Topics in Quantitative Risk Management

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1. Basel II and its risk management consequences
2. An overview of credit risk models
3. Modelling extremes: use and limitations
4. Modelling dependence beyond linear correlation

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Lecture 1: Basel II and its risk management consequences.

Some history (1):

- Basel Committee of Banking Supervision was established by the Central-Bank Governors of the Group of Ten at the end of 1974.

- The Basel Committee does not possess any formal supranational supervising authority, and hence its conclusions do not have legal force.

- It formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements - statutory or otherwise - which are best suited to their own national systems.

Some history (2):

- 1988: the First Basel Accord on Banking Supervision (the Accord, Basel I) initiated an important step toward an international minimum capital standard.

Formula (ingredients):

- Risk-weighted amount (RWA)
  $= \sum(\text{ assets } \times w_a) + \sum(\text{ credit equivalents } \times w_{ce})$
  for some weights w_a, w_{ce} determined in the Accord
- Tier 1, 2, 3 capital defines Regulatory Capital (RC)
- Cooke Ratio: RC $\geq$ 8% RWA

(1988 Accord: only using Tier 1 and 2 with at least 50% in Tier 1)

Some history (3):

- Problems: granularity, netting, derivatives
  improve market risk management
- 1993: G-30 report on off-balance products, Value-at-Risk (VaR) is born: J.P. Morgan Weatherstone 4^{15} report
  RiskMetrics
- 1996: Amendment to Basel I
  Market risk for debt and equity positions

Ingredients:

- mark-to-market, netting
- on- and off-balance positions
- flexibility of methodology used for MR: internal (VaR) models

Some history (4):

- For instance for market risk:

$$\text{RC}(t) = \max\left(\text{VaR}^{10}_{99\%}(t-1), \frac{k}{60}\sum_{i=1}^{60}\text{VaR}^{10}_{99\%}(t-i)\right)$$

where $3 \leq k \leq 4$ depending on model quality (backtesting)
(more details: credit, no-action P & L, specific risk ...)

- Institutions should have a strong risk management group (!)
- Capital allocation, limit setting and verification, performance measurement (RARORAC, ...)
- Stress testing
- Cooke ratio also may include Tier 3 capital towards market risk

Some history (5):

- Between 1998 - 2000: Basel I + Amendment (also referred to as the BIS 1998 rules) are in place worldwide

Quantitative Risk Measurement and Management are born

- Some references:

    - www.bis.org
      (BIS = Bank of International Settlements in Basel)
    - M. Crouchy, D. Galai, R. Mark (2001) Risk Management. McGraw Hill.
    - Risk Magazine

Basel II (1):

- Crouchy et al. (p. 68): "The BIS 1998 rules are generally accepted to be flawed"
  - no portfolio effects, diversification
  - lack of granularity (loan to a corporate counterparty generates five times the amount of risk as does a loan to an OECD bank, regardless of their respective creditworthiness)
  - all corporate borrowers pose an equal credit risk (AA versus B-rated corporation)
  - handling of revolving credit agreements (under one year)
  - no allowance for netting
- As a consequence: misallocation of capital
  regulatory arbitrage

Basel II (2):

- Solution: use of internal credit VaR models
  - CreditMetrics (J.P. Morgan)
  - CreditRisk+ (CSFB)
  - Moody's / KMV

  (See Lecture 2 for more details)
- Consequently: A New Capital Adequacy and Credit Risk Modelling Framework, the 1999 Consultative Papers (= Basel II)
- Integrated Framework: Regulatory Capital, Economic Capital, Credit VaR, Operations VaR

Basel II (3):

- Key ingredients (Crouchy et al., pp. 71-89):
  - maintain "today's" total level of capital
  - level playing field across countries
  - comprehensive approach to risks: introduce as new classes interest rate risk and operational risk
  - focus on internationally active banks
- Three pillar framework:
  - Pillar I: minimum capital requirement (quantitative)
  - Pillar II: supervisory review process (qualitative)
  - Pillar III: market discipline (disclosure)

Basel II (4):

- How can Pillar I be achieved?

  - For credit risk: standardized approach
    internal ratings-based approach
    credit risk modelling

  - For operational risk: basic indicator approach
    standardized approach
    advanced measurement approach

Basel II (5):

- A general comment: the regulatory system has become extremely complicated,

"In one respect, the result of such an ambitious undertaking (Basel II) was probably predictable. The process has generated a product of vast complexity - putting to shame the US Internal Revenue Code, long the World's record holder for complexity. Thousands of pages of task force and working group papers, years in the making, have given rise to hundreds of pages of rules, guidelines, and standards saturated with arcane mathematical formulae. They're not written by or for bankers - or for that matter, by or for conventional bank examiners. They're written for mathematicians and economists - "quants"."

John D. Hawke, Jr. (March 3, 2003)
Comptroller of the Currency

Research consequences for mathematical finance and quantitative risk management:

- obvious
- credit risk methodology, credit derivatives
- portfolio modelling across and within risk types
- aggregation of risks (coherence, copulae, ...)
- stress testing (EVT)
- more dynamic models (beyond one-period RM)
- capital allocation
- new risk classes: e.g. operational risk
- RM beyond banking: insurance, general corporations ... Shiller (?)

Two examples:

- operational risk
- risk aggregation

References to related work:

www.math.ethz.ch/finance
/$\sim$embrechts
www.risklab.ch

Operational Risk (1):

- Definition:

  "The risk of losses resulting from inadequate or failed internal processes, people and systems or from external events"

- Examples:
  - Barings, £ 700, - Mio
  - Allied Irish, US$ 700, - Mio
  - Bank of New York, US$ 140, - Mio
  - USA total (2001): US$ 50, - Bio
  - CR (51 %), MR (23 %), OR (16 %), other (10 %)

Operational Risk (2):

- However not Business Risk !
- Recall Pillar I, II and III.
- Pillar I proposals:

  - BIA: $\text{RC(OR)} = \alpha \ \text{GI}$

  - SA: $\text{RC(OR)} = \sum_{i=1}^{8} \beta_i \ \text{GI}_i$

  - AMA: $\text{RC}_1\text{(OR)} = \sum_{i=1}^{8} \sum_{k=1}^{7} \gamma_{i,k} e_{i,k}$

    $\text{RC}_2\text{(OR)} = \sum_{i=1}^{8} \rho_{i,\alpha}$

- Recall: overall RC equal !

Operational Risk (3):

- A stylized model

  - data base: $\left\{X_k^{t,i} : t = 1, \ldots, T; i = 1, \ldots, s; k = 1, \ldots, N^{t,i}\right\}$

    where t (years), s (loss types), $N^{t,i}$ (total number of losses in year t for loss type i)

  - truncation and censoring

  - $L_t = \sum_{i=1}^{s} L_{t,i} \qquad , \ t = 1, \ldots, T$

    $L_{t,i} = \sum_{k=1}^{N^{t,i}} X_k^{t,i} \qquad , \ i = 1, \ldots, s$

Operational Risk (4):

- Model F_{L_t}, $F_{L_{t,i}}$

- Calculate risk measures (e.g. for F_{L_t}):

$$\text{OR-VaR}_{T+1}^{1-\alpha} = F_{L_{T+1}}^{\leftarrow}(1-\alpha),\ \alpha \text{ small}$$

$$\text{OR-CVaR}_{T+1}^{1-\alpha} = E(L_{T+1} \mid L_{T+1} > \text{OR-VaR}_{T+1}^{1-\alpha})$$

- Discussion (via stylized facts)

Risk Aggregation (1):

- $X_1, \ldots, X_d$ one-period risks

  - credit positions of d obligors
  - different risk types
  - operational risks for different business lines
  - insurance losses (multi-line)

- $\Psi(\mathbf{X})$ a (risky) financial position ($\mathbf{X} = (X_1, \ldots, X_d)'$)

  - $S_d = \sum_{i=1}^{d} X_i, \quad M_d = \max(X_1, \ldots, X_d)$

  - $M_d I_{\{S_d > q_\alpha\}}$

Risk Aggregation (2):

- $\sum_{i=1}^{d}(X_i - k_i)^+$ (excess-loss, combined Europeans)
- $(\sum_{i=1}^{d} X_i - k)^+$ (stop-loss, Asian)
- $I_{\{M_d > l\}}$ (digital)
- general basket options, credit derivatives

\- a risk measure ρ

- VaR_α, CVaR_α
- $E(\cdot)^k$

Risk Aggregation (3):

- F (the distribution function)

The problem

- Given the distribution functions $F_1, \ldots, F_d$ of the marginal risks $(X_1, \ldots, X_d)$ (model, empirical, ...)
- and some ideas of dependence between $X_1, \ldots, X_d$
- estimate $\rho(\Psi(\mathbf{X}))$

Clearly this problem is in general not well-defined, hence find bounds:

$$\rho_L(\Psi(\mathbf{X})) \leq \rho(\Psi(\mathbf{X})) \leq \rho_U(\Psi(\mathbf{X}))$$

Risk Aggregation (4):

- A typical example:

  - $X_1, \ldots, X_d \sim N(0,1)$
  - no information on dependence
  - $\Psi(\mathbf{X}) = S_d = X_1 + \cdots + X_d$
  - $\rho = \mathrm{VaR}_\alpha$

- Calculate:

$$\rho_U(\Psi(\mathbf{X})) = \sup_{F \text{ df of } \mathbf{X}} \mathrm{VaR}_\alpha(S_d)$$

Risk Aggregation (5):

- A (surprising?) fact for $d = 2$:
    - there exist infinitely many joint dfs F on $\mathbf{X}$ so that $\mathrm{VaR}_\alpha(X_1 + X_2) > \mathrm{VaR}_\alpha(X_1) + \mathrm{VaR}_\alpha(X_2)$
    - the "diversification gap" $\mathrm{VaR}_\alpha(X_1 + X_2) - (\mathrm{VaR}_\alpha(X_1) + \mathrm{VaR}_\alpha(X_2))$ can be quantified (numerically)
    - for F (joint df of $\mathbf{X}$) comonotone, VaR_α is additive and yields maximal correlation

- Many more results hold (Embrechts-Hoeing-Juri, Fin. and Stoch. (2), 2003)

Conclusion:

- Importance of historical perspective on RM
- Evolution Basel I $\longrightarrow$ Basel II
- Highly complex
- Interesting (finance-) mathematical problems
- Need for practicality
- It is all happening now: Basel II $\sim$ 2005/6
- More and more holistic approach, IRM

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Lecture 2: An overview of credit risk models.

Recall from Lecture 1:

- The history from Basel I to Basel II
- The three Pillar concept in Basel II
- More granularity for credit risk management within Basel II
- For this lecture: focus on Pillar I advanced internal models

Organization of presentation:

- Slides “Credit Risk Models: An Overview” (P. Embrechts, R. Frey, A. McNeil)
- Slides “Modeling Distributions: Extreme Value Theory and Copulae” (Paul Embrechts)
- Slides “Ruin, Operational Risk and How Fast Stochastic Processes Mix” (Paul Embrechts)

These slides cover also Lectures 3 and 4.