Firm's Value-Based Credit Risk Models

Paul Embrechts

London School of Economics
Department of Accounting and Finance

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Basic Idea

Black and Scholes (1973) and Merton (1974):

Shares and bonds are derivatives on the firm's assets.

Limited liability gives shareholders the option to abandon the firm, to *put it to the bondholders*.

Bondholders have a short position in this put option.

Accounting identity:

$$\text{Assets} = \text{Equity} + \text{Liabilities}$$
$$V = E + \overline{B}$$

Balance Sheet

Assets		Liabilities	
Assets (Value of Firm)	V	Equity (Shares)	E
		Debt (Bonds)	$\overline{B}$
	V		$E + \overline{B}$

Note: A real-world balance sheet may not give all the correct numbers because of the accounting rules.

Specification of the Bonds

- Zero Bonds with maturity T and face value (total) K
- Price $\overline{B}(t,T)$ at time t per bond
- Firm is solvent at time T, *if*
$$V \geq K$$
- Payoff of the bond:

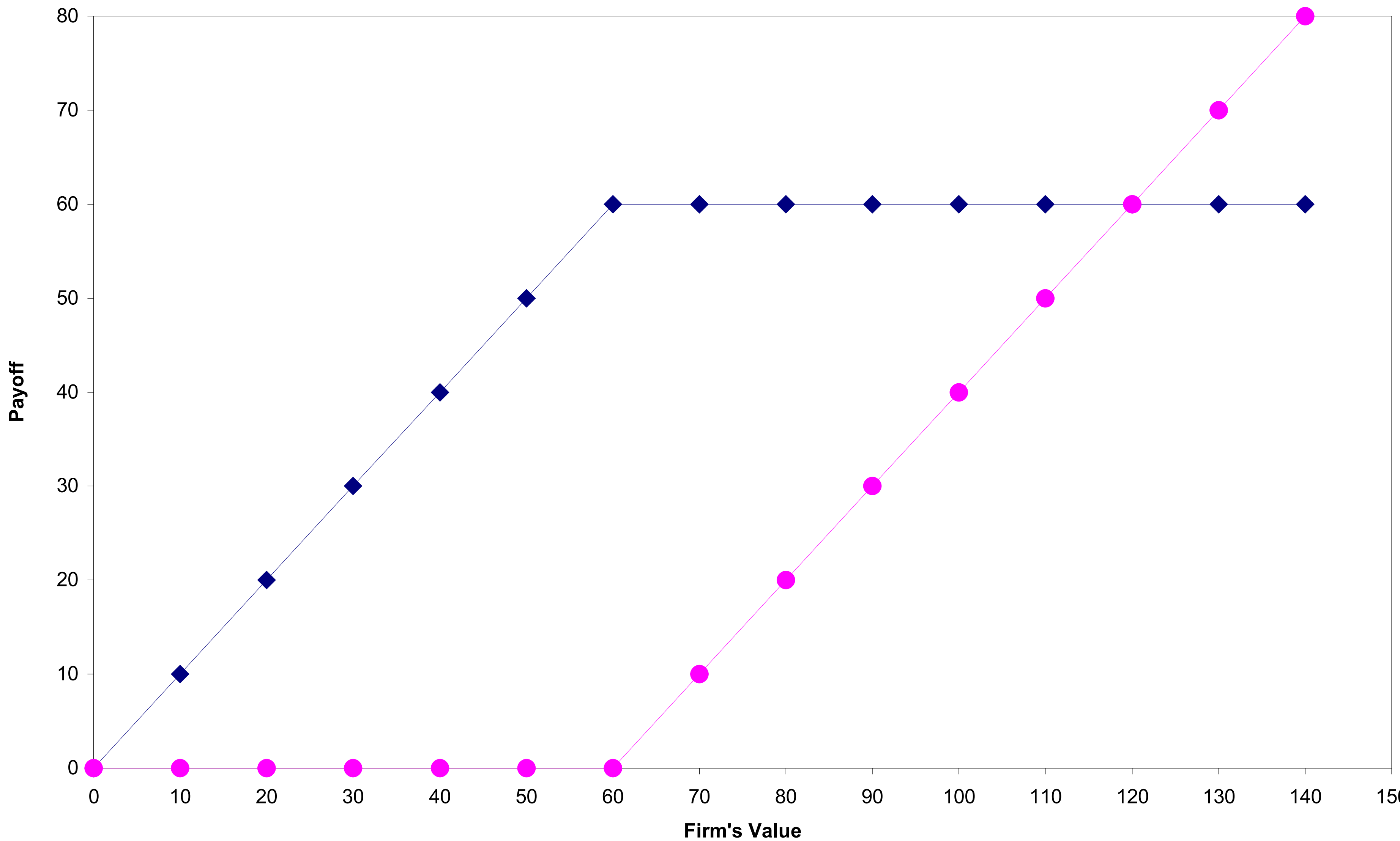
$$\overline{B}(T,T) = \begin{cases} K & \text{if solvent:} \quad V \geq K, \\ V & \text{if default:} \quad V < K \end{cases}$$

Specification of the Share

- Price E, no dividends
- Payoff as 'Residual Claim':
  Gets the remainder of the firm's value after paying off the debt (like Call Option):

$$E(T) = \begin{cases} V - K & \text{if solvent:} \quad V \geq K, \\ 0 & \text{if default:} \quad V < K \end{cases}$$

limited liability for $V < K$.


Payoff
Firm's Value
0
10
20
30
40
50
60
70
80
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Valuation with Option Pricing Theory

The value of the whole firm is *tradeable* and available as hedge instrument:

$$V = E + \overline{B}$$

Assume lognormal dynamics for the firm's value:

$$dV = \mu V\,dt + \sigma V\,dW$$

Then $\overline{B}$ and E must satisfy the Black-Scholes p.d.e.:

$$0 = \frac{\partial}{\partial t}\overline{B} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2}{\partial V^2}\overline{B} + rV\frac{\partial}{\partial V}\overline{B} - r\overline{B}.$$

($r =$ risk-free interest rate) -

Solution for the share price (Black-Scholes formula)

$$E(t,V) = VN(d_1) - Ke^{-r(T-t)}N(d_2)$$

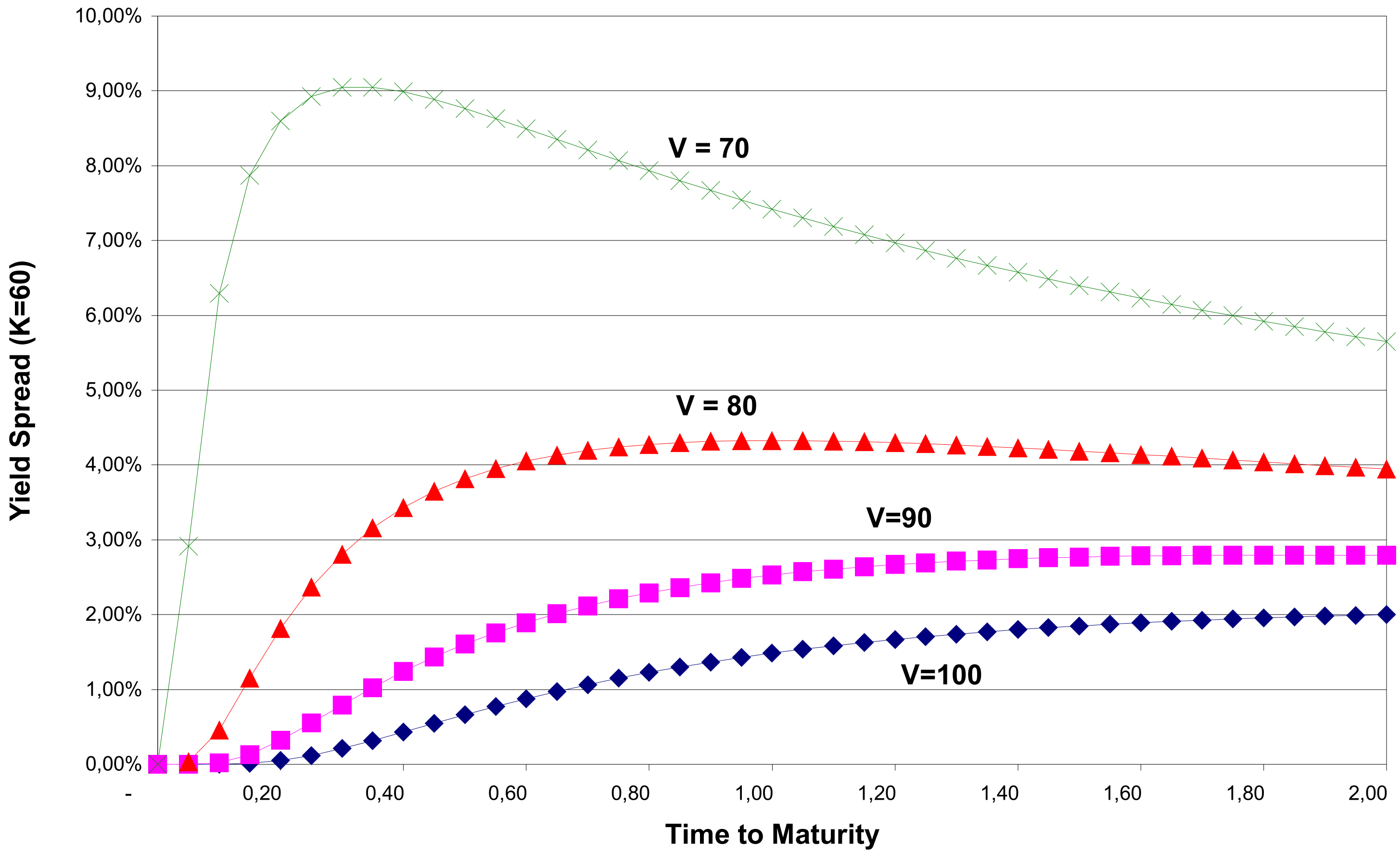
$$d_{1;2} = \frac{\ln(V/K) + (r \pm \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

Value of the bond:

$$\overline{B}(t,T) = V - E(t,V) = Ke^{-r(T-t)}N(d_2) + VN(-d_1)$$

Survival probability: $N(d_2)$
Expected recovery payoff: $VN(-d_1)$

Credit Spreads Implied by the BS/M Model



Hedging

Set up portfolio

$$\Pi = \overline{B}(V,t) + \Delta E(V,t)$$

by Itô's lemma:

$$\begin{aligned} d\Pi &= d\overline{B} + \Delta dE \\ &= \left(\frac{\partial \overline{B}}{\partial t} + \frac{1}{2}\frac{\partial^2 \overline{B}}{\partial V^2} + \Delta \frac{\partial E}{\partial t} + \frac{1}{2}\Delta \frac{\partial^2 E}{\partial V^2} \right) dt + \left(\frac{\partial \overline{B}}{\partial V} + \Delta \frac{\partial E}{\partial V} \right) dV. \end{aligned}$$

To eliminate the stochastic dV-term choose

$$\Delta = -\frac{\frac{\partial \overline{B}}{\partial V}}{\frac{\partial E}{\partial V}}.$$

Key Assumptions and Limitations

- Can observe (or imply) the firm's value V
  Critical! Possible solution see KMV-Approach

- The firm's value follows a lognormal random walk
  Can be relaxed at the cost of tractability (e.g. Zhou).

- Only zero-coupon debt
  Can be relaxed at the cost of tractability. (e.g. Geske)

- Default only at T: *Easy to relax. See Black-Cox and others.*

- Constant interest-rates r
  Very simple if independence between interest-rates and V. Otherwise see Briys-de Varenne and Longstaff-Schwartz.

Extensions:

- Default before maturity T of the bonds.
- Default-free interest rates r stochastic.
- Different Securities: Coupon-Bonds, Callable Bonds, Convertibles . . . .
- More details in capital structure: Multiple Claims Maturity, Seniority . . . .

Model can be extended in analogy to equity option models:

BSM model for default risk	$\leftrightarrow$	BSM model for equity options
Early default	$\leftrightarrow$	Barrier options
complicated capital structure	$\leftrightarrow$	exotic options with complicated payoff functions
callability, convertibiliy	$\leftrightarrow$	American or Bermudan options

Can use the same numerical methods in both approaches.

Default before maturity

Idea: (Black and Cox 1976)

Default when firm's value less than discounted value of liabilities.

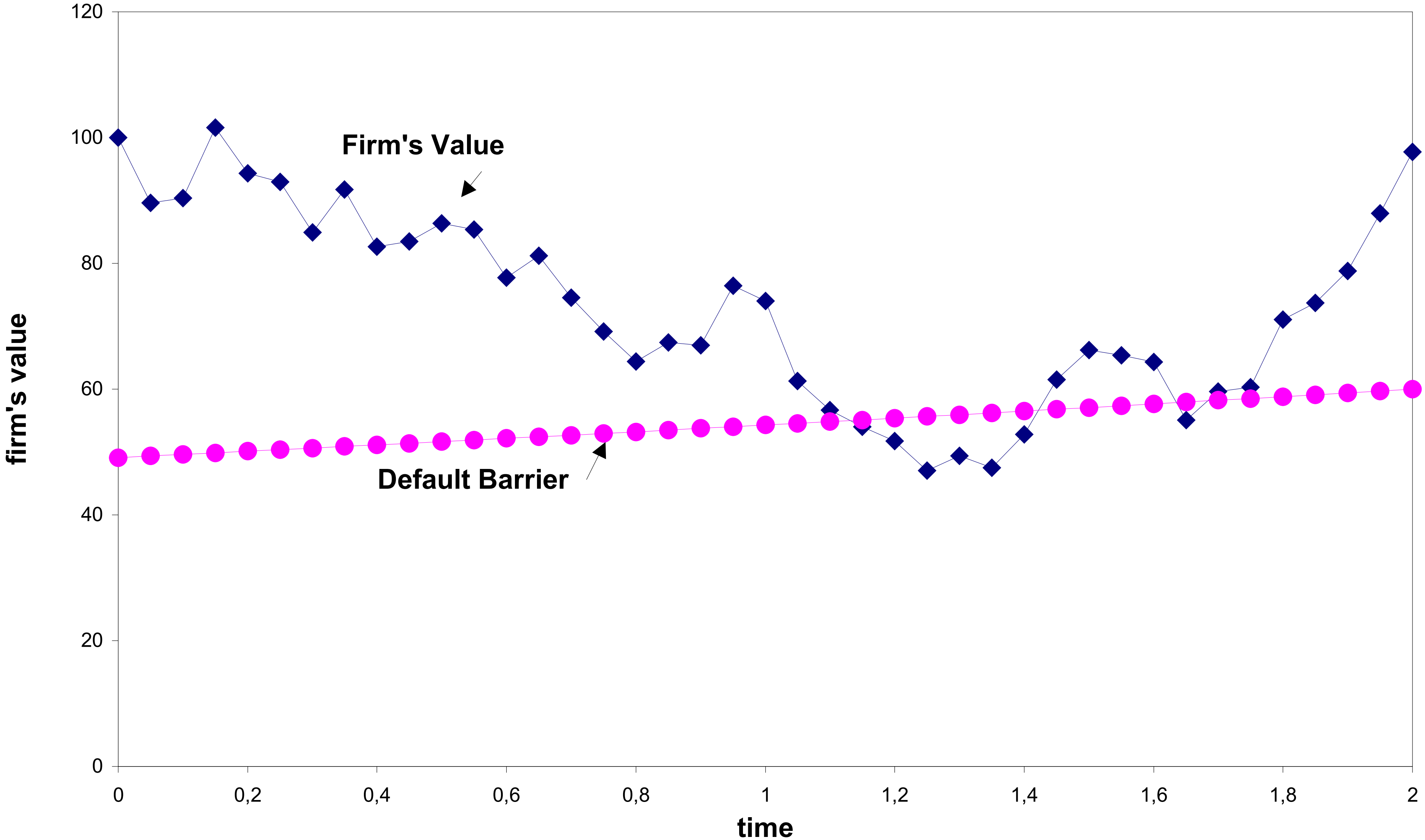
$$V(\tau) \leq \bar{S}B(\tau, T) = \bar{S}e^{-r(T-\tau)}$$

$\tau =$ time of default

Alternatively:
Constant Barrier: Default, as soon as

$$V(\tau) \leq \bar{S}$$

Default as soon as insufficient collateral.



firm's value
120
100
80
60
40
20
0
Firm's Value
Default Barrier
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
2
time

Bankruptcy Costs

Need bankruptcy costs to have loss in default. (Otherwise the firm's value would be sufficient for a full payoff of the bonds.)

Payoff of bonds:

K at maturity T, if there was no default previously: $\tau > T$

$cB(\tau, T)$ at τ, if default before maturity: $\tau \leq T$.

c as in recovery models for intensity models.

Strategic Default

Leland (1994), Leland / Toft (1996), Mella-Barral / Perraudin (1997)

Aim:
Qualitative study of corporate-finance effects

- endogenous capital structure and default barrier
- optimal capital structure
- effects of different bankruptcy regimes
- asset substitution effect / incentives to managers

Time-independent capital structure.

Leland (1994)

Value of firm's *assets*:

$$dV = rV\,dt + \sigma V\,dW$$

Debt:

- infinite maturity, (aggregate) coupon C, market price $\overline{B}$
- debt generates a *tax benefit* of $\tau C dt$
  (this is the only reason, why debt is issued at all)

Default:

- at barrier V_B of V
- debt recovers $(1-\alpha)V_B$
- bankruptcy cost αV_B (disadvantage of debt)
- equity can choose V_B

Debt Valuation

- Cdt : coupon payments to debt
- financed by issuance of new equity (dynamics of V remain unchanged)

O.d.e. for debt as a claim on V:

$$0 = \frac{1}{2}\sigma^2 V^2 \frac{\partial^2}{\partial V^2}\overline{B} + rV\frac{\partial}{\partial V}\overline{B} - r\overline{B} + C$$

with boundary conditions
$\overline{B} \to C/r$ as $V \to \infty$
$\overline{B} = (1-\alpha)V_B$ at $V = V_B$. -

Solution to the bond pricing equation

$$\overline{B} = (1 - p_B)\frac{C}{r} + p_B(1 - \alpha)V_B$$

where

$$p_B = \left(\frac{V}{V_B}\right)^{-X}, \qquad X = 2r/\sigma^2$$

is the present value of receiving 1 at default.

Then:

$p_B(1 - \alpha)V_B =$ present value of bankruptcy costs
$(1 - p_B)\tau C =$ present value of tax benefits

The Value of the Firm

$$\begin{aligned}[\text{Market Value of the Firm}] = & \ [\text{Value of Equity}] \\ & + [\text{Value of Debt}] \\ = & \ [\text{Value of Assets}] \\ & + [\text{Value of Tax Break}] \\ & - [\text{Value of Bankruptcy Costs}]\end{aligned}$$

Can solve for the value of equity

$$E = V - (1-\tau)C/r + ((1-\tau)C/r - V_B)p_B$$

Equity will choose V_B to maximize this.

Results

- Equity holders will *hold off* bankruptcy . . .
- . . . until the value of the *gamble for resurrection* is less than the cash needed to keep the firm alife.
- Bankruptcy at asset levels significantly *below* the outstanding debt level
- Empirical tests (Anderson / Sundaresan (1999)): at least as good as other models

Other Securities than Shares and Bonds

By solving the pricing equation with different boundary- and final conditions, more complicated securities can be priced.

Need to take care of:

- consistent development of firm's value
- payoff structures

- Coupon payments:
  Firm's value decreases after coupon is paid out

$$V(T_i+) = V(T_i-) - C$$

  Bond price decreases after coupon is paid out

$$\overline{B}_c(T_i+, V, r) = \overline{B}_c(T_i-, V - C, r) - C.$$

- Convertible Bonds: One bond for α shares

$$\overline{B}(t, V, r) \geq \alpha E(t, V, r),$$

or (with $V = E + \overline{B}$)

$$\overline{B}(t, V, r) \geq \frac{\alpha}{1+\alpha} V.$$

- Callable Bonds:
bonds can be called for B^*:

$$\overline{B}(t, V, r) \leq B^*$$

Empirics: Pricing Accuracy

Eom, Helwege, Huang (2000)

- One-shot pricing of a cross-section of corporate bonds with asset-based models.
- Substantial pricing errors in all models.
- Merton Model:
  - ⋆ generally underestimates spreads
  - ⋆ by a significant amount (80% of the spread)
  - ⋆ parameter variations do not help much
- Geske Model: similar to Merton model:
  severe underestimation of spreads
- Longstaff-Schwartz:

- ⋆ overestimates spreads severely for risky bonds
- ⋆ but could not raise spreads enough for good quality credits
- ⋆ still slightly better than Merton

- Leland-Toft:
  coupon size drives variation in predicted spreads

- all models have problems for short maturities or high quality

- Very poor predictive power in all cases: mean absolute errors in spreads are more than 70% of the true spread

KMV's Distance to Default

- **Default Point:** ('Asset value, at which the firm will default) between total liabilities and short-term liabilities' (implicitly assuming: Barrier Modell)

- **Distance to Default:** Summary Statistic for credit quality

$$[\text{Distance to Default}] = \frac{[\text{Market Value of Assets}] - [\text{Default Point}]}{[\text{Market Value of Assets}] \times [\text{Asset Volatility}]}$$

- **Expected Default Frequency** = Frequency, with which firms of the same distance to default have defaulted in history.
  Calibration to historical data, leaving the modelling framework.

Linking the Firm's Value Model to Market Variables

Model	**Market**
V	*unobservable*
K	total debt
σ_V	*unobservable*
r	*observable*
T	user choice
some **Outputs**	
E	market capitalisation
$\sigma_E = \frac{\partial E}{\partial V} \frac{V}{E} \sigma_V$	equity volatility

Can use market capitalisation and equity volatility to calibrate.

Advantages of Firm's Value Models

- ✔ Relationships between different securities of same issuer
- ✔ Convertible bonds
- ✔ Collateralized Loans
- ✔ default correlation between different issuers can be modelled realistically.
- ✔ Fundamental orientation
- ✔ well-suited for theoretical questions (corporate finance)

Disadvantages of Firm's Value Models

- ✘ observability of firm's value: calibration, fitting
- ✘ bonds are not inputs but outputs
  defaultable bonds are far from being fundamentals
- ✘ all data is rarely available
- ✘ souvereign issuers cannot be priced
- ✘ often complex and unflexible
- ✘ unrealistic short-term spreads

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