DYNAMIC

DEPENDENCE STRUCTURES

FOR MULTIVARIATE HIGH-FREQUENCY

DATA IN FINANCE

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ACKNOWLEDGEMENT

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REFERENCES

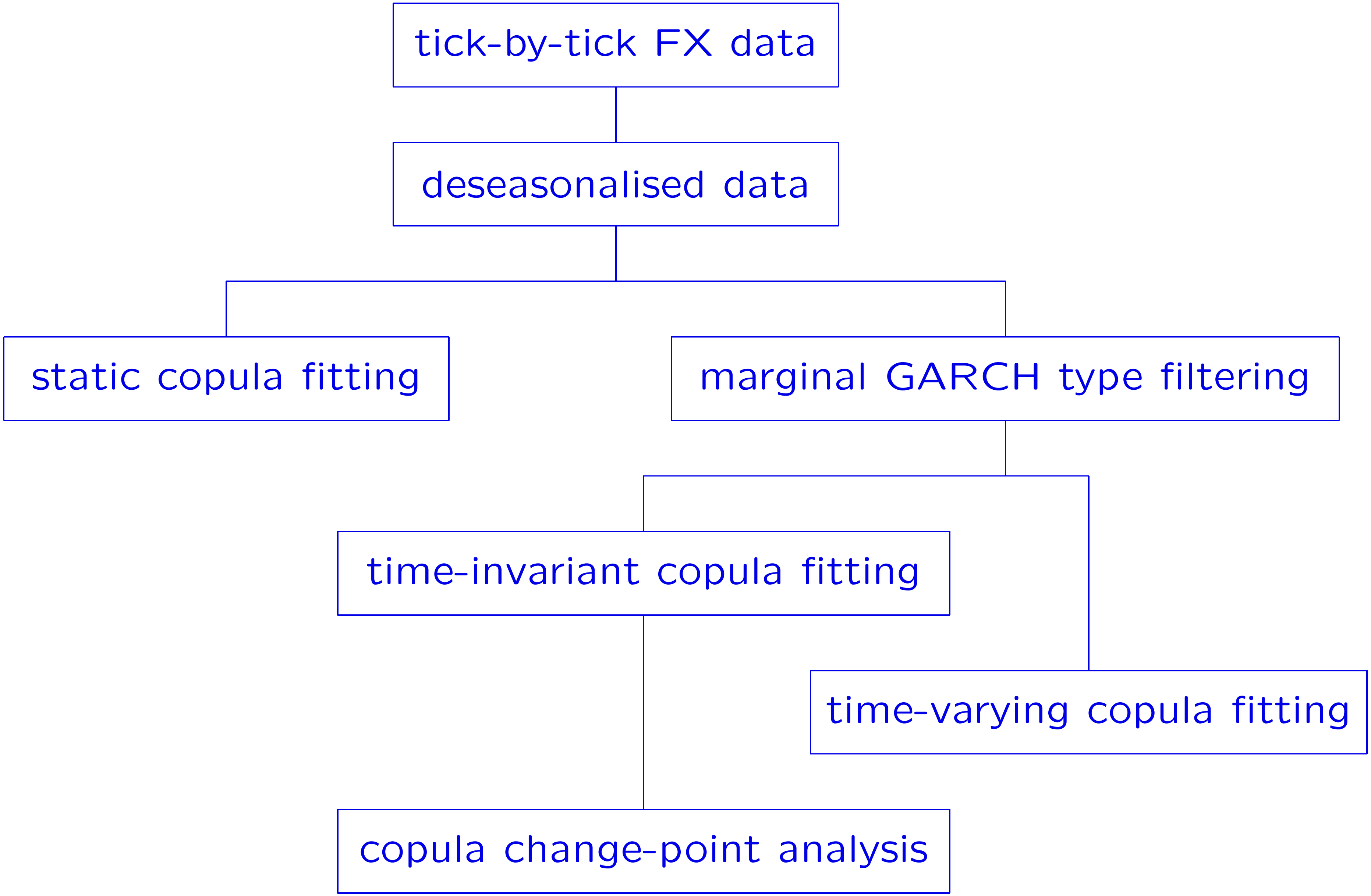
[1] Breymann, W., Dias, A., and Embrechts, P. (2003). Dependence structures for multivariate high-frequency data in finance. *Quant. Finance*, 3:1–14.

[2] Dias, A., and Embrechts, P. (2003). Dynamic copula models for multivariate high-frequency data in finance. Research paper, ETH-Zürich.

OVERVIEW

- Motivation
- The FX data
- Deseasonalisation
- Static copula fitting
- Dynamic dependence structure modelling
  - Time-invariant copula model
  - Tail dependence analysis
  - Time-varying copula model
  - Copula change-point detection
- Conclusion

OVERVIEW OF THE FX COPULA ANALYSIS



MOTIVATION

- The Goal:
  Studying the **dynamic dependence structure across time scales**

- Why?
  - The change of the behavior as a function of the time horizon may contain important information
  - Improves extrapolation from small to large time horizons

- Requires:
  Characterising dependence for horizons from minutes to months

- Here:
  Restriction to high-frequency region (**1 hour – 1 day**)

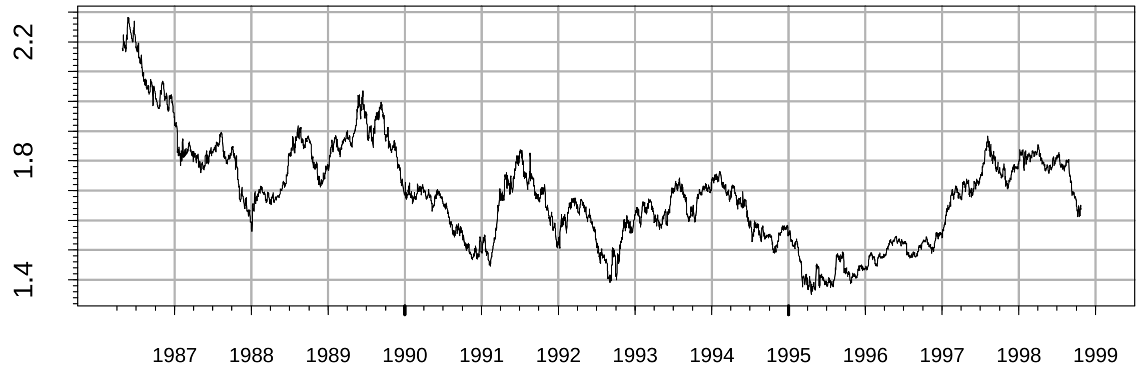
- Peculiarities of high-frequency data are taken into account

THE DATA

- Tick–by–tick bid and ask quotes
- Period: Febr 1986–Dec 1998
- Collected and filtered by Olsen Data
- Irregularly spaced
- About 10 million data points for a single series
- Regularisation to 5 min. time series by linear interpolation
- Reduction to logarithmic middle prices:

$$\xi_{\alpha,t} = \frac{\log\left(p_{\alpha,t}^{Bid} \cdot p_{\alpha,t}^{Ask}\right)}{2}$$

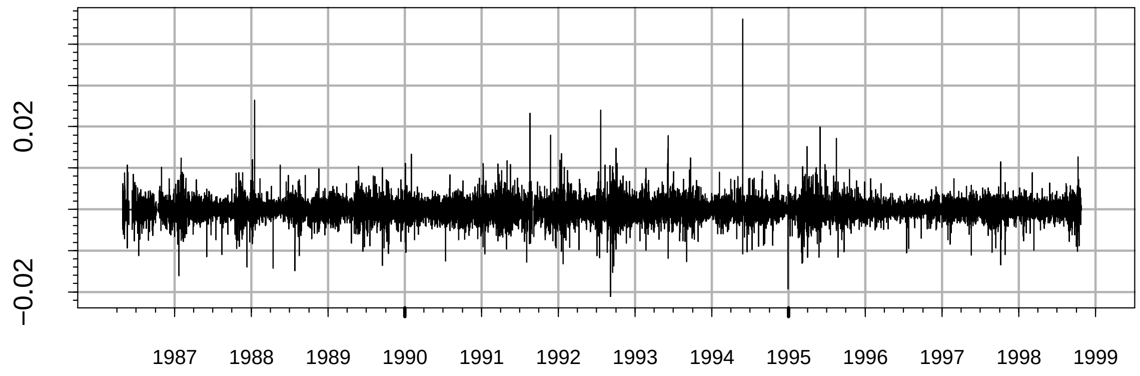
FX PRICES FOR USD/DEM AND USD/JPY

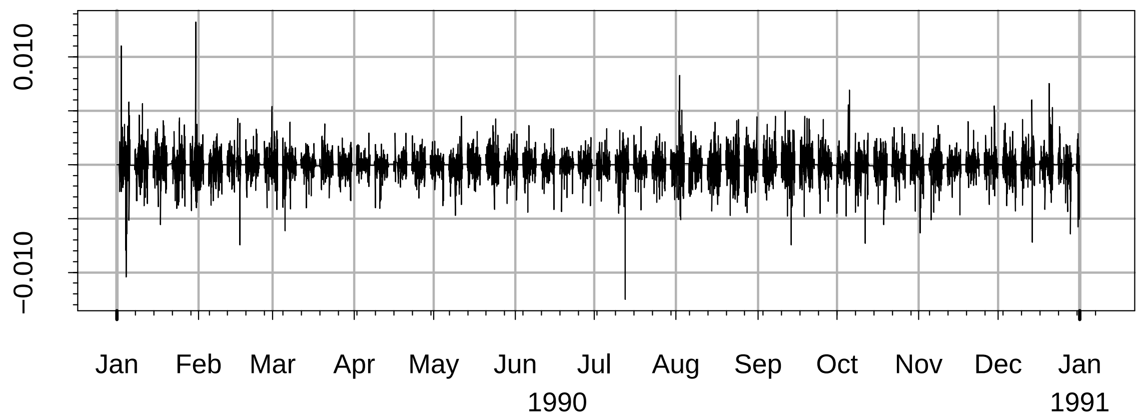


DESEASONALISATION OF FINANCIAL DATA

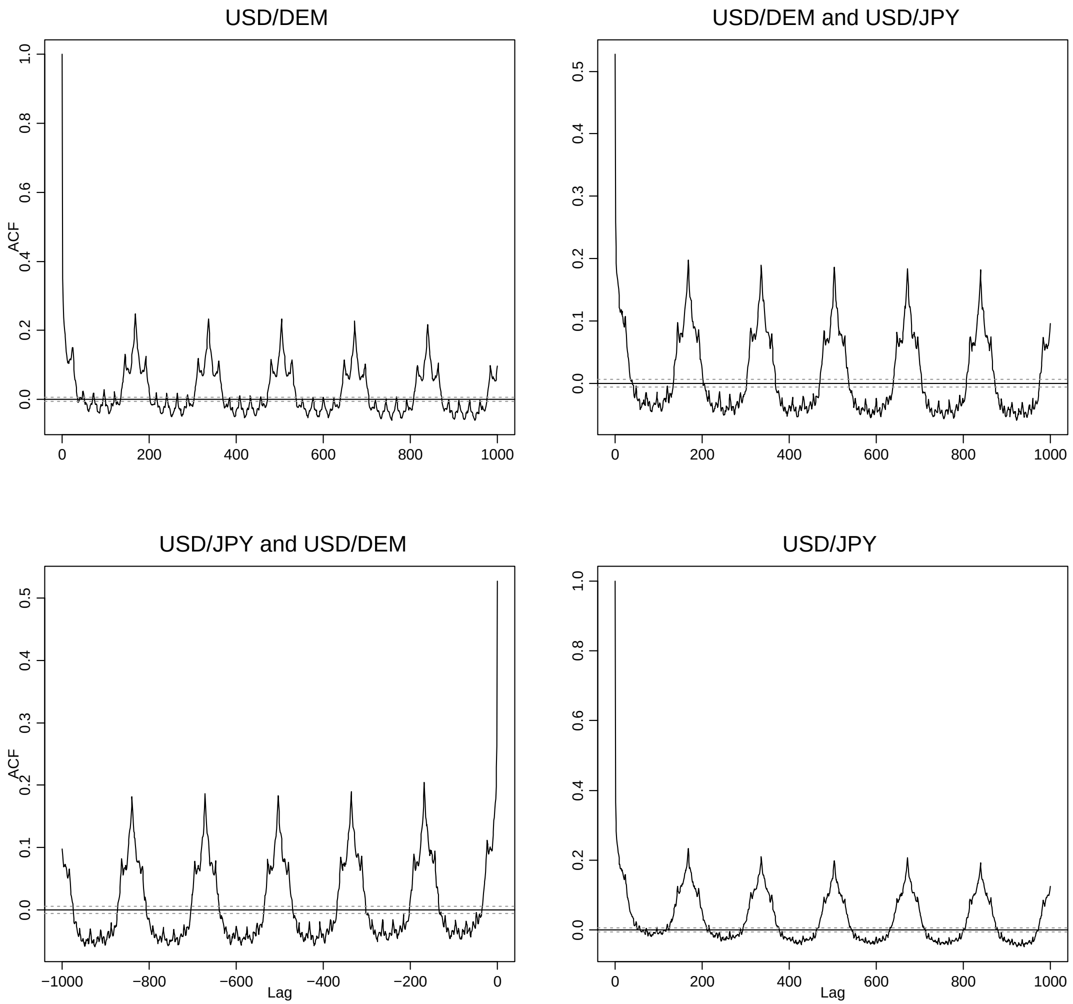
- High frequency financial data present strong seasonalities
- Main periodicities: daily and weekly
- Seasonalities cover more subtle statistical properties
- Affected by Daylight Saving Time (DST)
- Theory of stochastic processes favors time transformation to an activity-based time scale, but:
  $\longrightarrow$ Loss of synchronicity in the multivariate case
- Instead:
  $\longrightarrow$ Volatility weighting based on weekly activity pattern

HOURLY RETURNS OF USD/DEM





AUTOCORRELATION FUNCTIONS OF ABSOLUTE RETURNS



MODELLING REQUIREMENTS

- The flexibility of modelling arbitrary patterns that display abrupt volatility changes ($\rightarrow$ Japanese lunch break)

- Taking into account slow temporal changes in the habits of the market participants, institutional changes, etc

- Keeping track of Daylight Saving Time (DST) to take into account the one hour displacement between DST and non–DST periods

- The modelling of the geographical decomposition of market activity to take into account local public holidays and other irregularities

THE VOLATILITY PATTERN

- Integrated squared volatility: $V_t^2 = \sum_{t' \le t} (v_{t'}[\delta])^2$

- Volatility wrt horizon ΔT:

$$\Delta V_t^2[\Delta T] \equiv V_t^2 - V_{t-\Delta T}^2 = \sum_{i=0}^{n-1} (v_{t-i\delta}[\delta])^2 = (v_t[\Delta T])^2$$

- Deseasonalised returns:

$$x_t[\Delta T] = \frac{\xi_t - \xi_{t-\Delta T}}{\sqrt{\Delta V_t^2[\Delta T]}}.$$

- $\delta = 5$ minutes: elementary time step; $n = \Delta T/\delta$

- Aggregation property:

$$x_t[\Delta T] = \frac{x_{t-\Delta T_2}[\Delta T_1] \sqrt{\Delta V_{t-\Delta T_2}^2[\Delta T_1]} + x_t[\Delta T_2] \sqrt{\Delta V_t^2[\Delta T_2]}}{\sqrt{\Delta V_t^2[\Delta T]}}$$

COMPUTING THE VOLATILITY PATTERN

- Decomposition of the volatility:

$$v_t^2[\delta] = a_t \left(v_\tau^{(d)}[\delta] \right)^2$$

  - with relative market activity factor a_t and
  - volatility averaged over DST period d conditional to the time in the week, $\tau = t \mod (1 \text{ week})$:

$$\left(v_\tau^{(d)}[\delta] \right)^2 = \frac{1}{N_d} \sum_{i=1}^{N_d} (r_{t_i+\tau}[\delta])^2$$

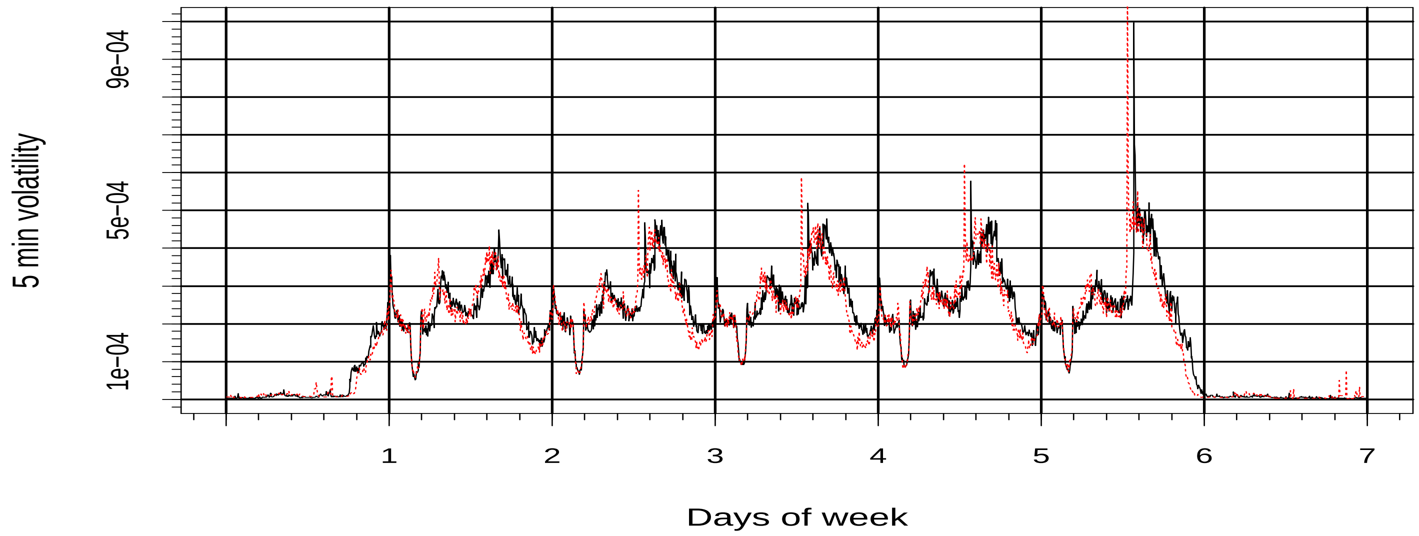
- Weekend volatility:

$$v^{(w)}[\delta] = \left| r_{t_w^{(end)}}[\Delta T_w] \right| \sqrt{\frac{\delta}{\Delta T_w}},$$

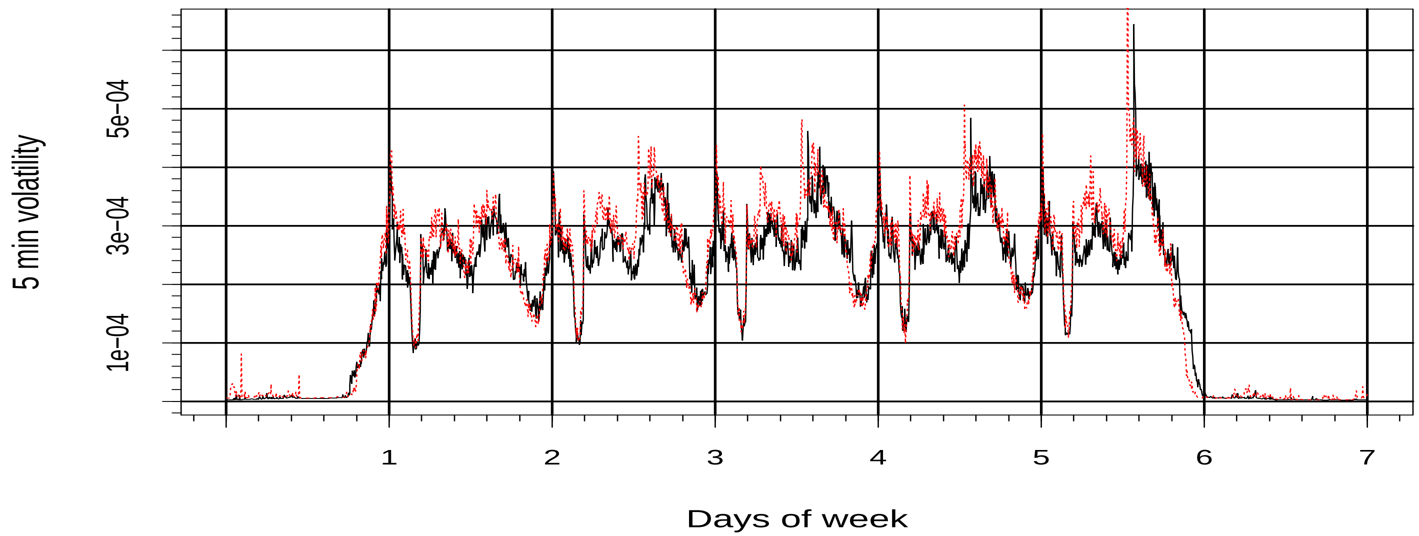
  - with weekend length $\Delta T_w = t_w^{(end)} - t_w^{(start)}$

THE WEEKLY VOLATILITY PATTERN

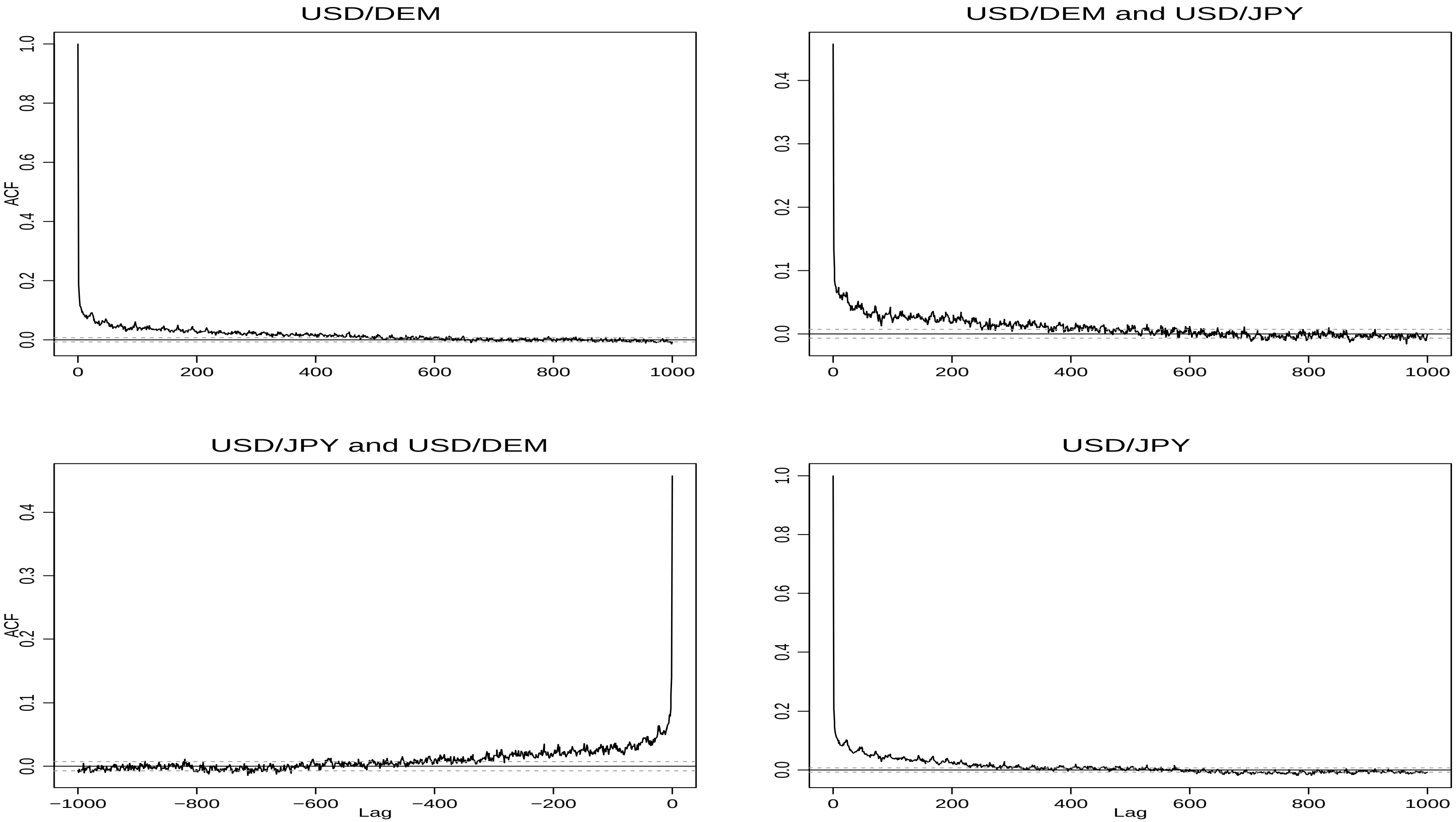
USD/DEM



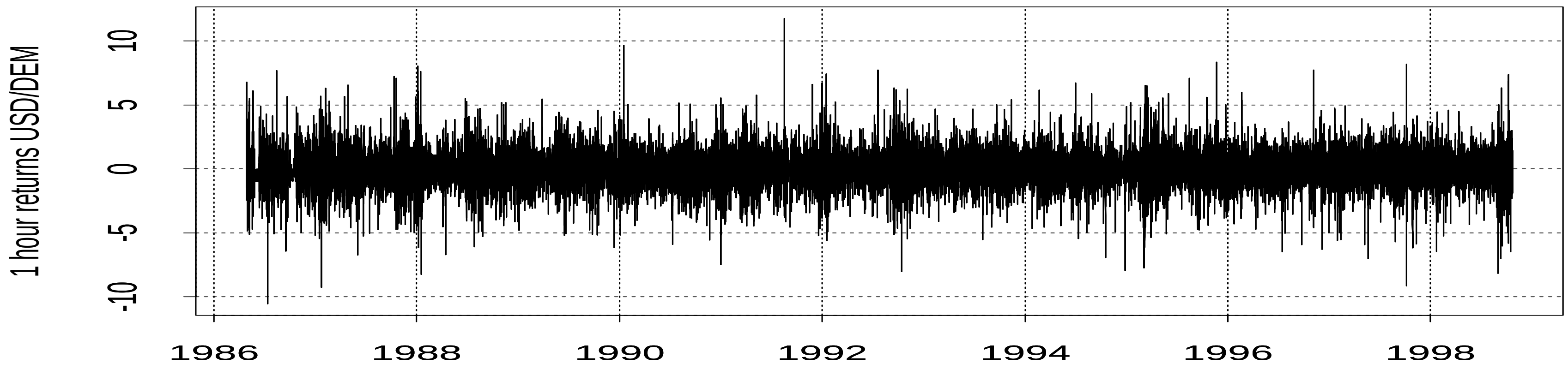
USD/JPY

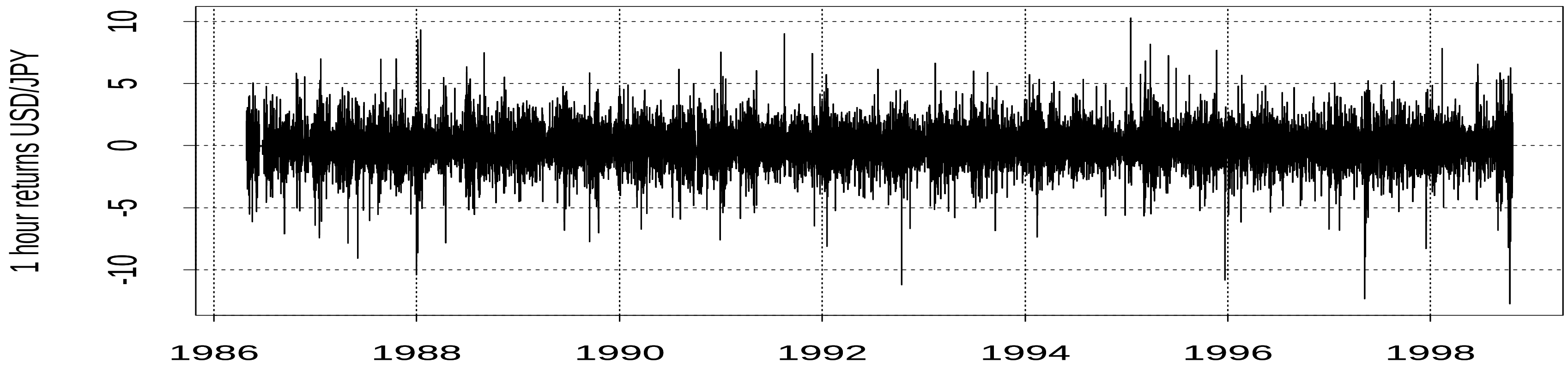


ACF OF DESEASONALISED HOURLY ABSOLUTE RETURNS



DESEASONALISED RETURNS





DEPENDENCE STRUCTURE MODELLING

- Static copula fitting
- Marginal GARCH type filtering
- Time-invariant copulas across time scales
  - Copula fitting
  - Goodness of fit and ellipticality test
  - Tail coefficient
  - Spectral measure and bivariate excesses
- Multivariate (matrix-diagonal) GARCH
- Time-varying copulas across time scales
- Change-point detection

DEPENDENCE STRUCTURE

$\mathbf{X} = (X_1, \ldots, X_d)$ d-dimensional vector of risk factors

- Marginal distributions (assumed continuous)

$$F_1(x_1) = P\left(X_1 \le x_1\right), \ldots, F_d(x_d) = P(X_d \le x_d)$$

- Joint distribution

$$\begin{aligned} F_{\mathbf{X}}(\mathbf{x}) &= P\left(X_1 \le x_1, \ldots, X_d \le x_d\right) \\ &= P\left(F_1(X_1) \le F_1(x_1), \ldots, F_d(X_d) \le F_d(x_d)\right) \\ &= P\left(U_1 \le F_1(x_1), \ldots, U_d \le F_d(x_d)\right) \end{aligned}$$

$(U_1, \ldots, U_d) \overset{d}{\sim} C$ on $[0,1]^d$: the copula

- Copula representation (Sklar):

$$F_{\mathbf{X}}(\mathbf{x}) = C\left(F_1(x_1), \ldots, F_d(x_d)\right)$$

$$C(u_1, u_2, \ldots, u_d) = F_{\mathbf{X}}\left(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d)\right)$$

FAMILIES OF COPULAS

- Gaussian copula for correlation ρ:

$$C_\rho^{Ga}(u,v) = \int_{\infty}^{\Phi^{-1}(u)} \int_{\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp\left\{-\frac{(s^2 - 2\,\rho s\,t + t^2)}{2(1-\rho^2)}\right\} \mathrm{d}s\,\mathrm{d}t$$

- t–copula for ν degrees of freedom and correlation ρ:

$$C_{\nu,\rho}^{t}(u,v) = \int_{\infty}^{t_\nu^{-1}(u)} \int_{\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \left\{1 + \frac{(s^2 - 2\,\rho s\,t + t^2)}{\nu(1-\rho^2)}\right\}^{-\frac{(\nu+1)}{2}} \mathrm{d}s\,\mathrm{d}t$$

- Clayton copula:

$$C_\beta^{Cl}(u,v) = \max\left[-\left\{(-\log u)^{1/\beta} + (-\log v)^{1/\beta}\right\}^{\beta}, 0\right]$$

FAMILIES OF COPULAS

- Gumbel copula:

$$C_\beta^{Gu}(u,v) = \exp\left[-\left\{(-\log u)^{1/\beta} + (-\log v)^{1/\beta}\right\}^{\beta}\right]$$

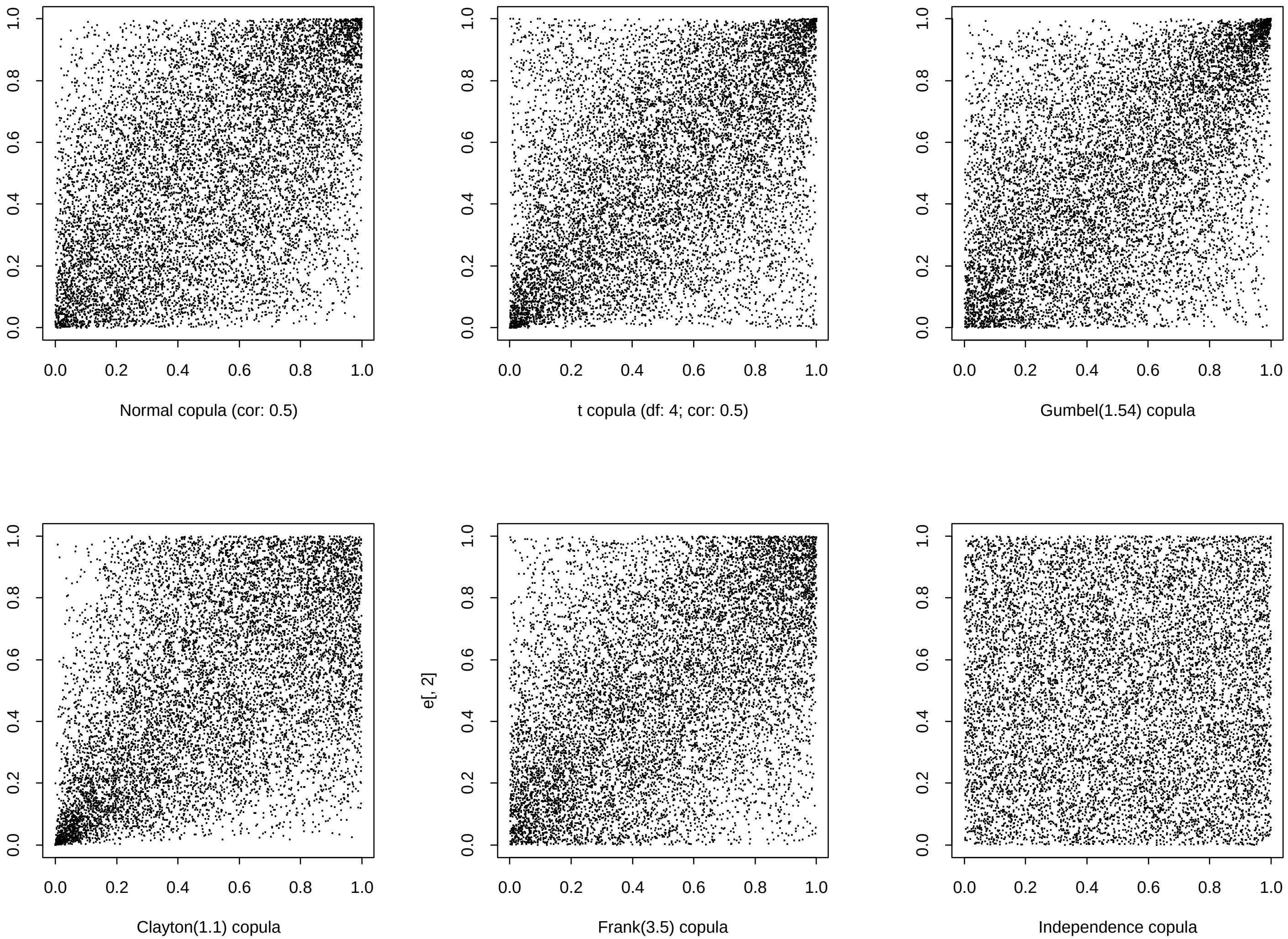
- Mixture of Gumbel and survival Gumbel copulas:

$$\begin{aligned} C(u_1,u_2;\boldsymbol{\theta}) &= \theta_3 C^{Gu}(u_1,u_2;\theta_1) + (1-\theta_3)(u_1+u_2-1+C^{Gu}(1-u_1,1-u_2;\theta_2)) \\ &= \theta_3 u_1^{2^{1/\theta_1}} + (1-\theta_3)\left(2u_1 - 1 + (1-u_1)^{2^{1/\theta_2}}\right) \end{aligned}$$

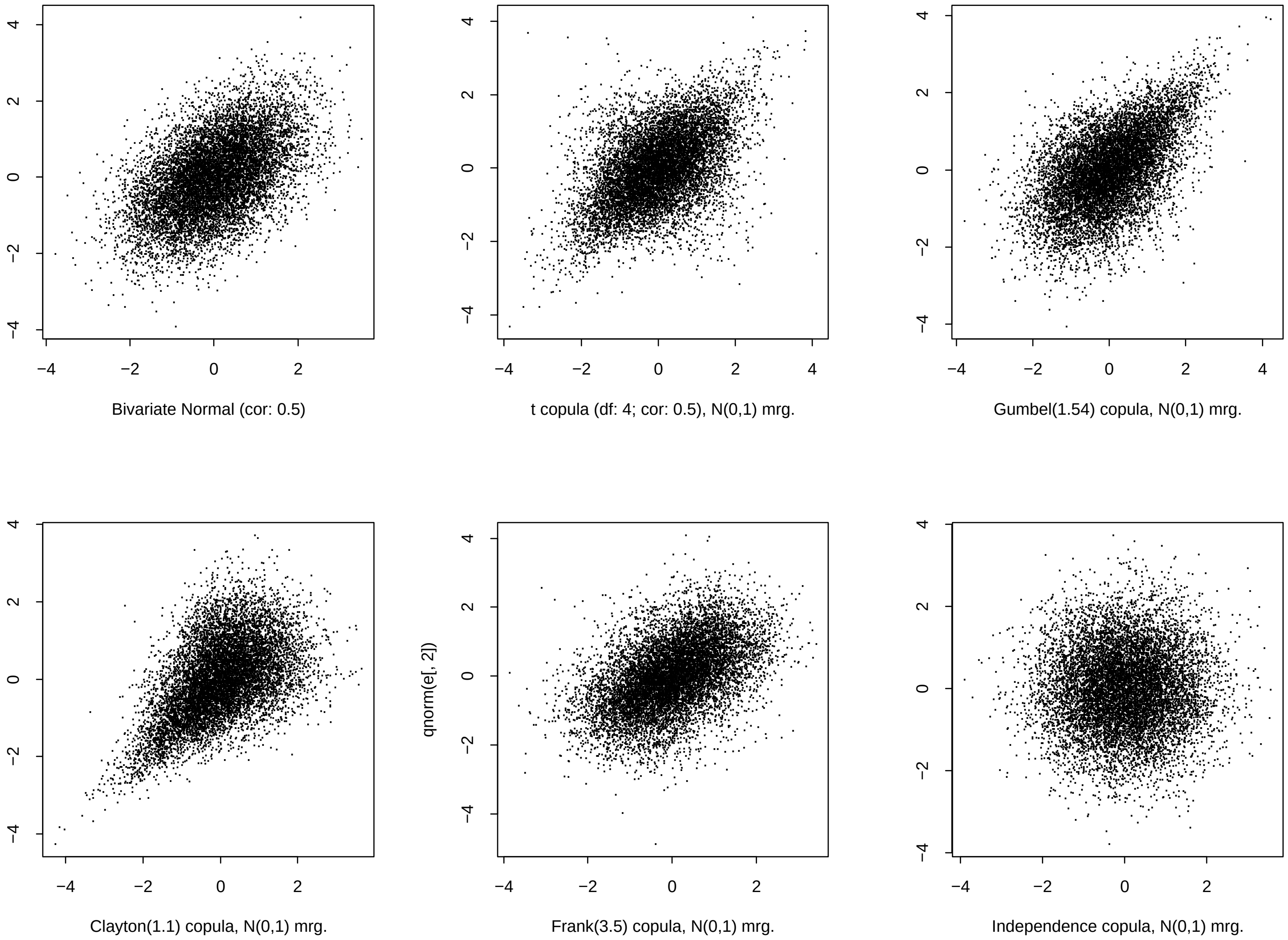
- Frank copula:

$$C_\beta^{Fr}(u,v) = -\frac{1}{\beta}\log\left[1 + \frac{(e^{-\beta u}-1)(e^{-\beta v}-1)}{e^{-\beta}-1}\right]$$

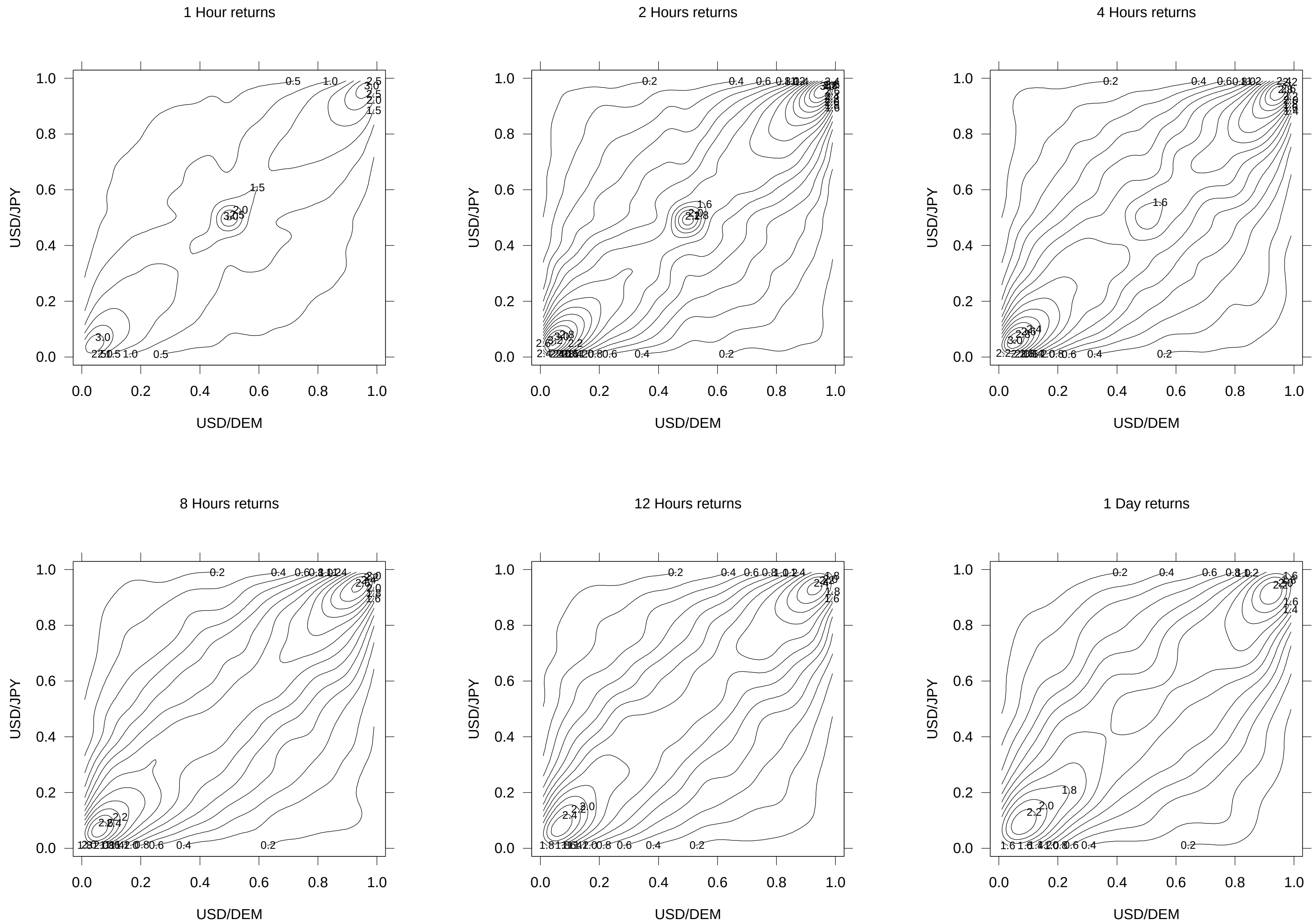
COPULA DENSITIES FOR SELECTED COPULAS



COPULA DENSITIES WITH NORMAL MARGINS



CONTOUR-PLOTS OF DESEASONALISED USD/DEM AND USD/JPY RETURNS



FITTING STATIC COPULA MODELS

SUMMARY OF THE RESULTS

- Use pseudo-likelihood method
- t-copula fits best across all frequencies
- Persistence of tail dependence
- Data are non-elliptical for higher frequencies

Reference: [1]

MARGINAL GARCH TYPE FILTERING

Univariate ARMA-GARCH model:

$$X_t = \mu_t + \epsilon_t$$

$$\mu_t = \mu + \sum_{i=1}^{p_1} \phi_i \left(X_{t-i} - \mu\right) + \sum_{j=1}^{q_1} \theta_j \epsilon_{t-j}$$

$$\epsilon_t = \sigma_t Z_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p_2} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q_2} \beta_j \sigma_{t-j}^2$$

where $Z_t \sim t_\nu$

USD/DEM

Frequency	p_1	q_1	p_2	q_2	$\hat{\nu}$ ($\widehat{s.e.}$)
1 hour	–	–	1	1	3.693 (0.054)
2 hours	2	2	2	1	3.708 (0.044)
4 hours	–	5	1	1	3.975 (0.105)
8 hours	2	4	1	1	4.679 (0.234)
12 hours	1	–	1	1	5.385 (0.326)
1 day	1	–	1	1	5.797 (0.556)

USD/JPY

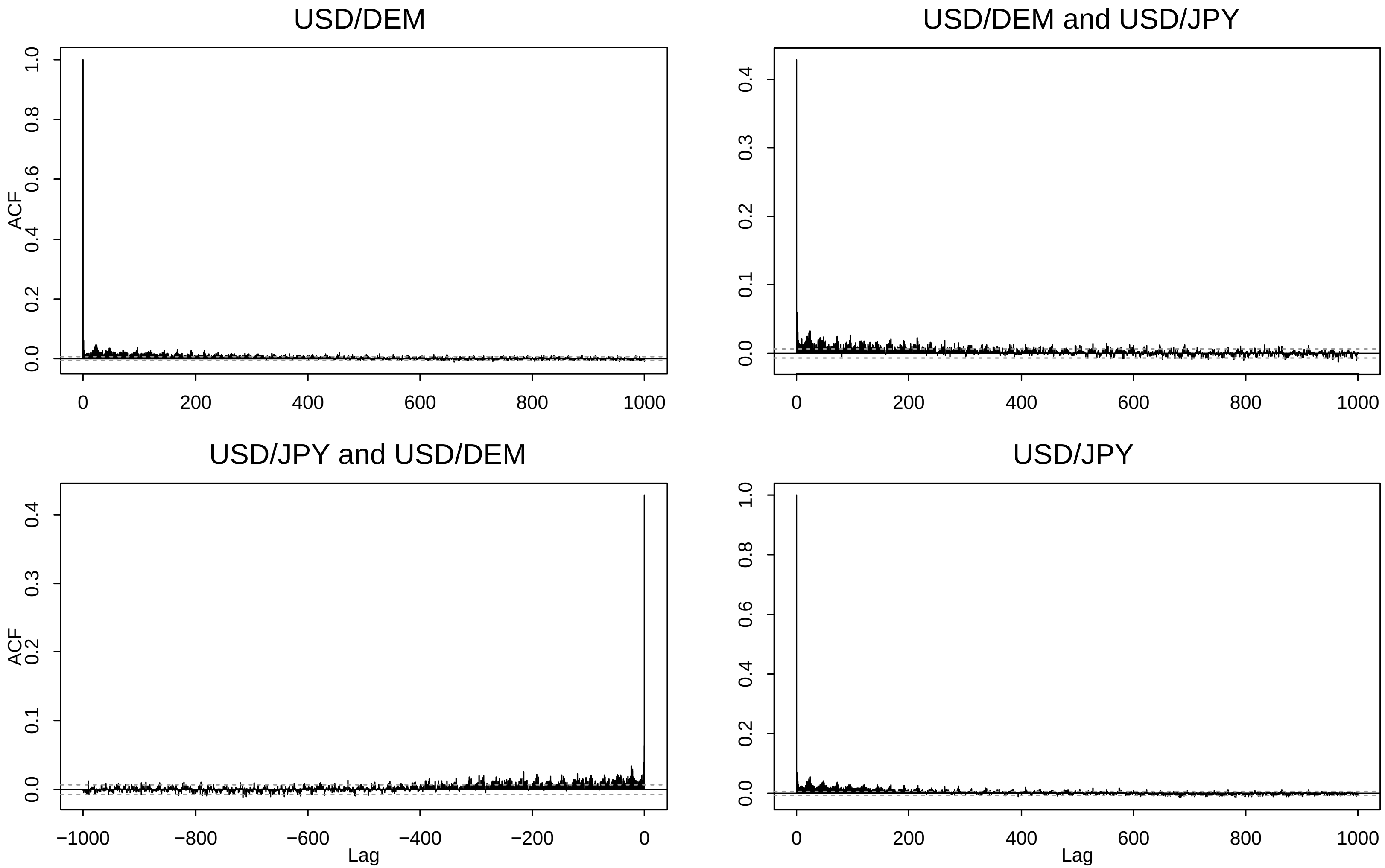
Frequency	p_1	q_1	p_2	q_2	$\hat{\nu}$ ($\widehat{s.e.}$)
1 hour	–	–	1	1	3.654 (0.052)
2 hours	1	–	2	1	3.759 (0.077)
4 hours	4	4	2*	1	3.819 (0.109)
8 hours	2	2	1*	1	4.357 (0.195)
12 hours	1	–	1*	1	4.574 (0.251)
1 day	10	–	1*	1	4.889 (0.412)

Marginal residuals: $\hat{z}_t = (x_t - \hat{\mu}_t)/\hat{\sigma}_t$

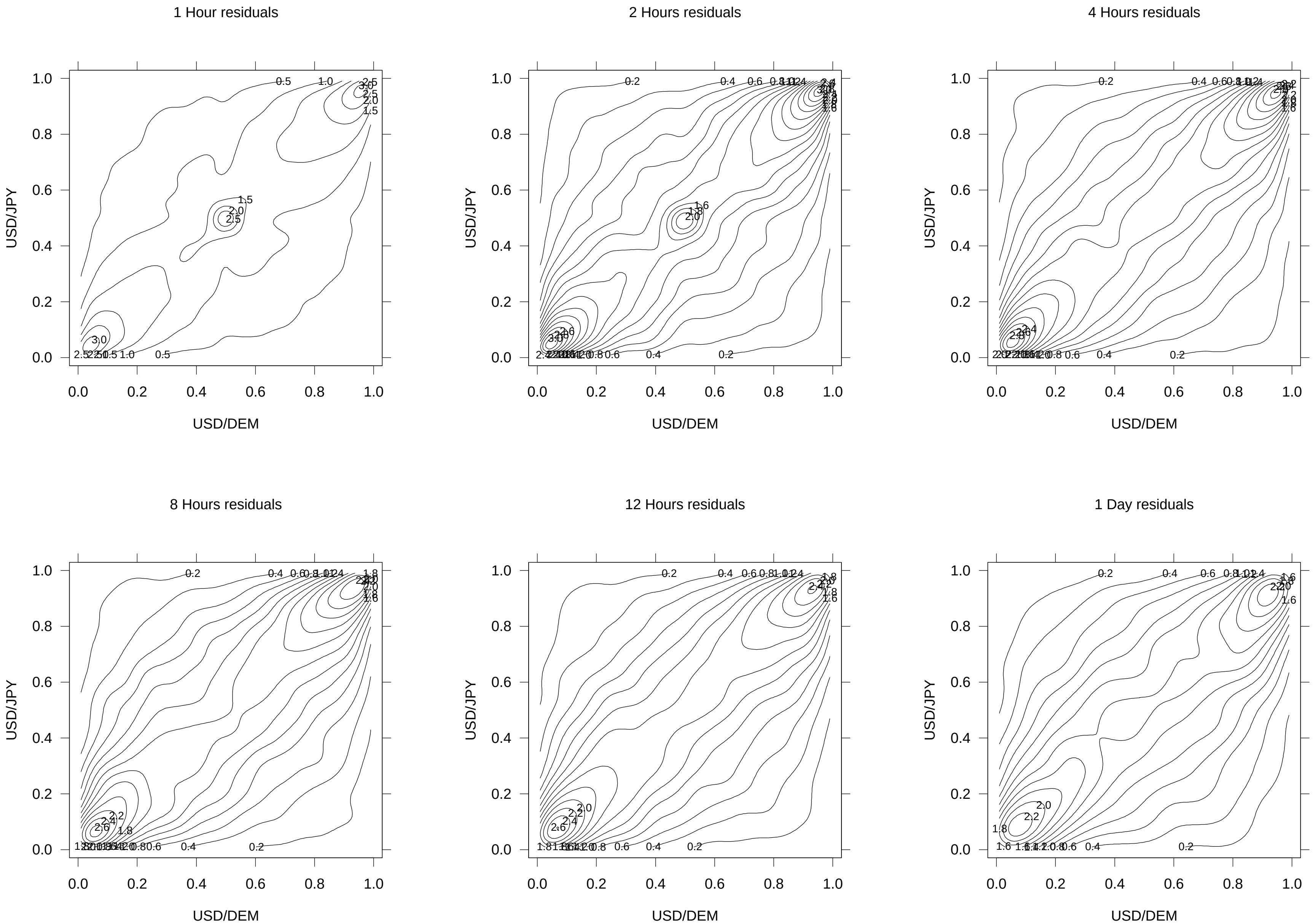
* leverage effect: $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p_2} \alpha_i(|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i})^2 + \sum_{j=1}^{q_2} \beta_j \sigma_{t-j}^2$

Sample autocorrelograms

Absolute values of the one hour USD/DEM and USD/JPY residuals

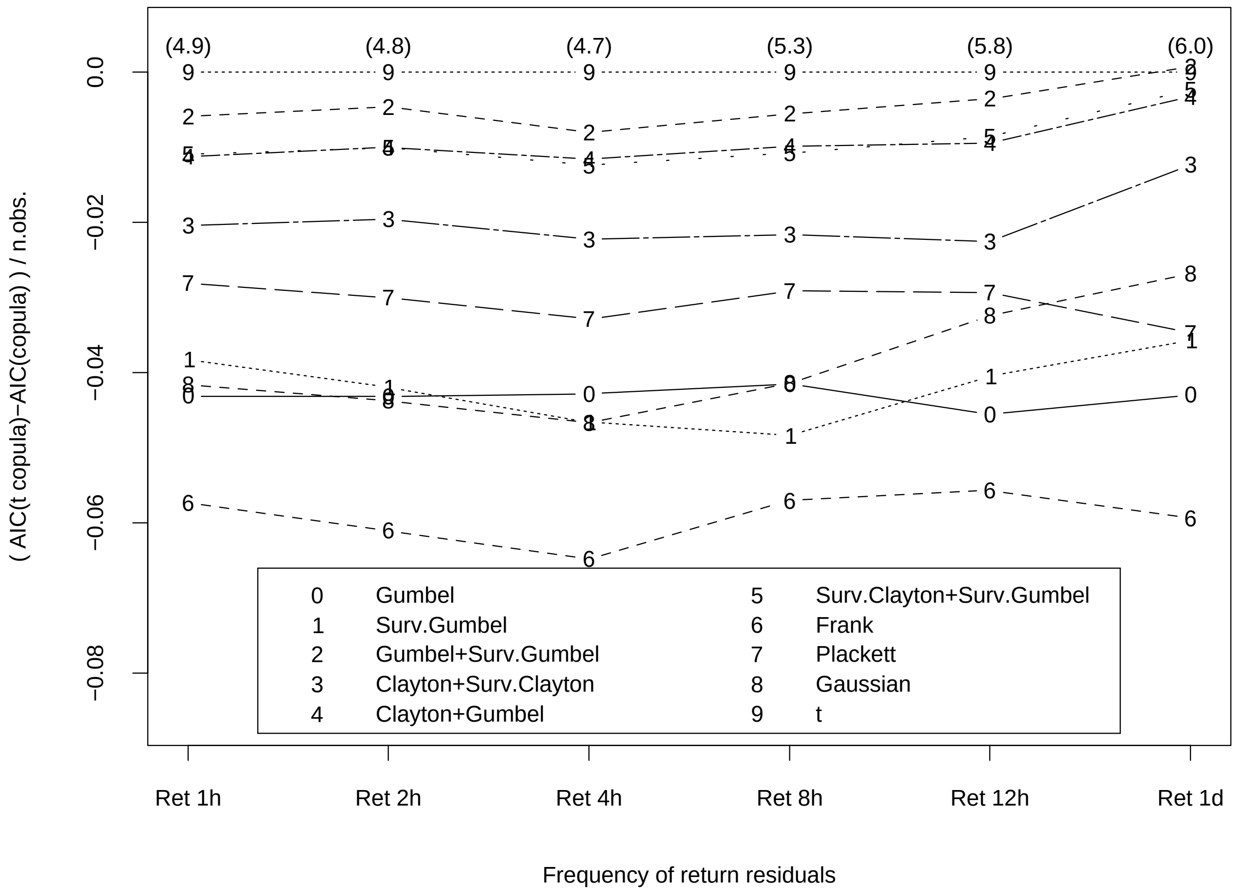


CONTOUR-PLOTS OF FILTERD USD/DEM AND USD/JPY RETURNS



GOODNESS OF FIT

FOR DIFFERENT FREQUENCIES RELATIVE TO THE t-COPULA



GOODNESS OF FIT

of the t and Gumbel misture models fitted to the residuals

Frequency	sample size	t-model	Gumbel mixture
1 hour	78,239	0	0
2 hour	39,119	0	0
4 hour	19,559	0.0348	0.0006
8 hour	9,779	0.3808	0.1079
12 hour	6,519	0.2471	0.1949
1 day	3,259	0.7211	0.6775

ELLIPTICALITY TEST
FOR THE BIVARIATE RESIDUALS

Frequency	Original margins	t margins
1 hour	0	0
2 hours	0	0
4 hours	0	0.348
8 hours	0.001	0.069
12 hours	0.145	0.501
1 day	0.389	0.451

TAIL-DEPENDENCE ANALYSIS

- Tail-dependence coefficient

  Let X_1 and X_2 be random variables with distributions F_1 and F_2, such that

  $$\lim_{u \to 1^-} P\left(X_2 \geq F_2^{-1}(u) | X_1 \geq F_1^{-1}(u)\right) = \lambda_U$$

  exists. If $\lambda_U \in (0, 1]$ then (X_1, X_2) has upper tail-dependence coefficient λ_U and (X_1, X_2) has no upper tail dependence if $\lambda_U = 0$

- Spectral analysis

  Assumption: Multivariate regular variation of (X_1, X_2)

- Asymptotic clustering

  Bivariate Archimedean excesses have a Clayton copula (asymptotically)

[3] Juri and Wüthrich (2002). Copula convergence theorems for tail events. Insurance Math. Econom., 30: 405–420

TAIL-DEPENDENCE COEFFICIENT ESTIMATES FOR THE RESIDUALS

	t copula	Gumbel mixture	
Frequency	λ	λ_L	λ_U
1 hour	0.242	0.209	0.250
2 hour	0.261	0.207	0.269
4 hour	0.273	0.265	0.225
8 hour	0.261	0.216	0.289
12 hour	0.247	0.288	0.224
1 day	0.240	0.286	0.226

SPECTRAL ANALYSIS

- Suppose that the d-dimensional random vector $\mathbf{X}$ has a **regularly varying tail distribution**, i.e., the tail behaviour of $\mathbf{X}$ is characterised by a tail index α and the limit

$$\frac{P(\| \mathbf{X} \| > tx, \mathbf{X}/ \| \mathbf{X} \| \in \cdot)}{P(\| \mathbf{X} \| > t)} \quad \xrightarrow{v} \quad x^{-\alpha} P(\Theta \in \cdot),$$

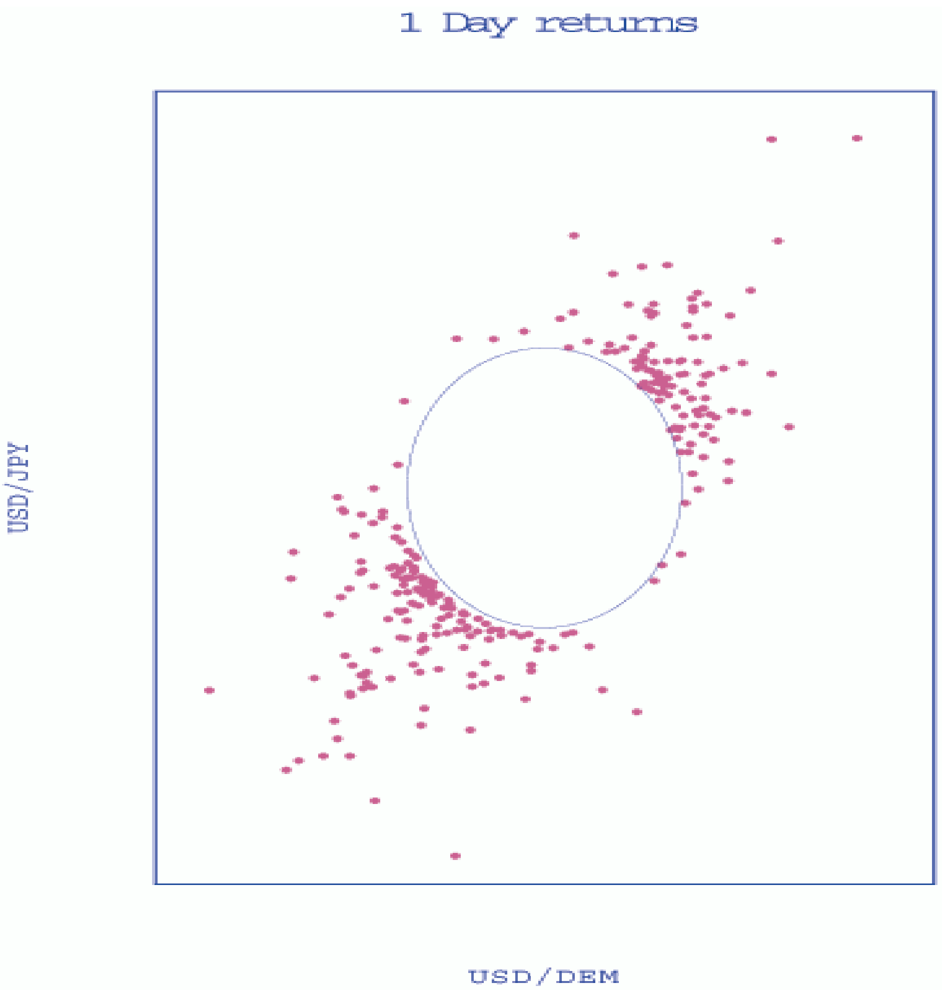
where $x > 0$, $t \to \infty$, exists. The distribution function of Θ is the **spectral distribution** of $\mathbf{X}$

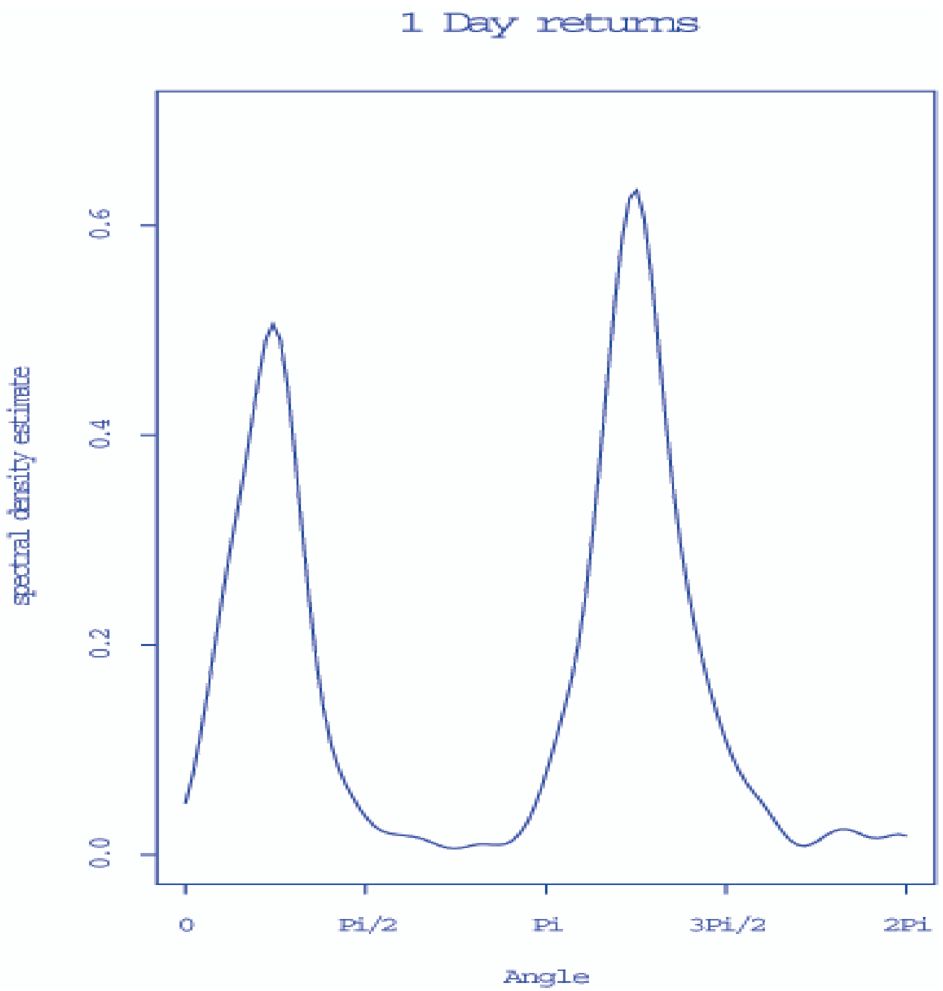
- **Estimator:**

$$\widehat{P}(\Theta \in S) = \frac{1}{k_n} \sum_{i=1}^{n} \epsilon_{\mathbf{x}_i/\|\mathbf{x}\|_{k_n,n}}(V(S))$$

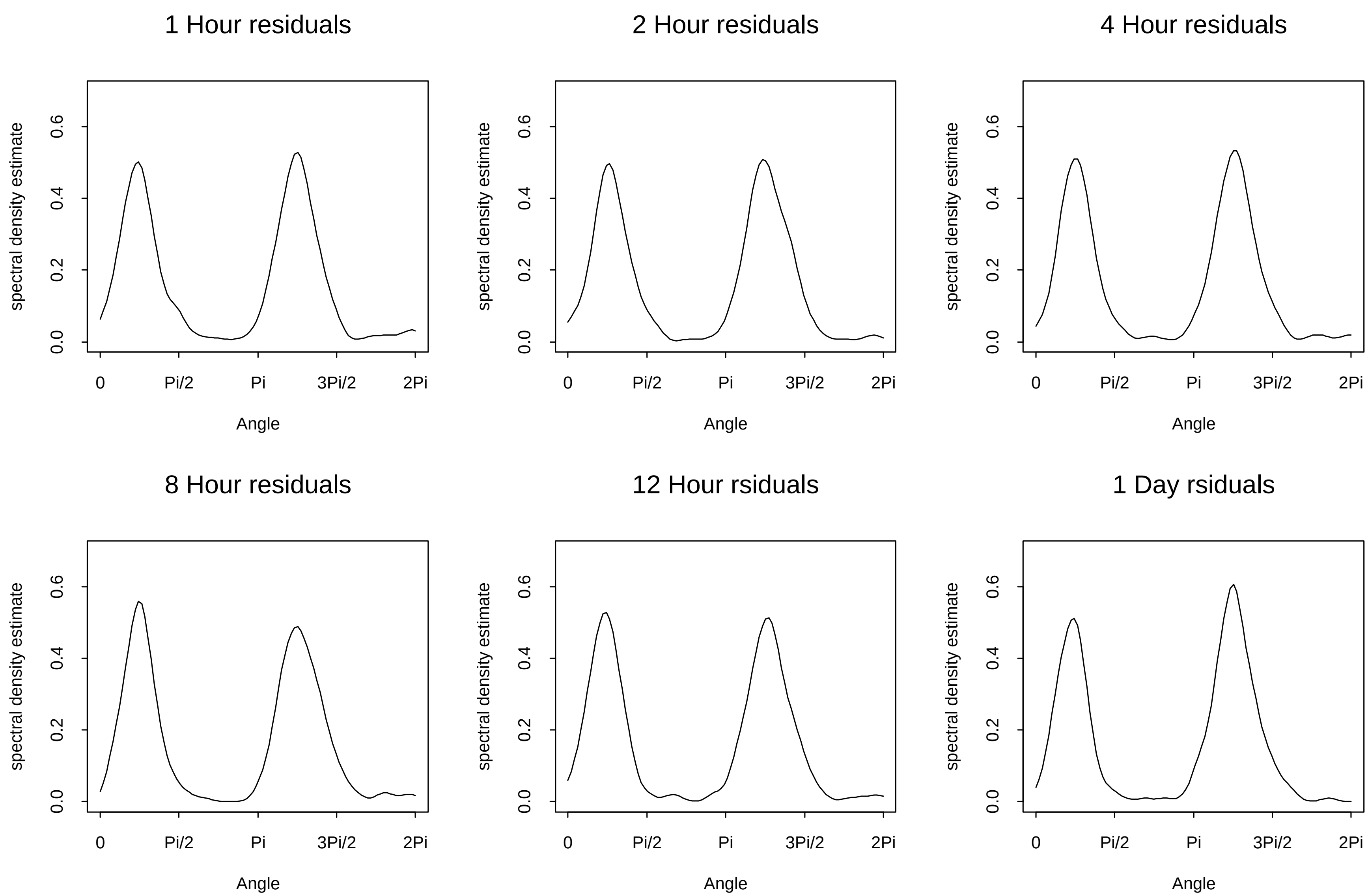
where $V(S) = \{\mathbf{x} \in \mathbb{S}_{+}^{d-1} : \mathbf{x}/ \| \mathbf{x} \| \in S\}$

SPECTRAL MEASURE ESTIMATION





SPECTRAL ANALYSIS
FOR THE BIVARIATE RESIDUALS



ASYMPTOTIC CLUSTERING OF BIVARIATE EXCESSES

- Extreme tail dependence copula relative to a threshold t:

$$C_t(u,v) = P(U \leq F_t^{-1}(u), V \leq F_t^{-1}(v) | U \leq t, V \leq t)$$

  with conditional distribution function

$$F_t(u) := P(U \leq u | U \leq t, V \leq t),\ 0 \leq u \leq 1$$

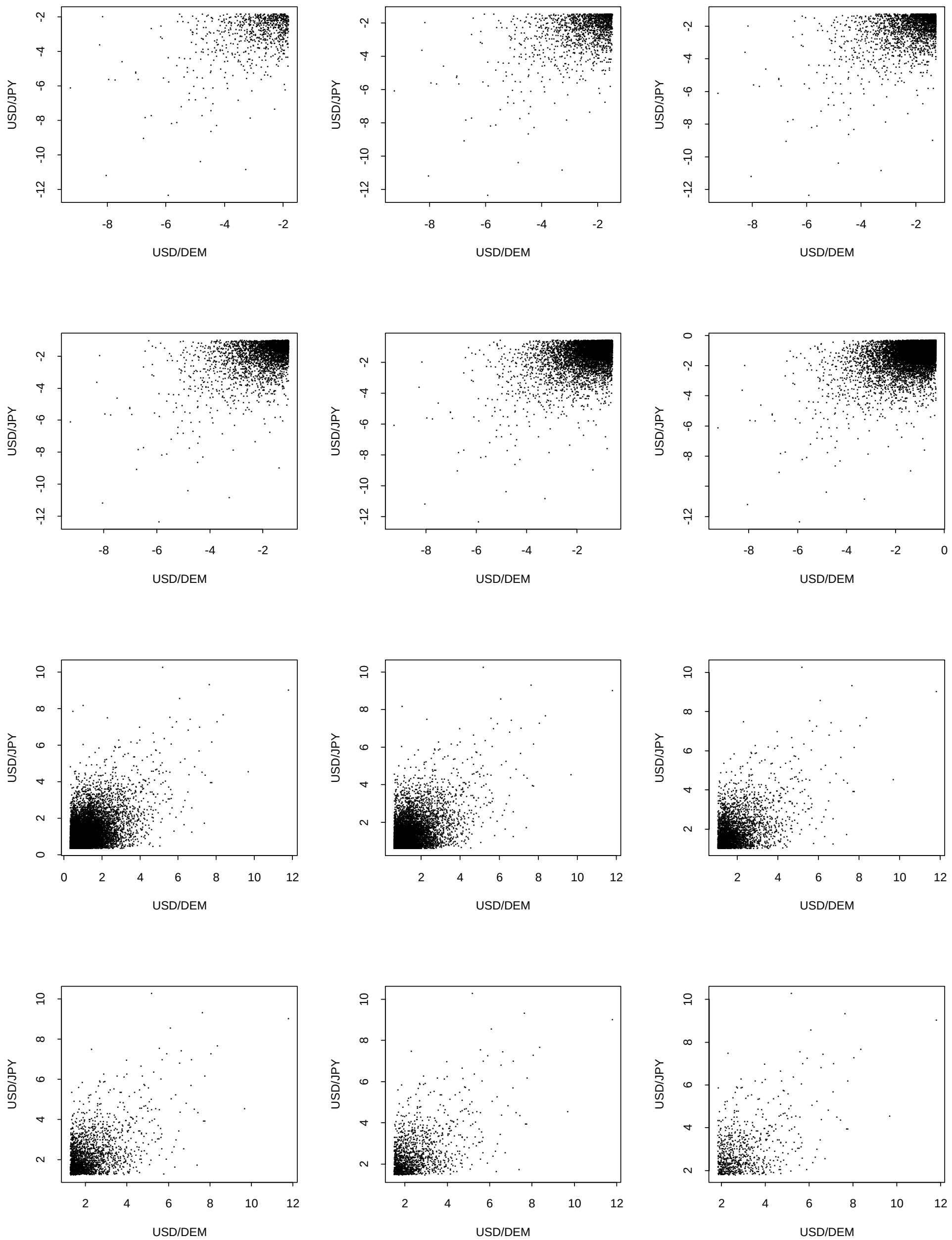
- **Archimedean** copulas: $\exists$ cont., strictly decreasing function $\psi : [0,1] \mapsto [0,\infty]$ with $\psi(1) = 0$, s.t.

$$C(u,v) = \psi^{[-1]}(\psi(u) + \psi(v))$$

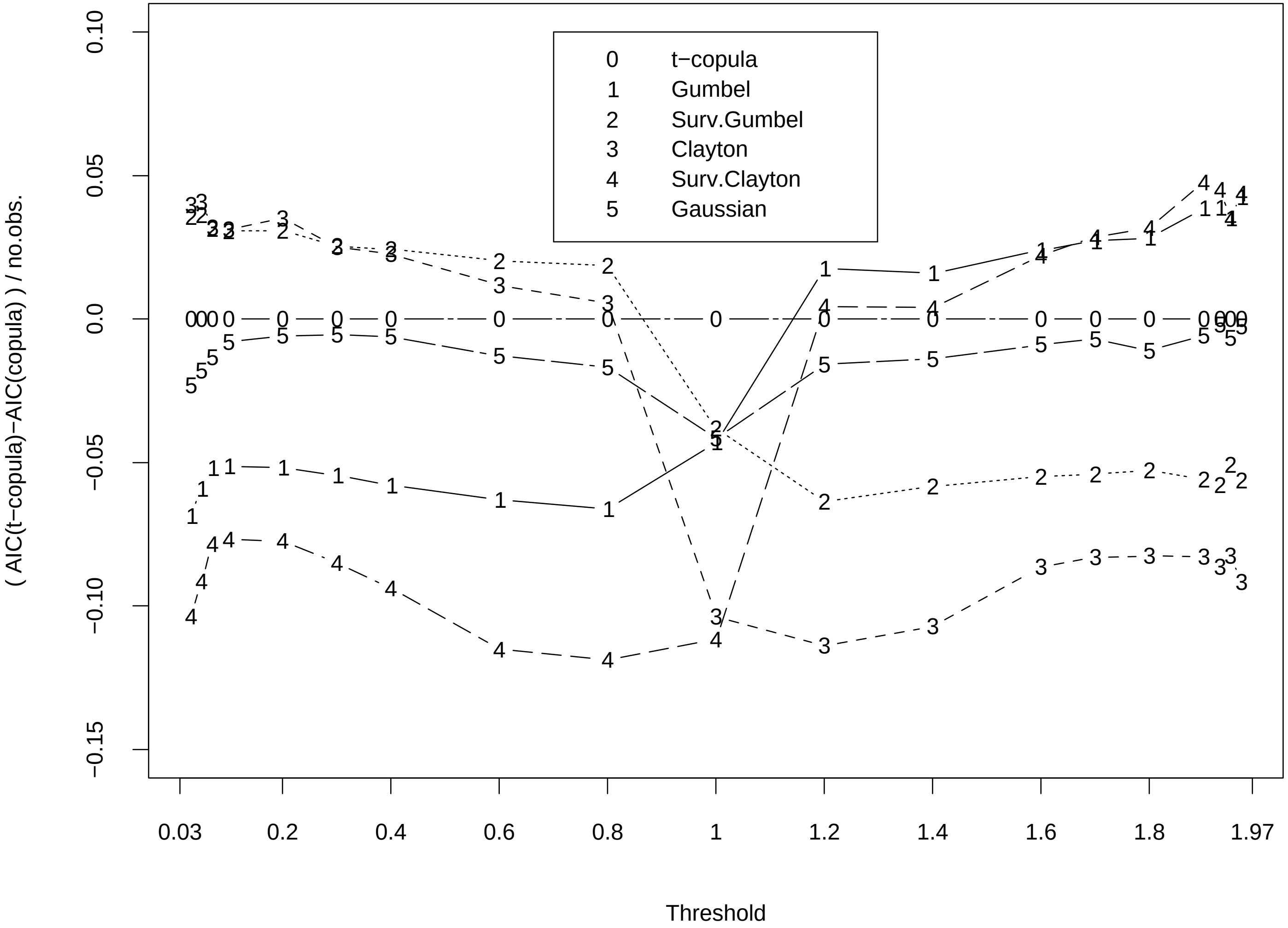
- For "sufficiently regular" Archimedean copulas (Juri and Wüthrich (2002)):

$$\lim_{t \to 0+} C_t(u,v) = C_\alpha^{Clayton}(u,v)$$

BIVARIATE EXCESSES FOR DIFFERENT THRESHOLDS



COPULA FITTING OF MULTIVARIATE EXCESSES



MULTIVARIATE (MATRIX-DIAGONAL) GARCH

d–dimensional matrix–diagonal GARCH process with an AR component:

$$\begin{aligned}
\mathbf{X}_t &= \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t \\
\boldsymbol{\mu}_t &= \boldsymbol{\mu} + \sum_{i=1}^{p_1} M_i \left(\mathbf{X}_{t-i} - \boldsymbol{\mu}\right) \\
\boldsymbol{\epsilon}_t &= \Sigma_t^{1/2} \mathbf{Z}_t \\
\Sigma_t &= A_0 A_0^t + \sum_{i=1}^{p_2} (A_i A_i^t) \otimes (\boldsymbol{\epsilon}_{t-i} \boldsymbol{\epsilon}_{t-i}^t) + \sum_{j=1}^{q} (B_j B_j^t) \otimes \Sigma_{t-j}
\end{aligned}$$

where

A_i and B_j are lower triangular $d \times d$ matrices

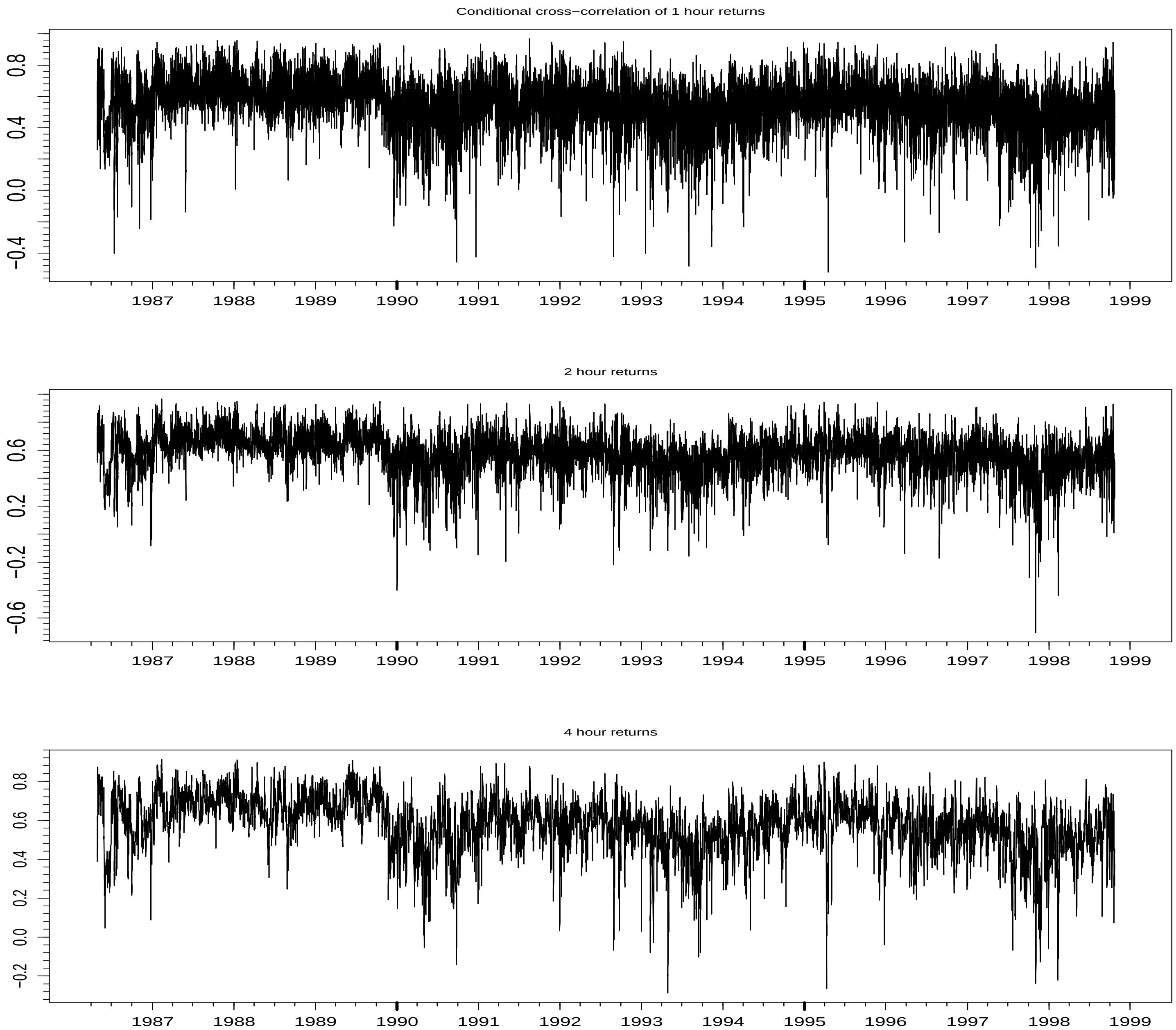
M_i is a full matrix in $\mathbb{R}^{d \times d}$

$(\mathbf{Z}_t)_{t \in \mathbb{Z}}$ is an iid vector sequence with zero mean vector and unit variances

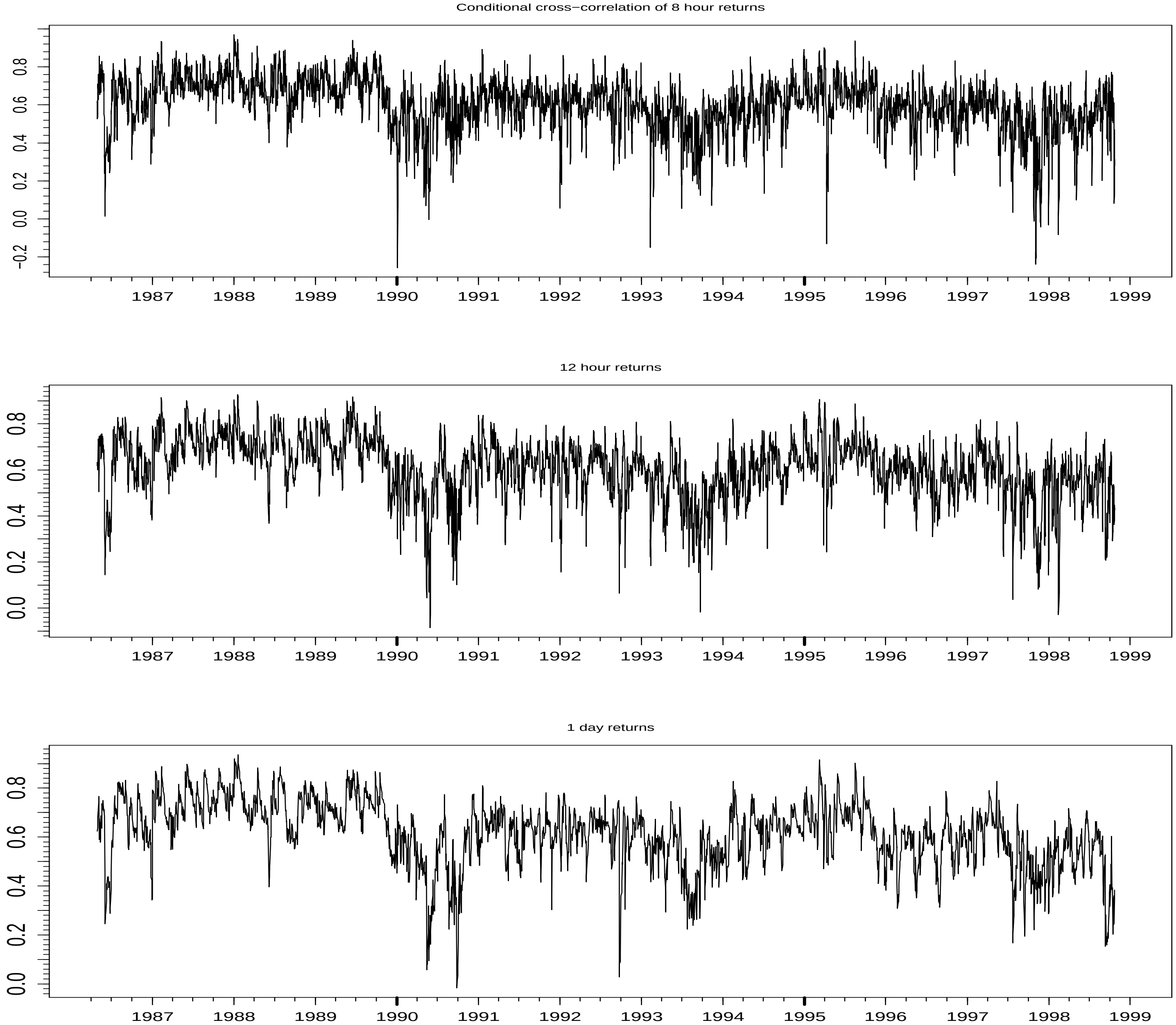
Σ_t is the conditional covariance matrix of the vector $\boldsymbol{\epsilon}_t$

$\otimes$ is the Hadamard product

CONDITIONAL CROSS-CORRELATION ESTIMATED BY MATRIX-DIAGONAL GARCH



CONDITIONAL CROSS-CORRELATION ESTIMATED BY MATRIX-DIAGONAL GARCH



TIME-VARYING COPULAS ACROSS TIME SCALES

- Multivariate GARCH model with time-varying copula:

$$
\begin{aligned}
\mathbf{X}_t &= \mathbf{c} + \boldsymbol{\epsilon}_t \\
\boldsymbol{\epsilon}_t &= \boldsymbol{\sigma}_t \mathbf{Z}_t \\
\boldsymbol{\sigma}_t^2 &= A_0 + \sum_{i=1}^{p} A_i \otimes (\boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}_{t-i}^t) + \sum_{j=1}^{q} B_j \otimes \boldsymbol{\sigma}_{t-j}^2
\end{aligned}
$$

where A_i and B_j are diagonal $d \times d$ matrices

- Assume that $\mathbf{Z}_t$ has a bivariate t-copula with time dependent parameters

$$
\begin{aligned}
&\nu_t = \nu \qquad \text{for all } t, \\
&\rho_t = h^{-1}(r_0 + r_1 z_{1,t-1} z_{2,t-1} + s_1 h(\rho_{t-1})),
\end{aligned}
$$

where $h(\cdot)$ is Fisher's transformation for the correlation

$$
h(\rho) = \log\left(\frac{1+\rho}{1-\rho}\right).
$$

Dynamic copula modelling

- Standardised residual return series obtained from the univariate filtering:

$$\{(\widehat{z}_{1,t}, \widehat{z}_{2,t}) : t = 0, \ldots, n\}$$

- Standardized residual returns mapped into $[0, 1]^2$:

$$\left\{ \left(t_{\widehat{\nu}_1} \left(\sqrt{\frac{\widehat{\nu}_1}{\widehat{\nu}_1 - 2}}\, \widehat{z}_{1,t} \right), t_{\widehat{\nu}_2} \left(\sqrt{\frac{\widehat{\nu}_2}{\widehat{\nu}_2 - 2}}\, \widehat{z}_{2,t} \right) \right), t = 1, \ldots, n \right\}.$$

Dynamic copula results

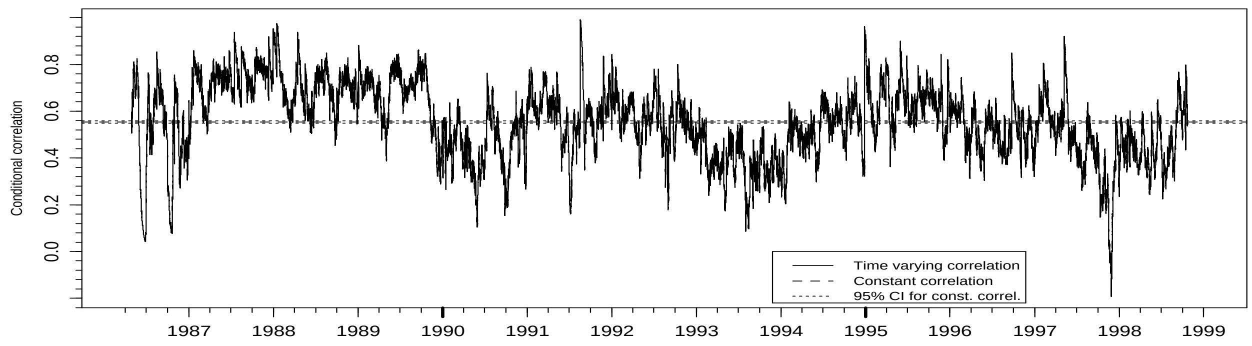
Time frequency	Parameter Estimates ($\widehat{s.e.}$) non–dynamic		dynamic	
1 hour	$\widehat{\nu}$	4.935 (0.108)	$\widehat{\nu}$	6.330 (0.167)
	$\widehat{\rho}$	0.558 (0.002)	$\widehat{r}_0$	0.0005 (0.0002)
			$\widehat{r}_1$	0.0193 (0.0010)
			$\widehat{s}_1$	0.9921 (0.0005)
	AIC	-31517.70	AIC	-34488.72
2 hours	$\widehat{\nu}$	4.822 (0.147)	$\widehat{\nu}$	6.203 (0.230)
	$\widehat{\rho}$	0.580 (0.003)	$\widehat{r}_0$	-0.0004 (0.0002)
			$\widehat{r}_1$	0.0128 (0.0009)
			$\widehat{s}_1$	0.9952 (0.0004)
	AIC	-17192.73	AIC	-19349.29
4 hours	$\widehat{\nu}$	4.669 (0.195)	$\widehat{\nu}$	6.072 (0.313)
	$\widehat{\rho}$	0.592 (0.005)	$\widehat{r}_0$	-0.0008 (0.0002)
			$\widehat{r}_1$	0.0147 (0.0011)
			$\widehat{s}_1$	0.9947 (0.0004)
	AIC	-9085.848	AIC	-10262.23

Dynamic copula results

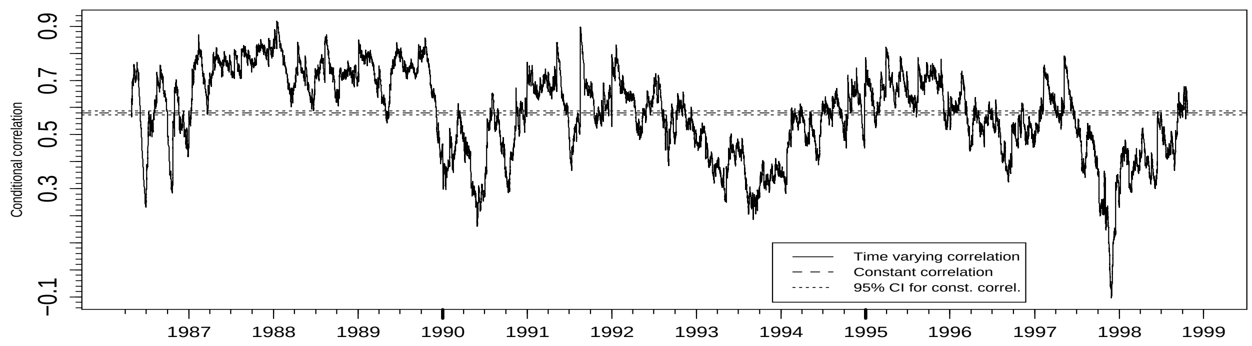
Time frequency	Parameter Estimates ($\widehat{s.e.}$) non–dynamic		dynamic	
8 hours	$\widehat{\nu}$	5.296 (0.339)	$\widehat{\nu}$	7.206 (0.584)
	$\widehat{\rho}$	0.612 (0.006)	$\widehat{r}_0$	0.0005 (0.0005)
			$\widehat{r}_1$	0.0173 (0.0014)
			$\widehat{s}_1$	0.9927 (0.0006)
	AIC	-4813.6	AIC	-5456.312
12 hours	$\widehat{\nu}$	5.830 (0.499)	$\widehat{\nu}$	8.053 (0.884)
	$\widehat{\rho}$	0.620 (0.008)	$\widehat{r}_0$	0.0002 (0.0008)
			$\widehat{r}_1$	-0.0249 (0.0023)
			$\widehat{s}_1$	0.9901 (0.0010)
	AIC	-3299.16	AIC	-3744.28
1 day	$\widehat{\nu}$	5.945 (0.758)	$\widehat{\nu}$	8.573 (1.455)
	$\widehat{\rho}$	0.619 (0.011)	$\widehat{r}_0$	-0.0023 (0.0017)
			$\widehat{r}_1$	-0.0343 (0.0041)
			$\widehat{s}_1$	0.9846 (0.0021)
	AIC	-1644.549	AIC	-1881.760

Time–varying cross–correlations estimated by the time–varying copula–based model

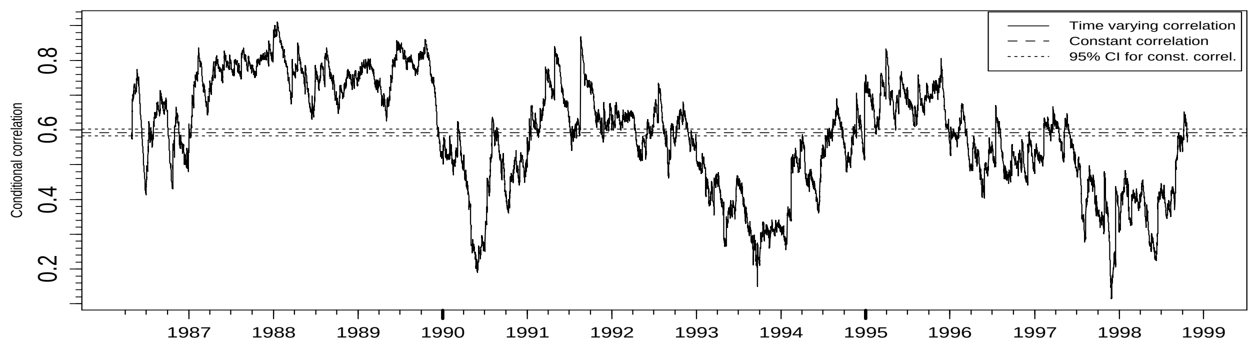
Conditional cross–correlation of 1 hour returns



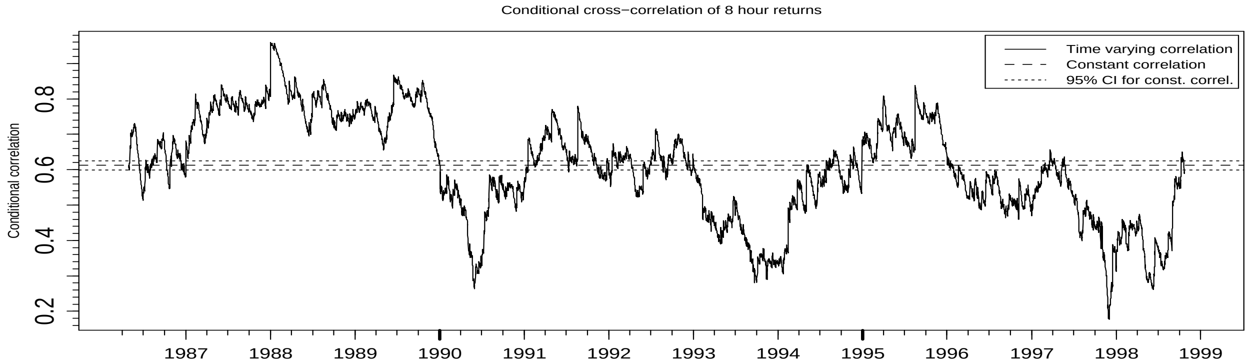
Conditional cross–correlation of 2 hour returns

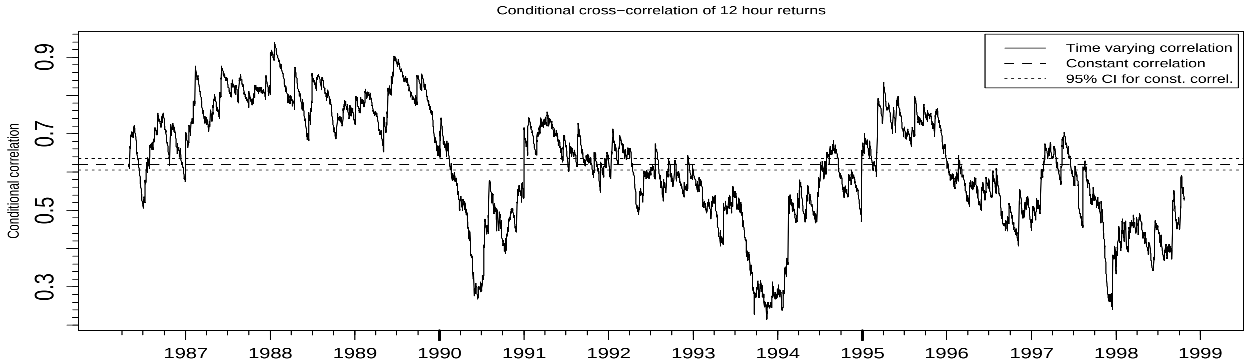


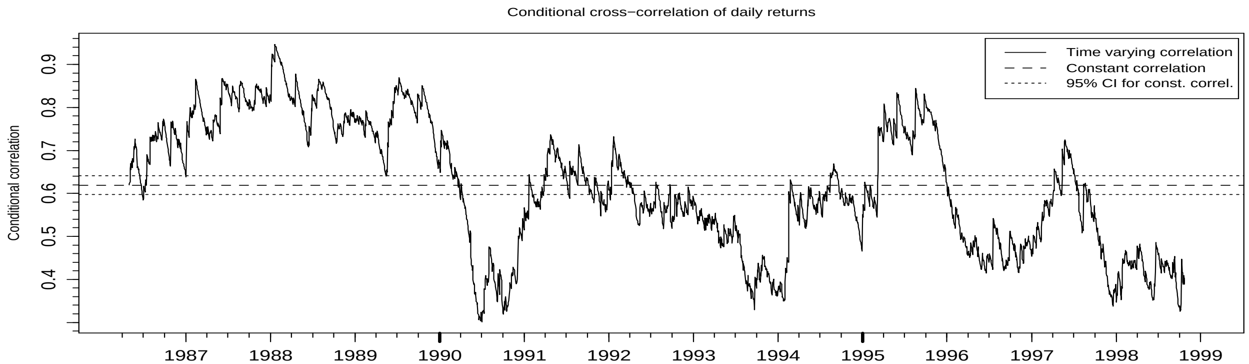
Conditional cross–correlation of 4 hour returns



Time–varying cross–correlations estimated by the time–varying copula–based model







THE GENERAL CHANGE–POINT PROBLEM

$\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ independent, from

$$C(\mathbf{x}; \boldsymbol{\theta}_1, \boldsymbol{\eta}_1), \ldots, C(\mathbf{x}; \boldsymbol{\theta}_n, \boldsymbol{\eta}_n)$$

$$H_0 : \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \ldots = \boldsymbol{\theta}_n \qquad \text{and} \qquad \boldsymbol{\eta}_1 = \boldsymbol{\eta}_2 = \ldots = \boldsymbol{\eta}_n$$

$$H_A : \boldsymbol{\theta}_1 = \ldots = \boldsymbol{\theta}_{k^*} \neq \boldsymbol{\theta}_{k^*+1} = \ldots = \boldsymbol{\theta}_n \quad \text{and} \quad \boldsymbol{\eta}_1 = \boldsymbol{\eta}_2 = \ldots = \boldsymbol{\eta}_n$$

Likelihood ratio test

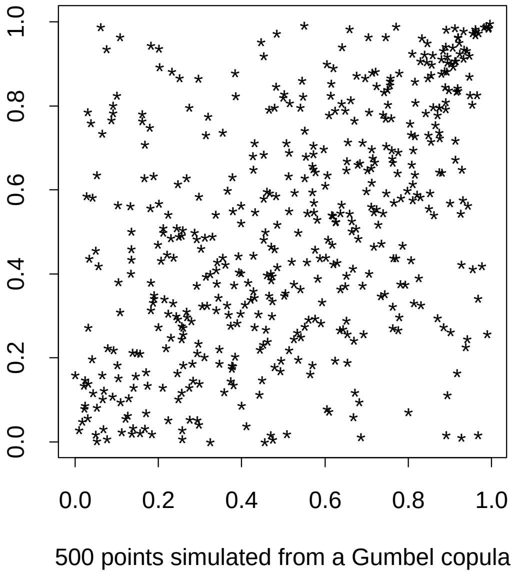
$$Z_n = \max_{1 \le k < n} (-2 \log(\Lambda_k))$$

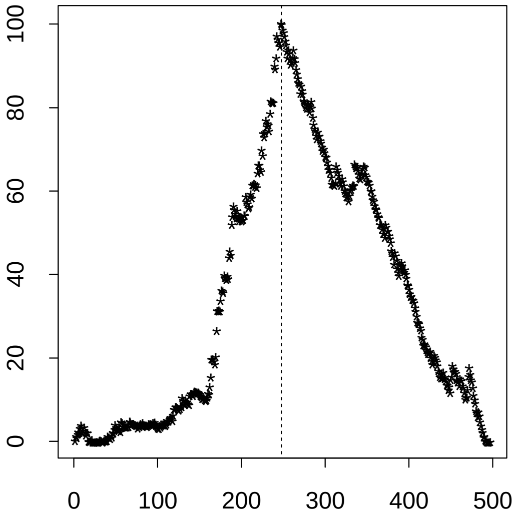
with

$$\Lambda_k = \frac{\sup\limits_{(\boldsymbol{\theta},\boldsymbol{\eta}) \in \Theta^{(1)} \times \Theta^{(2)}} \prod\limits_{1 \le i \le n} f(\mathbf{X}_i; \boldsymbol{\theta}, \boldsymbol{\eta})}{\sup\limits_{(\boldsymbol{\theta},\boldsymbol{\theta}',\boldsymbol{\eta}) \in \Theta^{(1)} \times \Theta^{(1)} \times \Theta^{(2)}} \prod\limits_{1 \le i \le k} f(\mathbf{X}_i; \boldsymbol{\theta}, \boldsymbol{\eta}) \prod\limits_{k < i \le n} f(\mathbf{X}_i; \boldsymbol{\theta}', \boldsymbol{\eta})}$$

CHANGE-POINT DETECTION

A SIMULATED EXAMPLE





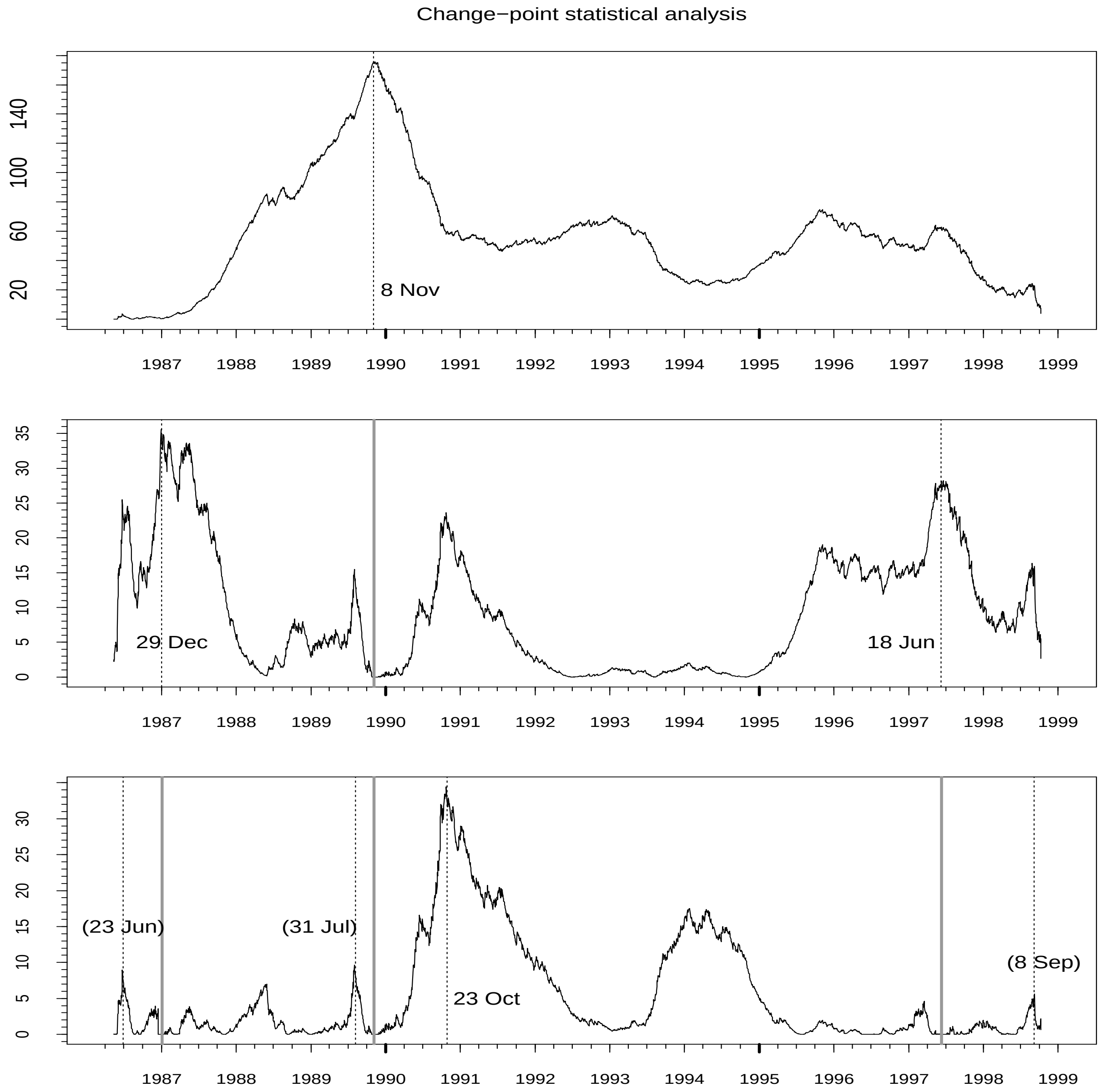
SEVERAL PROBLEMS

- The power of the test
- Boundary changes
- Size of the change
- Confidence intervals

Change-point analysis for USD/DEM and USD/JPY spot rate residuals

$z_{n\,obs}^{1/2}$	n	$P\left(Z_n^{1/2} > z_{n\,obs}^{1/2}\right)$	$H_0(0.95)$	Time of change
13.26	3 259	0	reject	8 Nov. 1989
5.96	923	0.0000004	"	29 Dec. 1986
5.31	2 336	0.0000143	"	18 June 1997
2.99	176	0.0689621	not rej.	(23 June 1986)
3.10	747	0.0709747	"	(31 July 1989)
5.86	1 985	0.0000007	reject	23 Oct. 1990
2.36	351	0.3380491	not rej.	(8 Sep. 1998)
2.78	1 736	0.1873493	"	(21 Oct. 1996)
2.86	249	0.1061709	"	(21 March 1990)

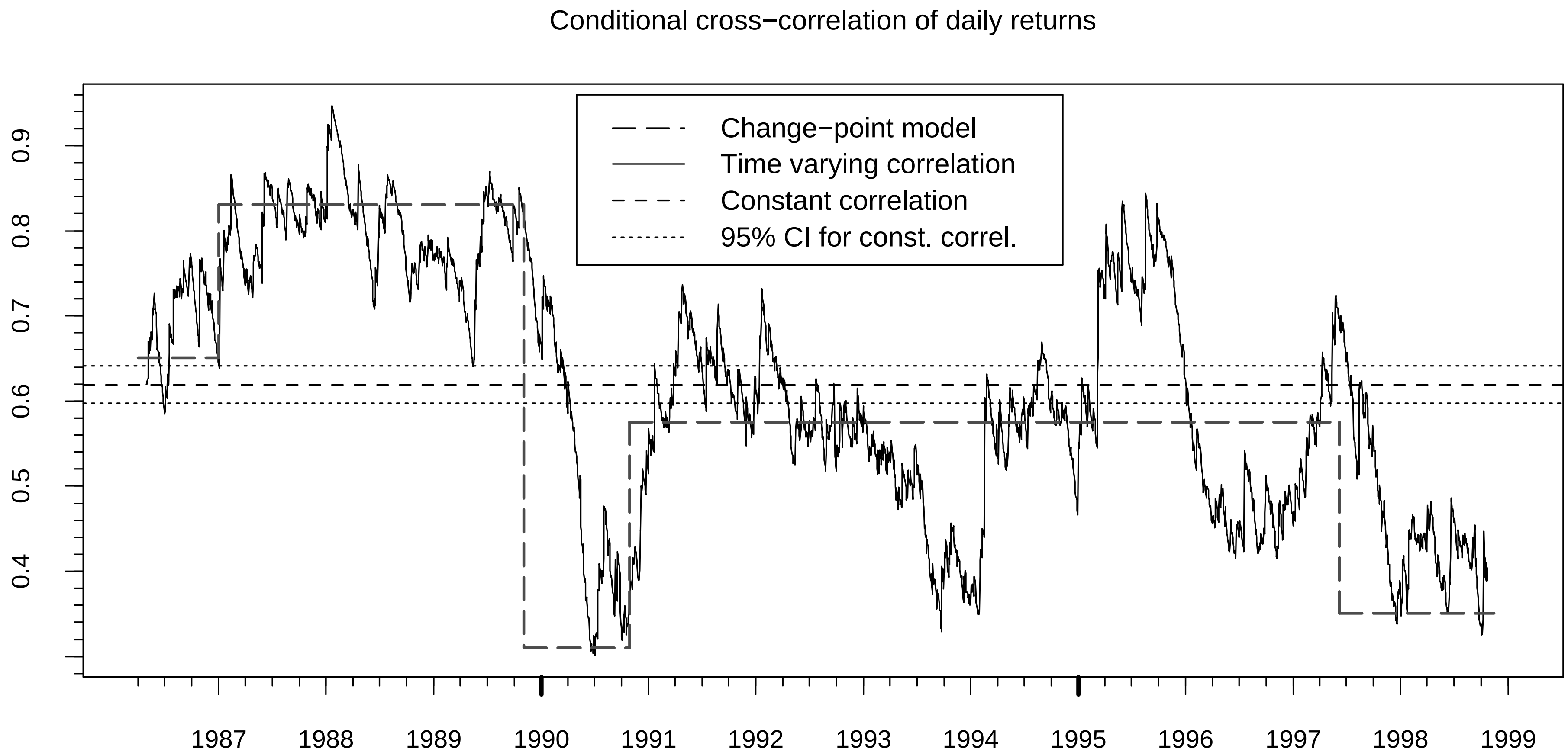
Change-point analysis for USD/DEM and USD/JPY spot rate residuals



Change-points detected on the USD/DEM and USD/JPY spot rate residuals

- December, 29 1986
- November, 8 1989: fall of the Berlin wall
- October, 23 1990: burst in the Japanese asset price bubble
- June, 18 1997: beginning of the Asia crisis

Estimated t-copula conditional correlation of daily returns on the FX USD/DEM and USD/JPY spot rates



Summary and further work

Summary:

- Deseasonalisation for multivariate tick-by-tick data
- Static versus dynamic copula fitting
- t-copula is overall fine
- Persistent tail-dependence
- Tested for ellipticality
- Change-point analysis

Further work:

- $d > 2$, other models
- Deseasonalisation, multivariate time change
- Tests for parameter constancy
- Link to multivariate EVT
- General Juri and Wüthrich (2002) result