is Fisher's transformation

$$h(\rho) = \log\left(\frac{1+\rho}{1-\rho}\right), \qquad -1 < \rho < 1,$$

Z_{it} are the innovations in (1) and $\gamma < 1$. We use ρ_t as our dynamic correlation parameter. Throughout the paper we call specification (3) the Fisher dynamics. This specification ensures that ρ_t is between -1 and 1. As a consequence of the regression-type dynamics of the innovations z_{it} in (3), when both innovations have the same sign we have a positive contribution to the correlation. When innovations have opposite signs, a negative contribution results. This agrees with evidence found by other authors that dependence is stronger when returns have the same sign than when returns have opposite signs; see for instance Jondeau and Rockinger (2006). We do not split the dynamics in the case of both positive and both negative returns because, according to the tests in Section 5, there is no evidence of asymmetry in the dependence structure for these data.

Specifications evolving from the vech-diagonal model introduced by Bollerslev et al. (1988) typically assume a time-varying covariance dynamics similar to the third line equation in (1), where ρ_t replaces σ_{it}^2 and $\epsilon_{it}\epsilon_{jt}$ replaces ϵ_{it}^2. In this setting we consider, for comparison with our model, the time-varying correlation dynamics proposed by Tse and Tsui (2002). Tse and Tsui (2002) define a dependence parameter ρ_t satisfying the equation:

$$\rho_t = (1 - \beta - \gamma)\rho + \beta\xi_{t-1} + \gamma\rho_{t-1} \tag{4}$$

where β and γ are non-negative, $\beta + \gamma \leq 1$, and

$$\xi_t = \frac{\sum_{h=0}^{m-1} z_{1t-h}z_{2t-h}}{\sqrt{\sum_{h=0}^{m-1} z_{1t-h}^2 \sum_{h=0}^{m-1} z_{2t-h}^2}}$$

with $m \geq 2$. In this paper we call specification (4) the Tse-Tsui dynamics. In the empirical study we use the Gaussian and the t-copula models each with Fisher dynamics and with Tse-Tsui dynamics.

Patton (2006b) introduces a model for the symmetrized Joe-Clayton copula parameters (upper and lower tail parameter), defined dynamically as

$$\tau_t = \Lambda\left(\omega + \beta\tau_{t-1} + \alpha\frac{1}{10}\sum_{j=1}^{10} |F_1(z_{1t-j}) - F_2(z_{2t-j})|\right) \tag{5}$$

where the logistic transformation $\Lambda(x) = (1 + e^{-x})^{-1}$ keeps τ_t in its interval domain $(0, 1)$. In the same paper, the Gaussian copula correlation parameter is modeled with dynamics similar to (5) where ρ_t replaces τ_t, $\Phi^{-1}(F_1(z_{1t-j})).\Phi^{-1}(F_2(z_{2t-j}))$ replaces $|F_1(z_{1t-j}) - F_2(z_{2t-j})|$ and $\tilde{\Lambda}(x) =$

$(1 - e^{-x})(1 + e^{-x})^{-1}$ replaces Λ, where Φ represents the standard normal distribution function.

The dynamics we propose for the correlation parameter differs from Tse and Tsui (2002) because we use the Fisher transformation to ensure the correlation estimates to be in $[-1, 1]$ and because we only use one lag of the past observations. The Patton (2006b) dynamics are based on ten past observations. Additionally, we regress the Fisher transformed correlation on the lagged Fisher transformed correlation while in Patton (2006b) the transformed correlation is regressed on a non-transformed correlation.

The t-copula parameters are the degrees of freedom, ν, and the correlation. We assume that the degrees of freedom are constant over time, as for the univariate margins, and allow the correlation to be time-varying. Recall that the innovations of the univariate or multivariate GARCH models, with Student-t distributed innovations, have constant degrees of freedom even for those multivariate GARCH models which allow for time-varying correlation. A generalization of the procedure for time-varying ν_t is definitely possible for the copula as well as for the margins.

2.2. Estimation

For a random sample $(\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_T)$, the distribution of $\mathbf{Z}_t$ can be written as

$$F(\mathbf{z}_t; \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\theta}_t) = C(F_1(z_{1t}; \boldsymbol{\alpha}_1), F_2(z_{2t}; \boldsymbol{\alpha}_2); \boldsymbol{\theta}_t)$$

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where we assume that each F_i is absolutely continuous with density f_i; the vectors $\boldsymbol{\alpha}_i$ parameterize the marginal distribution functions and the time-varying parameter $\boldsymbol{\theta}_t$ parameterizes the copula family.

Assuming that C has density c, given by

$$c(u_1, u_2; \boldsymbol{\theta}) = \frac{\partial^2 C(u_1, u_2; \boldsymbol{\theta})}{\partial u_1 \, \partial u_2}, \quad (u_1, u_2) \in [0, 1]^2,$$

the density of $\mathbf{Z}_t$ is given by:

$$f(\mathbf{z}_t; \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\theta}_t) = c(F_1(z_{1t}; \boldsymbol{\alpha}_1), F_2(z_{2t}; \boldsymbol{\alpha}_2); \boldsymbol{\theta}_t) f_1(z_{1t}; \boldsymbol{\alpha}_1) f_2(z_{2t}; \boldsymbol{\alpha}_2).$$

The log-likelihood function of the model is therefore given by

$$l(\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\theta}_t) = \sum_{t=p+1}^{T} \left(\log c(F_1(z_{1t}; \boldsymbol{\alpha}_1), F_2(z_{2t}; \boldsymbol{\alpha}_2); \boldsymbol{\theta}_t) + \sum_{i=1}^{2} \log f_i(z_{it}; \boldsymbol{\alpha}_i) \right), (6)$$

with p as in (1).

Numerical maximization of (6) yields the maximum likelihood estimates of the model. However, the optimization of the likelihood function with possibly many parameters is numerically difficult and time consuming. It is more tractable to estimate first the model parameters of the margins and then the dependence model parameters using the estimates from the first step. This two-step estimation procedure, also known as inference for margins (see for instance Joe (1997)) has been used in semi-parametric modeling, as in Andreou and Ghysels (2003), or in a full-parametric modeling context as in Engle and Sheppard (2001), Patton (2006b) and Jondeau and Rockinger

(2006). The marginal log-likelihood functions

$$l(\boldsymbol{\alpha}_i) = \sum_{t=p+1}^{T} \log f_i(z_{i,t}; \boldsymbol{\alpha}_i), \qquad i = 1, 2,$$

are independently maximized, yielding the estimates $\hat{\boldsymbol{\alpha}}_1$ and $\hat{\boldsymbol{\alpha}}_2$. The final function to maximize is

$$l(\boldsymbol{\theta}_t) = \sum_{t=p+1}^{T} \log c(F_1(z_{1t}; \hat{\boldsymbol{\alpha}}_1), F_2(z_{2t}; \hat{\boldsymbol{\alpha}}_2); \boldsymbol{\theta}_t). \tag{7}$$

From this, estimates for the dependence parameter $\boldsymbol{\theta}_t$ are obtained. Statistical properties of this estimation procedure are to be found in Chen and Fan (2005), Joe (2005) and Patton (2006a).

3. Data

The data consist of observations of spot exchange rates for the U.S. dollar, the Euro, and the Japanese Yen, covering the sample period October 1, 2000 until October 1, 2008, equally-spaced at sixty-minutes. The data set was obtained from Olsen Financial Technologies GmbH. The sixty-minute prices are obtained by linear interpolation between the average of the bid and ask tick quotes immediately before and immediately after the sixty-minute time stamps across the 24-hour day.

We compute the returns from prices of 1 U.S. dollar expressed in Euro and Yen, this is, EUR/USD and JPY/USD. Sixty-minute returns are computed

as the first difference of the logarithm of the sixty-minute prices. Analogously, from the corresponding frequency equally-spaced logarithmic prices, we compute two, four, eight, twelve hours and one day returns, yielding twelve series in all.

High-frequency data is subject to market microstructure frictions. By choosing sixty-minute prices as the highest frequency we achieve a compromise between estimation bias originating in high-frequency bid/ask bounce and variance which increases as the sampling frequency decreases.

We exclude all the returns from Friday 21:00 GMT until Sunday 21:00 GMT and Bank Holidays in order to simplify the inference and avoid modeling weekend and Bank Holiday effects. After deleting these slow trading activity days the data set consists of returns spanning $2,016$ days in total.

It is known that seasonal components, associated with working hours of exchanges worldwide, distort the estimation of volatility models, in particular GARCH type models as we use here.[5] To overcome this problem we compute the intraday seasonal pattern and use it to deseasonalize the sixty-minute returns. We compute a seasonal factor corresponding to each sixty-minute interval of the day as the average of the squared returns observed every day

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[5]See for instance Martens et al. (2002).

for that time interval,

$$s_i^2 = \frac{1}{T} \sum_{t=1}^{T} \tilde{x}_{it}^2 \qquad i = 1, 2, \ldots, 24,$$

where T is the number of days in the sample and $\tilde{x}_{it}$ is the raw return in the ith interval of day t. The sixty-minute deseasonalized returns are computed as

$$x_{it} = \frac{\tilde{x}_{it}}{s_i} \qquad i = 1, 2, \ldots, 24 \text{ and } t = 1, 2, \ldots, T. \tag{8}$$

We compute the deseasonalized returns for the other intraday frequencies similarly to (8), where the corresponding seasonal factor is the sum of the sixty-minute factors in that frequency time interval,

$$\bar{s}_i^2 = \sum_{t=0}^{24/\delta-1} s_{i\,24/\delta-t}^2 \qquad i = 1, \ldots, \delta,$$

where $\delta = 12, 6, 3, 2$ for 2-hour, 4-hour, 8-hour and 12-hour return frequency respectively. When clear from the context we refer to the deseasonalized returns simply as returns.

4. The EUR/USD and JPY/USD univariate returns at different time horizons

We first analyze the unconditional distribution of the returns at the several time horizons. The modeling of the univariate dynamics and conditional distribution follows. This section ends with model specification tests.

4.1. Univariate modeling

Tables 1 and 2 contain summary statistics of the returns. Observe that neither exchange rate returns shows a significant trend. The returns' skewness is mostly negative for the various time horizons although not revealing strikingly asymmetric returns. At all time horizons, both series show excess kurtosis higher than zero, corresponding to a normally distributed variable.

From the values obtained for the kurtosis we can see that, for both spot rates, the unconditional univariate distributions are clearly heavy tailed for shorter time horizons and become more thin tailed as the time horizon increases. This feature has been found in the financial econometric literature; see for instance Nekhili et al. (2002) where similar tail behavior was found for the JPY/USD exchange rate returns.

The estimated linear correlation decreases as the time horizon decreases. This feature is usually justified in the literature by the non-synchronicity of trades. The fact that trades are not synchronized causes an increasing downwards correlation bias as the time horizon shortens.

[Insert Table 1 here]

[Insert Table 2 here]

Concerning the conditional distributions, the Ljung-Box test statistics up to the twentieth order indicate serial correlation; see Table 3. We test for the

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presence of conditional heteroscedasticity using the LM test introduced by Engle (1982). The test statistics obtained reveal the presence of time-varying conditional variance at all the time horizons.

[Insert Table 3 here]

The results obtained from the tests on the univariate return time series justify, in our discrete-time setting, modeling the time-varying volatility with a GARCH specification assuming Student-t innovations. These model assumptions are checked in Section 4.2 with the appropriate specification tests. Tables 4 and 5 give the order of the models fitted and the corresponding parameter estimates. Note that the t-distributions fitted at the two and four hours time horizons may have infinite kurtosis ($\hat{\nu} < 4$) and so the fourth moment does not exist.

[Insert Table 4 here]

[Insert Table 5 here]

From the fitted AR-GARCH model parameters we recover the standardized residuals or filtered returns $\hat{z}_t$ for each univariate time series $(x_1, x_2, \ldots, x_T)$. That is

$$\hat{z}_t = (x_t - \hat{\mu}_t)/\hat{\sigma}_t, \qquad t = 1, 2, \ldots, T. \tag{9}$$

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Once the univariate models are selected and fitted, the dynamics and the goodness-of-fit of our assumed Student-t density need to be checked. In Table 6 we report the p-values of the Ljung-Box test and of the Anderson-Darling goodness-of-fit test for a Student-t density.

[Insert Table 6 here]

The Ljung-Box test for the residuals indicates that there is no autocorrelation at all time horizons except for the one hour EUR/USD and four hour JPY/USD. For the squared values of the residuals we have that the two hour EUR/USD and the one and eight hour JPY/USD time horizons fail the autocorrelation test at a significance level of 5%.

Table 6 also reports the p-values of the Student-t model goodness-of-fit test for the marginal distributions. The goodness-of-fit of the marginal densities, according to the Anderson-Darling test, are not rejected at all time horizons. The linear correlation between the residuals increases with the time horizons as observed for the returns (Table 2).

5. The EUR/USD and JPY/USD dependence structure at different time horizons

In order to fit the copula models we use the standardized residuals $\{(\hat{z}_{1t}, \hat{z}_{2t}) : t = 0, \ldots, T\}$ defined in (9) together with the estimated degrees of freedom

$\hat{\nu}_i$ from the marginal AR-GARCH modeling above; see Tables 4 and 5. These data are used in equation (7) where F_i is the Student-t distribution assumed for the GARCH innovations.

The copula families used in modeling the data are: Student-t, Frank, Plackett, Gaussian, Gumbel and Clayton; for details on these classes see Joe (1997), Embrechts et al. (2002) and Nelsen (2006). This choice of copula models is partly based on previous analyses, tractability, flexibility and to allow for a fairly broad class with respect to extremal clustering and possible asymmetry. If there is asymmetry in the dependence structure we need a copula able to model this feature. Of particular interest are the the upper and lower tail dependence coefficients, as defined in McNeil et al. (2005) page 209. These range from zero in case of asymptotic independence, to one in case of perfect asymptotic tail dependence. The Clayton copula family has lower tail dependence ranging from asymptotic independence to perfect asymptotic tail dependence. Hence, it is convenient to use the mixture Clayton with the survival Clayton model to investigate the existence and asymmetry of upper and lower tail dependence.

Denoting the Clayton copula family with parameter $\theta > 0$ by $C^{\mathrm{Cl}}(\cdot, \cdot; \theta)$, the mixture copula with vector parameter $\boldsymbol{\theta}$ has a distribution function of the form

$$C(u_1, u_2; \boldsymbol{\theta}) = \theta_3 \, C^{\mathrm{Cl}}(u_1, u_2; \theta_1) + (1 - \theta_3) \, C^{\mathrm{Cl}}(u_1, u_2; \theta_2). \qquad (10)$$

The AIC values in Table 8 allow us to rank the several models. The best non-dynamic copula models are the Clayton mixture and the t-copula for the one, two and four hour time horizons. For the remaining, longer, time horizons the best non-dynamic models are the Placket and the t-copula. The t-copula continues being the best when we introduce dynamics in the model parameter. The second best dynamic model at all time horizons is the Gaussian. The dynamics used with each copula parameter are listed in Table 7. Further support for the t-based models is to be found in Breymann et al. (2003), Daul et al. (2003), Demarta and McNeil (2005), Rosenberg and Schuermann (2006), Pesaran et al. (2004) and Platen and Heath (2006).

[Insert Table 7 here]

For the Clayton mixture model defined in (10), we test the null hypothesis $H_0 : \theta_1 = \theta_2$ and $\theta_3 = 0.5$ versus the alternative $H_A : \theta_1 \neq \theta_2$ and $\theta_3 \neq 0.5$. A high p-value indicates that a three parameter asymmetric Clayton mixture is not significantly better than its one parameter symmetric version. For the six time horizons, from the one hour to the daily, we obtain as p-values 0.8211, 0.1166, 0.2004, 0.8148, 0.4208 and 0.3377 respectively. The p-values obtained strongly favor the symmetric model at all time horizons. The estimated coefficient of tail dependence corresponding to the Clayton parameter for the six time horizons, from the one hour to the daily, are 0.4569, 0.4267, 0.4074, 0.4240, 0.4099 and 0.3956, respectively, around the middle of the tail

coefficient range $(0, 1)$.

It is not surprising that exchange rates show symmetric dependence in the data, given that its definition is always against another currency. This fact makes symmetric copula families to be potentially good models for exchange rates dependence. This is not the case in other financial data, such as stock returns for instance, where downside dependence is often stronger than upside dependence.

In order to compare the relative performance of the dynamic models compared to their non-dynamic versions, we perform a likelihood ratio test for each model and time horizon. The test statistics and p-values obtained are in Table 8.

[Insert Table 8 here]

The Gaussian and the t-copula dynamic models outperform their non-dynamic versions at all time horizons. The dynamic Frank model is significantly better than the non-dynamic model at all time horizons except for the eight hour one. The dynamic Clayton mixture outperforms at the eight and two hour time horizons. The dynamic Gumbel beats the non-dynamic Gumbel at the eight hour time horizon and the Clayton at one and two hours.

In summary, the Gaussian and the t-copula models stand out as the best models consistently across all time horizons and their dynamic versions always add significant modeling power. It is remarkable that at all time

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horizons the Gaussian and t-copula models with Fisher dynamics lead to a better AIC than the Gaussian and t-copula models with Tse-Tsui dynamics.

The parameter estimates and asymptotic standard errors obtained for all Student-t and Gaussian copula models are reported in Table 9 and Table 10. These include the parameter estimates for the corresponding non-dynamic model.

[Insert Table 9 here]

[Insert Table 10 here]

The estimate for α in the t-copula model with Fisher dynamics (3) can be considered to be zero for two hours, twelve hours and daily returns. But β and γ are definitely different from zero for both Gaussian and t-copula models with Fisher and Tse-Tsui dynamics, at all time horizons. In other words, the estimated correlation depends on the marginal returns and on the correlation from the previous time period.

The degrees of freedom estimated for the dependence structure is always larger for the dynamic Student-t copula models than for the static t-copula model, as can be seen from a comparison of the values listed in Table 9. Hence, a non-dynamic modeling might induce a spurious heavier conditional tail.

In contrast to multivariate GARCH models, with Gaussian or multivariate Student-t conditional distribution, copula-GARCH models allow us com-

plete freedom to choose different conditional marginal distributions. In the case of the fitted Student-t copula models we compare the degrees of freedom estimated for each marginal conditional distribution (see Tables 4 and 5) and for the copula (see Table 9). For the daily time horizon, for instance, we estimate $\hat{\nu} = 15.8$ for the EUR/USD, $\hat{\nu} = 9.2$ for the JPY/USD and $\hat{\nu} = 8.4$ for the t-copula with Fisher dynamics. Although for our data we cannot conclude that the estimates are statistically different for most of the time horizons, using three parameters for the degrees of freedom must surely contribute to the excellence of the results in the specification tests.

For comparison with the copula-GARCH model we estimate the BEKK model introduced by Engle and Kroner (1995). We model the conditional mean of each return series using the same AR specification as in the marginal modeling in Section 4. Then we estimate a bivariate BEKK model on the residuals. We fit a BEKK model at each of the six time horizons assuming (bivariate) Student-t innovations.

Table 11 lists the corresponding autocorrelation and goodness-of-fit tests. All time horizons pass the autocorrelation test except the one hour time horizon residuals. The one, two and eight hour squared residuals fail the same test for both rates at the 5% level. Only the JPY/USD residuals for time horizons four, eight and twelve hours pass the goodness-of-fit test for a Student-t distribution. The other nine series fail the test.

[Insert Table 11 here]

The goodness-of-fit results for the distribution of the residuals are substantially worse than for the copula-GARCH model (see Anderson-Darling test results in Tables 6 and 11). The p-values for testing the Student-t distribution are higher for the copula-GARCH model in ten out of twelve series. The failure of the Student-t distribution might be due to the asymmetry revealed by the estimated skewness (higher than for the original returns) especially for the JPY/USD residuals; see Table 12. The existence of excess kurtosis in the AR-BEKK residuals is consistent with a heavy tailed Student-t distribution.

[Insert Table 12 here]

We conclude that the copula-GARCH model is better specified than the flexible BEKK model.

6. Evaluating and comparing the correlation estimates at different time horizons

From the estimated parameters for the dynamic correlation models we compute the time-varying estimated correlation for the Gaussian and Student-t copula with Fisher dynamics (3) and Tse-Tsui dynamics (4), and for the BEKK model for all time horizons. As a benchmark we use two nonparametric measures of dependence: Kendall's tau coefficient and realized correlation.

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For all time horizons we compute rolling window estimates of Kendall's tau using a window size of 40 observations. Kendall's tau at day t is computed using the previous 39 pairs of EUR/USD and JPY/USD returns, and the return pair at t itself. Since for our data the best performing copula models belong to the elliptical family we transform Kendall's tau estimates, τ_{Kendall}, into linear correlation, ρ_τ, using the relationship

$$\tau_{\text{Kendall},t} = \frac{2}{\pi} \arcsin \rho_{\tau,t}, \tag{11}$$

which is valid for elliptical distributions; see Lindskog et al. (2003). The linear correlation estimated from Kendall's tau is plotted in Figure 1 together with the dynamic Gaussian and Student-t copula with Fisher dynamics (3) and daily frequency. Recall from Table 2 in Section 4 that the unconditional constant correlation estimate is 0.43. In the period 2007-2008 we can observe a significant drop in the correlation starting in the beginning of 2007, bottoming out in the third quarter with a consequent rise reaching the previous levels during the second quarter of 2008. Events having different effects in European and Japanese economies cause changes in the correlation between the two exchange rates. A delay in the effect of the sub-prime crisis on the Asian economies compared with the European can justify the decrease in the correlation observed until mid-2007. After mid-2007 all FX markets, including Euro and Japanese Yen, had suffered equal contagion from the crisis and correlation between Euro and Yen returned to pre-crisis levels.

[Insert Figure 1 here]

Given these results we can infer strongly that the conditional dependence is time-varying and, consequently, that its dynamics should be modeled. Clearly, assuming that the correlation is constant may cause substantial mispricing and errors in risk measurement.

In order to formally evaluate and benchmark our nonlinear model correlation estimates we follow Andersen et al. (2003). First, we evaluate the correlation estimates obtained from the different models using the Kendall's tau estimates. We perform the regression

$$\rho_{\tau,t} = b_0 + b_1 \rho_{\text{model},t} + u_t,$$

to project the correlation, $\rho_{\tau,t}$, obtained from Kendall's tau using (11) onto a constant and the various parametric model estimates, $\rho_{\text{model},t}$.

We plot in Figure 2 the regression of the estimates of Kendall's tau for the daily filtered returns, computed according to (11), on the Tse-Tsui dynamics, Fisher dynamics and BEKK model correlation estimates. From these plots we can anticipate superior results for the Fisher dynamics with Gauss and Student-t copula in the regression analysis. The regression of Kendall's tau transformed correlation on the Tse-Tsui dynamics and BEKK model estimates are visibly weaker.

[Insert Figure 2 here]

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We report in Table 13 the numerical results from the evaluation regressions. The results are consistent across models and time horizons. For all models the regression R^2 is highest for the four hours time horizon. For the four, eight, twelve hour and daily time horizons the two models with Fisher dynamics (Gaussian and t-copula) always have the highest R^2. For the two hour time horizon the two dynamic Gaussian copula models have the highest R^2. Finally the best R^2 for the one hour time horizon is for the Gaussian model with Tse-Tsui dynamics and for the BEKK model.

[Insert Table 13 here]

We further evaluate the correlation estimates from a different point of view. We aggregate the model correlation estimates from the different time horizons into daily and compare them with the daily realized correlation and Kendall's tau estimates. The day-t realized correlation is computed from the hourly time horizon returns as

$$\rho_{\mathrm{RC},t} = \frac{\sum_{i=1}^{24} x_{it} \cdot y_{it}}{\sqrt{\sum_{i=1}^{24} x_{it}^2 \ \sum_{i=1}^{24} y_{it}^2}}, \quad t = 1, 2, \ldots, T,$$

where x_{it} is the hour i EUR/USD return of day t, and y_{it} the JPY/USD return for the same hour and day. With the model correlation estimates at time horizon h, for $h = 1, 2, 4, 8, 12$ hours, we compute the day-t correlation

estimate as

$$\rho_{\text{model},t,h} = \frac{\sum_{i=1}^{24/h} \rho_{i,t,h} \, \sigma_{\text{EUR},i,t,h} \, \sigma_{\text{JPY},i,t,h}}{\sqrt{\sum_{i=1}^{24/h} \sigma_{\text{EUR},i,t,h}^2 \, \sum_{i=1}^{24/h} \sigma_{\text{JPY},i,t,h}^2}}, \quad t = 1, 2, \ldots, T,$$

where $\rho_{i,t,h}$ is the time horizon h estimated correlation for time period i of day t. For instance, $\rho_{3,t,2}$ is the two hour time horizon correlation estimated for 4–6am of day t. The $\sigma_{\text{EUR},i,t,h}^2$ and $\sigma_{\text{JPY},i,t,h}^2$ are the marginal variances for the same time period, obtained from the univariate time series modeling.

In order to compare the correlation estimates obtained from the different time horizons we use the regression

$$\rho_{\text{RC},t} = b_0 + b_1 \rho_{\text{model},t,h} + u_t. \tag{12}$$

to project the realized correlation $\rho_{\text{RC},t}$ on a constant and on the model estimates $\rho_{\text{model},t,h}$. We also regress the daily Kendall's tau correlation, $\rho_{\tau,t}$, on the daily model correlations obtained from the different time horizons as in (12). The results are reported in Table 14.

[Insert Table 14 here]

The R^2 for the realized correlation increases with the time frequency used to compute the daily correlations, and this consistently for all the models. The shorter the time horizon the higher the R^2. For each time horizon the BEKK model has the higher R^2. The t and Gaussian models with Fisher dynamics are the second best. Only for the one hour time horizon the second best R^2

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model is the Gaussian with Tse-Tsui dynamics. The conclusion here is that, when measured against the realized correlation, using shorter time horizons improves the dynamic daily correlation estimates.

When regressing the daily rolling window Kendall's tau correlation on the daily correlations estimated from different time horizons, the results (see Table 14) are still consistent across time horizons and models. The R^2 is higher when the correlations are estimated from longer time horizons. The highest R^2 is obtained from the t model with Fisher dynamics for all frequencies except one and eight hours where the Gaussian model with Fisher dynamics is better. The BEKK model has the lowest R^2 for all time horizons except for the eight hours where it ranks the second to last. When measured against the Kendall's tau correlation the results show no advantage in using shorter time horizons for modeling the daily correlation. Across models the Fisher dynamics perform consistently better than the Tse-Tsui dynamics and the BEKK model.

Implications for the choice of sampling frequency resulting from regressing the realized correlation are different to those resulting from regressing Kendall's tau. This may be due to the different conceptual nature of the benchmark models.

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7. Conclusion

The contribution of the paper is threefold. First of all, we want to contribute to the ongoing discussion between practitioners and academics in order to advance the methodological basis for risk measurement technology. There is a need to move beyond linear correlation and, through the notion of copula, our paper contributes to this goal. Second, we propose a parsimonious conditional dependence model, the copula with Fisher dynamics model, taking dynamics in the dependence structure into account and use it to analyze the dependence between Euro and Japanese Yen. Thirdly we show the superiority of the proposed model compared to models previously investigated in the literature.

Looking first at the univariate conditional distributions of the EUR/USD and JPY/USD returns we conclude they are well described by a Student-t distribution and this from hourly up to daily return frequencies. We observe some tendency for the tail of the distribution to become heavier as the time frequency increases.

To model dependence, comparing different copula models, the t-copula performs best for the time-invariant models and the Gaussian and t-copula models are the best for the time-varying models. Combining different copula families and dynamic specifications we conclude that the dependence between the EUR/USD and JPY/USD is time-varying. The best time-varying models

(Gaussian and t-copula) are statistically better than the corresponding time-invariant model. All these results are obtained consistently at all the time horizons considered.

Testing for different upper and lower tail dependence revealed no asymmetry, a natural feature of FX data.

We observe that the degrees of freedom of each Student-t margin and copula are different at all the time horizons considered. We find a heavier tail in the marginal exchange rates than in the dependence structure. Although not always statistically significant, this observation raises concerns about using a bivariate Student-t for the conditional distribution of the returns.

We find that the degrees of freedom estimated for the dynamic t-copula model are significantly higher than for the time-invariant model. The time-invariant model seems to overestimate the dependence tail heaviness.

The dependence paths estimated using the time-varying copula-GARCH models have very high significance at all time horizons when compared to two non-parametric measures of dependence: realized correlation and Kendall's tau.

We found a remarkable decrease in the dependence between the EUR/USD and JPY/USD returns during the first three quarters of 2007 followed by an increase till mid 2008 back to values observed before 2007. This decline in the correlation was possibly the result of a delay in the effects of the sub-prime

crisis on the Japanese Yen compared with the effect on the Euro. Once both currencies became equally influenced by the crisis their dependence went back to pre-crisis values.

We conclude that the proposed copula with Fisher transformation dynamics model is superior to alternative models previously tested in the literature and that its use should be considered in preference to these models in future work.

Acknowledgments

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Table 1: Summary statistics: mean and standard deviation

Time horizon	EUR/USD		JPY/USD	
	Mean	Std. dev.	Mean	Std. dev.
1 hour	-0.0143	0.9999	-0.0007	1.0000
2 hours	-0.0154	1.0131	-0.0005	1.0147
4 hours	-0.0175	1.0181	0.0032	0.9926
8 hours	-0.0271	0.9970	-0.0064	0.9714
12 hours	-0.0185	0.9476	0.0029	0.9546
1 day	-0.0459	1.0212	0.0061	0.9903

Notes: Sample mean and standard deviation of the EUR/USD and JPY/USD returns at the six time horizons.

Table 2: Summary statistics: skewness, kurtosis, and correlation

Time horizon	EUR/USD		JPY/USD		Linear correlation
	Skewness	Kurtosis	Skewness	Kurtosis	
1 hour	-0.0624	6.854	-0.0838	7.493	0.3878
2 hours	0.0044	5.287	-0.0149	4.876	0.4040
4 hours	-0.3074	7.391	-0.0641	3.066	0.4030
8 hours	-0.0430	2.978	-0.1743	2.349	0.4088
12 hours	-0.0433	2.244	-0.1211	2.080	0.4016
1 day	0.0703	0.711	-0.0967	1.070	0.4311

Notes: Sample skewness, excess kurtosis and linear correlation of the EUR/USD and JPY/USD returns at the six time horizons.

Table 3: Ljung-Box (L-B) and ARCH LM (LM) tests

Time horizon	EUR/USD				JPY/USD			
	L-B	p-value	LM	p-value	L-B	p-value	LM	p-value
1 hour	45.74	0.00	1 143.90	0.00	40.13	0.00	2 093.60	0.00
2 hours	48.04	0.00	556.70	0.00	55.30	0.00	820.54	0.00
4 hours	33.14	0.03	165.46	0.00	56.78	0.00	439.64	0.00
8 hours	35.53	0.01	172.03	0.00	33.95	0.02	257.78	0.00
12 hours	34.91	0.02	130.37	0.00	34.46	0.03	206.58	0.00
1 day	26.71	0.14	126.94	0.00	30.85	0.06	75.47	0.00

Notes: Ljung-Box (L-B) and ARCH LM (LM) test statistics up to the twentieth order for the EUR/USD and JPY/USD returns at the six time horizons. The null hypothesis of no serial correlation in returns is rejected at 5% level for all time horizons except for the daily. The null hypothesis of no ARCH effects is rejected for all time horizons at the 5% level.

Table 4: Results from the univariate modeling of EUR/USD

Time horizon	1 day	12 hours	8 hours	4 hours	2 hours	1 hour
Constant, $\hat{\mu}$	-0.0476 (0.0209)			-0.0116 (0.0068)	-0.0153 (0.0047)	-0.0119 (0.0033)
AR(1), $\hat{\phi}_1$		–	–	-0.0220 (0.0079)	–	–
AR(2), $\hat{\phi}_2$		–	–	0.0165 (0.0076)	–	–
GARCH constant, $\hat{\alpha}_0$	–	–	0.0013 (0.0058)	0.0197 (0.0006)	0.0007 (0.0003)	0.0007 (0.0002)
Lag 1 ϵ^2, $\hat{\alpha}_1$	–	0.0232 (0.038)	–	0.0196 (0.0026)	0.0162 (0.0020)	0.0753 (0.0098)
Lag 2 ϵ^2, $\hat{\alpha}_2$	0.0587 (0.0188)	–	0.0074 (0.0028)	–	–	-0.0623 (0.0082)
Lag 1 variance, $\hat{\beta}_1$	0.9617 (0.0071)	0.9763 (0.0037)	0.9664 (0.0078)	0.9813 (0.0025)	0.9844 (0.0021)	0.9866 (0.0022)
Degrees of freedom, $\hat{\nu}$	15.8061 (4.1700)	5.4046 (0.4550)	5.1034 (0.3284)	3.7330 (0.1266)	3.7480 (0.0881)	4.1117 (0.0713)

Notes: Maximum likelihood estimates and corresponding asymptotic standard errors (in parentheses) obtained from fitting AR-GARCH models to the return series EUR/USD at each time horizon.

Table 5: Results from the univariate modeling of JPY/USD

Time horizon	1 day	12 hours	8 hours	4 hours	2 hours	1 hour
Constant, $\hat{\mu}$	–	–	–	0.0169 (0.0072)	0.0111 (0.0049)	0.0078 (0.0034)
AR(1), $\hat{\phi}_1$	–	–	–	–	-0.0533 (0.0061)	–
GARCH constant, $\hat{\alpha}_0$	0.0280 (0.0102)	0.0088 (0.0036)	–	0.0079 (0.0031)	0.0045 (0.0016)	0.0092 (0.0025)
Lag 1 ϵ^2, $\hat{\alpha}_1$	0.0624 (0.0103)	–	0.0031 (0.0004)	0.0347 (0.0055)	0.0796 (0.0106)	0.0356 (0.0051)
Lag 2 ϵ^2, $\hat{\alpha}_2$	–	0.0517 (0.0134)	–	–	-0.0548 (0.0105)	–
Lag 1 variance, $\hat{\beta}_1$	0.9108 (0.0143)	0.9480 (0.0085)	0.9647 (0.0051)	0.9600 (0.0076)	0.9731 (0.0049)	0.9568 (0.0072)
Degrees of freedom, $\hat{\nu}$	9.2553 (1.6769)	5.8374 (0.4944)	4.3450 (0.2921)	4.4333 (0.1794)	3.7322 (0.0854)	4.0833 (0.0749)

Notes: Maximum likelihood estimates and corresponding asymptotic standard errors (in parentheses) obtained from fitting AR-GARCH models to the return series JPY/USD at each time horizon.

Table 6: Properties of the copula-GARCH model residuals (filtered returns)

Time	EUR/USD		JPY/USD		A-D test		Linear
horizon	z_t	z_t^2	z_t	z_t^2	EUR/\$	JPY/\$	correlation
1 hour	0.0063	0.1740	0.1361	0.0001	0.3242	0.1327	0.4198
2 hours	0.0489	0.0002	0.0479	0.2290	0.4471	0.1501	0.4298
4 hours	0.2085	0.2680	0.0016	0.2090	0.1069	0.1704	0.4244
8 hours	0.3894	0.8379	0.1257	0.0001	0.3044	0.1137	0.4091
12 hours	0.2622	0.1479	0.3806	0.4281	0.1105	0.5220	0.4253
1 day	0.7408	0.1560	0.1145	0.5418	0.6912	0.9918	0.4498

Notes: The first left four columns have the Ljung-Box autocorrelation test p-values up to the order twelve for the EUR/USD and JPY/USD residuals, z_t, and squared residuals, z_t^2, at the six time horizons. Columns five and six have the p-values from the Anderson-Darling (A-D) goodness of fit test. The null hypothesis is that the residuals follow a Student-t distribution. The right column displays the linear correlation between the EUR/USD and JPY/USD residuals.

Table 7: Dynamic copula parameter specification

Copula model	Parameter specification	
Clayton	$\theta_t = \alpha + \beta|z_{1t-1}z_{2t-1}| + \gamma\theta_{t-1}$	$\theta_t > 0$
Frank	$\theta_t = \alpha + \beta|z_{1t-1}z_{2t-1}| + \gamma\theta_{t-1}$	$\theta_t \in \mathbb{R}\backslash\{0\}$
Gauss/t Fisher	$\rho_t = h^{-1}(\alpha + \beta\,\text{sign}(z_{1t-1}z_{2t-1})\sqrt{|z_{1t-1}z_{2t-1}|} + \gamma h(\rho_{t-1}))$	$-1 \leq \rho_t \leq 1$
	$h(\rho) = \log((1+\rho)/(1-\rho))$	
Gauss/t Tse-Tsui	$\rho_t = (1 - \beta - \gamma)\rho + \beta\xi_{t-1} + \gamma\rho_{t-1}$	$-1 \leq \rho_t, \rho \leq 1$
	$\xi_{t-1} = \sum_{h=1}^{2} z_{1t-h}z_{2t-h}/\sqrt{\sum_{h=1}^{2} z_{1t-h}^2 \sum_{h=1}^{2} z_{2t-h}^2}$	$\beta + \gamma < 1$
Gumbel	$\theta_t = \alpha + \beta|z_{1t-1}z_{2t-1}| + \gamma\theta_{t-1}$	$\theta_t \geq 1$
Plackett	$\theta_t = (1 - \alpha - \beta)\theta + \alpha|z_{1t-1}z_{2t-1}| + \beta\theta_{t-1}$	$\theta_t, \theta \geq 0$

Notes: Parameter dynamics assumed for the different copula models used on modeling the EUR/USD and JPY/USD returns.

Table 8: Bivariate residuals modeling

Time horizon	Bivariate model	AIC		Log-likelihood ratio	
		static	dynamic	statistic	p-value
1 hour	Clayton	-7771.42	-9246.01	1478.59	0.00
	Frank	-8687.40	-10407.05	1723.65	0.00
	Gaussian Fisher	-9098.03	**-12647.45**	3553.42	0.00
	Gaussian Tse-Tsui	-9098.03	**-11937.74**	2841.71	0.00
	Gumbel	-9502.63	-9400.70	-99.92	1.00
	Clay & s.Clay	**-10246.42**	-5916.07	-4328.35	1.00
	Plackett	-9514.47	-9513.53	1.06	0.30
	t Fisher	**-11200.68**	**-13972.26**	2775.59	0.00
	t Tse-Tsui	**-11200.68**	**-13895.25**	2698.57	0.00
2 hours	Clayton	-4187.39	-4945.23	761.84	0.00
	Frank	-4577.34	-5543.39	970.04	0.00
	Gaussian Fisher	-4798.44	**-6758.40**	1963.96	0.00
	Gaussian Tse-Tsui	-4798.44	**-6434.96**	1638.52	0.00
	Gumbel	-5088.21	-4841.50	-242.70	1.00
	Clay & s.Clay	**-5453.88**	-6374.41	922.53	0.00
	Plackett	-5102.69	-5100.50	-0.19	1.00
	t Fisher	**-6084.28**	**-7585.54**	1505.27	0.00
	t Tse-Tsui	**-6084.28**	**-7554.79**	1474.51	0.00
4 hours	Clayton	-2186.47	-2183.67	1.20	0.55
	Frank	-2459.60	-3012.79	557.18	0.00
	Gaussian Fisher	-2479.79	**-3632.71**	1156.92	0.00
	Gaussian Tse-Tsui	-2479.79	**-3442.02**	964.23	0.00
	Gumbel	-2582.63	-2554.93	-23.70	1.00
	Clay & s.Clay	**-2787.72**	-2761.99	-23.72	1.00
	Plackett	-2736.84	-2734.44	-0.40	1.00
	t -Fisher	**-3105.85**	**-3990.58**	888.73	0.00
	t Tse-Tsui	**-3105.85**	**-3939.56**	837.71	0.00
8 hours	Clayton	-1003.97	-1006.14	6.17	0.05
	Frank	-1241.78	-1242.20	4.42	0.11
	Gaussian Fisher	-1209.32	**-1709.24**	503.92	0.00
	Gaussian Tse-Tsui	-1209.32	**-1662.27**	454.95	0.00
	Gumbel	-1217.20	-1220.23	7.03	0.03
	Clay & s.Clay	-1280.97	-1283.22	4.25	0.04
	Plackett	**-1355.58**	-1353.72	0.14	0.71
	t Fisher	**-1422.60**	**-1825.00**	406.40	0.00
	t Tse-Tsui	**-1422.60**	**-1805.93**	387.32	0.00
12 hours	Clayton	-707.36	-704.68	1.32	0.52
	Frank	-854.58	-850.52	-0.06	1.00
	Gaussian Fisher	-842.98	**-1189.56**	350.58	0.00
	Gaussian Tse-Tsui	-842.98	**-1119.11**	278.13	0.00
	Gumbel	-881.40	-878.38	0.98	0.61
	Clay & s.Clay	-922.55	-918.83	-1.72	1.00
	Plackett	**-947.95**	-945.77	-0.17	1.00
	t Fisher	**-1029.84**	**-1295.02**	269.18	0.00
	t Tse-Tsui	**-1029.84**	**-1267.28**	241.44	0.00
1 day	Clayton	-390.52	-387.71	1.1886	0.55
	Frank	-512.61	-549.68	41.07	0.00
	Gaussian Fisher	-475.58	**-600.16**	128.58	0.00
	Gaussian Tse-Tsui	-475.58	**-590.26**	116.67	0.00
	Gumbel	-467.69	-462.69	-0.99	1.00
	Clay & s.Clay	-494.47	-489.77	-2.70	1.00
	Plackett	**-546.84**	-543.09	-1.75	1.00
	t Fisher	**-538.11**	**-633.83**	99.71	0.00
	t Tse-Tsui	**-538.11**	**-628.47**	94.35	0.00

Notes: Bivariate copula models fitted to the residuals on EUR/USD and JPY/USD log-returns at the six different time horizons. The table lists estimates and asymptotic standard errors of parameters for the static and dynamic copula models.

Table 9: Estimates for the t-copula models

Time horizon		Parameter Estimates ($s.\hat{e}.$)			
		non-dynamic		t Fisher	t Tse-Tsui
1 hour	$\hat{\nu}$	4.6369 (0.1223)	$\hat{\nu}$	6.1284 (0.2026)	6.0224 (0.1964)
	$\hat{\rho}$	0.4209 (0.0041)	$\hat{\alpha}$	0.0005 (0.0002)	–
			$\hat{\beta}$	0.0338 (0.0021)	0.0095 (0.0006)
			$\hat{\gamma}$	0.9893 (0.0007)	0.9908 (0.0006)
2 hours	$\hat{\nu}$	4.0779 (0.1385)	$\hat{\nu}$	5.2857 (0.2222)	5.1875 (0.2151)
	$\hat{\rho}$	0.4312 (0.0058)	$\hat{\alpha}$	0.0008 (0.0004)	–
			$\hat{\beta}$	0.0447 (0.0034)	0.0137 (0.0010)
			$\hat{\gamma}$	0.9862 (0.0011)	0.9860 (0.0012)
4 hours	$\hat{\nu}$	4.2469 (0.2080)	$\hat{\nu}$	5.8504 (0.3728)	5.6622 (0.3530)
	$\hat{\rho}$	0.4450 (0.0080)	$\hat{\alpha}$	0.0030 (0.0011)	–
			$\hat{\beta}$	0.0746 (0.0071)	0.0236 (0.0023)
			$\hat{\gamma}$	0.9756 (0.0027)	0.9735 (0.0030)
8 hours	$\hat{\nu}$	5.4654 (0.4531)	$\hat{\nu}$	8.0167 (0.9059)	7.9486 (0.9093)
	$\hat{\rho}$	0.4431 (0.0111)	$\hat{\alpha}$	0.0087 (0.0032)	–
			$\hat{\beta}$	0.1042 (0.0142)	0.0401 (0.0055)
			$\hat{\gamma}$	0.9601 (0.0066)	0.9458 (0.0086)
12 hours	$\hat{\nu}$	4.5832 (0.4116)	$\hat{\nu}$	6.5264 (0.7888)	6.088 (0.6963)
	$\hat{\rho}$	0.4520 (0.0137)	$\hat{\alpha}$	0.0061 (0.0033)	–
			$\hat{\beta}$	0.0923 (0.0161)	0.0279 (0.0050)
			$\hat{\gamma}$	0.9663 (0.0070)	0.9663 (0.0071)
1 day	$\hat{\nu}$	6.1866 (0.9520)	$\hat{\nu}$	8.4611 (1.6871)	8.3670 (1.6592)
	$\hat{\rho}$	0.4821 (0.0178)	$\hat{\alpha}$	0.0089 (0.0056)	–
			$\hat{\beta}$	0.0922 (0.0190)	0.0400 (0.0097)
			$\hat{\gamma}$	0.9618 (0.0093)	0.9406 (0.0168)

Notes: Parameter estimates and asymptotic standard errors for the three t-copula models, non-dynamic, with Fisher transformation dynamic correlation and with Tse-Tsui dynamic correlation, fitted to the hourly up to daily returns on EUR/USD and JPY/USD rates.

Table 10: Estimates for the Gaussian copula models

Time horizon		Parameter Estimates ($\hat{s.e.}$)			
		non-dynamic		Gauss Fisher	Gauss Tse-Tsui
1 hour	$\hat{\rho}$	0.4141 (0.0034)	$\hat{\alpha}$	0.0006 (0.0002)	–
			$\hat{\beta}$	0.0326 (0.0018)	0.0121 (0.0008)
			$\hat{\gamma}$	0.9895 (0.0006)	0.9875 (0.0009)
2 hours	$\hat{\rho}$	0.4244 (0.0048)	$\hat{\alpha}$	0.0012 (0.0004)	–
			$\hat{\beta}$	0.0442 (0.0030)	0.0179 (0.0013)
			$\hat{\gamma}$	0.9859 (0.0010)	0.9808 (0.0016)
4 hours	$\hat{\rho}$	0.4310 (0.0068)	$\hat{\alpha}$	0.0034 (0.0010)	–
			$\hat{\beta}$	0.0727 (0.0062)	0.0293 (0.0025)
			$\hat{\gamma}$	0.9755 (0.0024)	0.9670 (0.0033)
8 hours	$\hat{\rho}$	0.4267 (0.0096)	$\hat{\alpha}$	0.0090 (0.0029)	–
			$\hat{\beta}$	0.0987 (0.0122)	0.0474 (0.0056)
			$\hat{\gamma}$	0.9609 (0.0058)	0.9376 (0.0087)
12 hours	$\hat{\rho}$	0.4358 (0.0117)	$\hat{\alpha}$	0.0091 (0.0037)	–
			$\hat{\beta}$	0.1034 (0.0161)	0.0372 (0.0064)
			$\hat{\gamma}$	0.9597 (0.0076)	0.9541 (0.0095)
1 day	$\hat{\rho}$	0.4612 (0.0159)	$\hat{\alpha}$	0.0105 (0.0058)	–
			$\hat{\beta}$	0.1031 (0.0186)	0.0497 (0.0102)
			$\hat{\gamma}$	0.9559 (0.0097)	0.9282 (0.0178)

Notes: Parameter estimates and asymptotic standard errors for the three Gaussian copula models, non-dynamic, with Fisher transformation dynamic correlation and with Tse-Tsui dynamic correlation, fitted to the hourly up to daily returns on EUR/USD and JPY/USD rates.

Table 11: Properties of the AR-BEKK model residuals

Time	EUR/USD		JPY/USD		A-D test	
horizon	z_t	z_t^2	z_t	z_t^2	EUR/USD	JPY/USD
1 hour	0.0092	0.0000	0.0001	0.0000	0.0000	0.0000
2 hours	0.0505	0.0003	0.1915	0.0070	0.0009	0.0489
4 hours	0.1873	0.2105	0.0733	0.7100	0.0002	0.2289
8 hours	0.2752	0.0002	0.7899	0.0107	0.0106	0.1264
12 hours	0.2919	0.1796	0.1956	0.4111	0.0105	0.1695
1 day	0.7459	0.0939	0.1069	0.7605	0.0003	0.0000

Notes: The first four left columns have the Ljung-Box autocorrelation tests p-values up to the order twelve for the EUR/USD and JPY/USD residuals, z_t, and squared residuals, z_t^2, at the six time horizons. Columns five and six have the p-values from the Anderson-Darling (A-D) goodness of fit test. The null hypothesis is that the residuals follow a Student-t distribution.

Table 12: Skewness, kurtosis, and correlation of the AR-BEKK model residuals

| Time horizon | EUR/USD | | JPY/USD | | Linear correlation |
	Skewness	Kurtosis	Skewness	Kurtosis	
1 hour	-0.1218	6.897	-0.1625	6.255	0.0142
2 hours	-0.0248	4.702	-0.1773	5.056	0.0210
4 hours	-0.2767	5.892	0.0152	5.365	-0.0165
8 hours	-0.0794	2.066	-0.2988	2.981	-0.0174
12 hours	-0.0272	1.576	-0.3870	2.355	-0.0084
1 day	0.0594	0.362	-0.3462	1.776	-0.0134

Notes: Sample skewness and kurtosis of the EUR/USD and JPY/USD returns at the six time horizons. The right column displays the linear correlation between the EUR/USD and JPY/USD residuals.

Table 13: Model evaluation on estimating correlation for different time horizons

	b_0	b_1	R^2
1 hour			
Gauss Fisher	-0.075 (0.001)	1.161 (0.002)	0.775
Gauss Tse-Tsui	-0.015 (0.001)	1.256 (0.003)	0.780
t Fisher	-0.059 (0.001)	1.221 (0.003)	0.748
t Tse-Tsui	-0.081 (0.001)	1.177 (0.003)	0.737
BEKK	-0.051 (0.001)	1.128 (0.002)	0.800
2 hours			
Gauss Fisher	-0.090 (0.001)	1.193 (0.003)	0.817
Gauss Tse-Tsui	-0.025 (0.001)	1.272 (0.003)	0.838
t Fisher	-0.074 (0.001)	1.158 (0.003)	0.814
t Tse-Tsui	-0.099 (0.001)	1.224 (0.004)	0.793
BEKK	-0.041 (0.001)	1.106 (0.003)	0.814
4 hours			
Gauss Fisher	-0.111 (0.001)	1.246 (0.003)	0.894
Gauss Tse-Tsui	-0.033 (0.001)	1.290 (0.004)	0.883
t Fisher	-0.098 (0.001)	1.214 (0.003)	0.894
t Tse-Tsui	-0.141 (0.002)	1.318 (0.004)	0.870
BEKK	-0.058 (0.002)	1.155 (0.004)	0.843
8 hours			
Gauss Fisher	-0.140 (0.002)	1.326 (0.006)	0.886
Gauss Tse-Tsui	-0.045 (0.003)	1.279 (0.007)	0.824
t Fisher	-0.126 (0.002)	1.283 (0.005)	0.885
t Tse-Tsui	-0.160 (0.003)	1.369 (0.007)	0.849
BEKK	-0.065 (0.003)	1.131 (0.007)	0.804
12 hours			
Gauss Fisher	-0.134 (0.003)	1.288 (0.007)	0.884
Gauss Tse-Tsui	-0.070 (0.003)	1.347 (0.008)	0.856
t Fisher	-0.124 (0.003)	1.258 (0.006)	0.892
t Tse-Tsui	-0.185 (0.004)	1.396 (0.008)	0.865
BEKK	-0.051 (0.004)	1.112 (0.008)	0.807
Daily			
Gauss Fisher	-0.136 (0.006)	1.315 (0.012)	0.848
Gauss Tse-Tsui	-0.073 (0.007)	1.286 (0.016)	0.759
t Fisher	-0.164 (0.006)	1.341 (0.011)	0.865
t Tse-Tsui	-0.201 (0.008)	1.422 (0.015)	0.801
BEKK	-0.021 (0.008)	1.053 (0.016)	0.668

Notes: Quality assessment of the competing models using the regression of the transformed Kendall's correlation on the dynamic copula models correlation estimates.

Table 14: Model evaluation on estimating daily correlation using data from different frequencies

	$\rho_{\tau,t} = b_0 + b_1\rho_{\mathrm{model},h,t} + u_t$			$\rho_{\mathrm{RC},t} = b_0 + b_1\rho_{\mathrm{model},h,t} + u_t$		
	b_0	b_1	R^2	b_0	b_1	R^2
1 hour						
Gauss-Fisher	0.061 (0.008)	1.006 (0.017)	0.632	-0.036 (0.010)	1.072 (0.023)	0.517
Gauss Tse-Tsui	0.121 (0.007)	1.066 (0.019)	0.610	0.029 (0.009)	1.131 (0.025)	0.610
t Fisher	0.073 (0.008)	1.064 (0.018)	0.619	-0.023 (0.010)	1.131 (0.025)	0.503
t Tse-Tsui	0.038 (0.008)	1.062 (0.017)	0.654	-0.034 (0.011)	1.069 (0.024)	0.477
BEKK	0.136 (0.009)	0.848 (0.019)	0.486	-0.047 (0.009)	1.124 (0.019)	0.612
2 hours						
Gauss Fisher	0.067 (0.007)	0.969 (0.016)	0.640	-0.008 (0.011)	0.980 (0.023)	0.466
Gauss Tse-Tsui	0.138 (0.007)	0.980 (0.018)	0.588	0.054 (0.010)	1.018 (0.024)	0.453
t Fisher	0.077 (0.007)	0.945 (0.015)	0.644	0.004 (0.011)	0.952 (0.022)	0.465
t Tse-Tsui	0.050 (0.008)	1.014 (0.016)	0.643	-0.010 (0.011)	0.993 (0.024)	0.442
BEKK	0.149 (0.008)	0.800 (0.017)	0.501	0.009 (0.009)	0.962 (0.020)	0.516
4 hours						
Gauss Fisher	0.099 (0.008)	0.872 (0.016)	0.587	0.026 (0.011)	0.878 (0.022)	0.425
Gauss Tse-Tsui	0.167 (0.007)	0.867 (0.018)	0.533	0.088 (0.010)	0.888 (0.024)	0.400
t Fisher	0.108 (0.008)	0.850 (0.016)	0.588	0.035 (0.011)	0.855 (0.022)	0.425
t Tse-Tsui	0.076 (0.008)	0.930 (0.017)	0.580	0.012 (0.012)	0.912 (0.024)	0.399
BEKK	0.148 (0.008)	0.784 (0.017)	0.514	0.047 (0.010)	0.853 (0.021)	0.438
8 hours						
Gauss Fisher	0.082 (0.008)	0.972 (0.018)	0.578	0.024 (0.012)	0.941 (0.026)	0.387
Gauss Tse-Tsui	0.171 (0.008)	0.874 (0.021)	0.467	0.092 (0.011)	0.895 (0.027)	0.348
t Fisher	0.093 (0.008)	0.938 (0.018)	0.572	0.033 (0.011)	0.912 (0.025)	0.387
t Tse-Tsui	0.090 (0.009)	0.952 (0.021)	0.500	0.019 (0.013)	0.948 (0.028)	0.353
BEKK	0.127 (0.008)	0.847 (0.017)	0.544	0.049 (0.011)	0.861 (0.023)	0.403
12 hours						
Gauss Fisher	0.033 (0.007)	1.013 (0.014)	0.705	0.030 (0.012)	0.858 (0.025)	0.360
Gauss Tse-Tsui	0.092 (0.007)	1.034 (0.016)	0.653	0.087 (0.012)	0.859 (0.027)	0.320
t Fisher	0.031 (0.006)	1.011 (0.013)	0.745	0.038 (0.012)	0.836 (0.024)	0.362
t Tse-Tsui	-0.014 (0.007)	1.113 (0.015)	0.712	0.012 (0.014)	0.896 (0.028)	0.327
BEKK	0.098 (0.007)	0.872 (0.014)	0.635	0.066 (0.011)	0.785 (0.022)	0.370
Daily						
Gauss Fisher	-0.136 (0.006)	1.315 (0.012)	0.848	-0.047 (0.015)	0.975 (0.030)	0.330
Gauss Tse-Tsui	-0.073 (0.007)	1.286 (0.016)	0.759	-0.005 (0.015)	0.966 (0.032)	0.304
t Fisher	-0.164 (0.006)	1.341 (0.011)	0.865	-0.065 (0.016)	0.989 (0.031)	0.333
t Tse-Tsui	-0.201 (0.008)	1.422 (0.015)	0.801	-0.095 (0.017)	1.055 (0.034)	0.312
BEKK	-0.021 (0.008)	1.053 (0.016)	0.668	-0.020 (0.014)	0.902 (0.027)	0.348

Notes: Quality assessment of the competing models using the daily transformed Kendall's correlation and realized correlation. The daily correlation estimates are obtained from the copula modeling at different time horizons. The non-parametric estimates are then projected on the parametric estimates.

Figure captions:

Figure 1: Time-varying cross-correlations estimated by Kendall's tau and by the time-varying copula-based models Student-t and Gaussian both with Fisher transform for the daily returns on the EUR/USD and JPY/USD spot rates.

Figure 2: Regression of the estimates of Kendall's tau for the filtered returns, computed according to (11), on the dynamic copula-GARCH and BEKK correlation estimates.
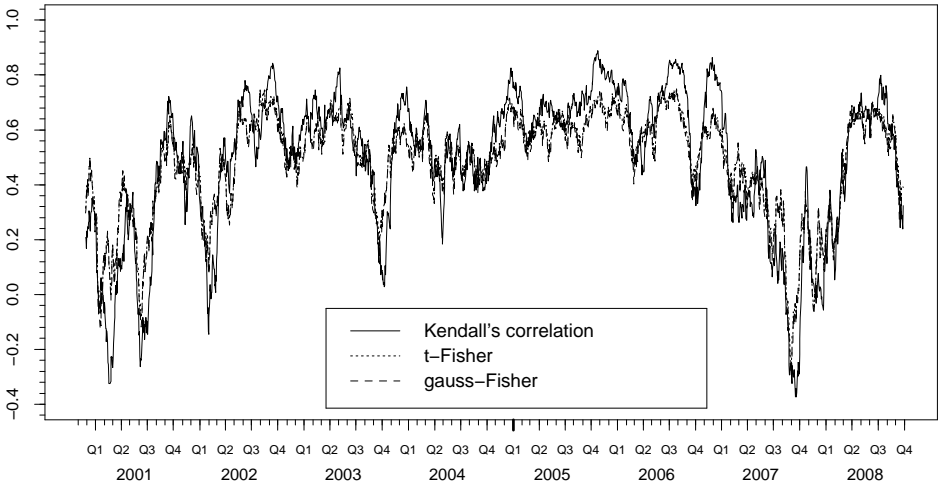
Figure 1:

Figure 2: