EVT and COPULAE: Essential Riskmanagement Tools or Just Fads?

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Contents

A. Introduction

Three statements:

- The Bell curve is wrong? (Private Banker)

- Did you use EVT as a tool in relationship to operational risk? (Moody's Analytical Framework for Operational Risk Management of Banks)

- Basel II for credit risk only uses the Gaussian copula (Dilip Madan)

Early warning and tools:

- 1997: P. Embrechts, C. Klüppelberg, T. Mikosch. *Modelling Extremal Events for Insurance and Finance*. Springer.

- 1988: P. Embrechts, A. McNeil, D. Straumann. Correlation and dependence in risk management: properties and pitfalls. (dependence = copula)

- 2004: P. Embrechts, R. Frey, A. McNeil. *Stochastic Methods for Quantitative Risk Management*. (Book project)

B. Modeling Distributions: Extreme Value Theory and Copulae

Some statements on extremes and correlation

- “A natural consequence of the existence of a lender of last resort is that there will be some sort of allocation of burden of risk of extreme outcomes. Thus, central banks are led to provide what essentially amounts to **catastrophic insurance coverage**. . . From the point of view of the risk manager, **inappropriate use of the normal distribution can lead to an understatement of risk**, which must be balanced against the significant advantage of simplification. From the central bank’s corner, the consequences are even more serious because we often need to concentrate on the left tail of the distribution in formulating lender-of-last-resort policies. Improving the characterization of the distribution of extreme values is of paramount importance”

  (Alan Greenspan, Joint Central Bank Research Conference, 1995)

Some statements on extremes and correlation

- “Extreme, synchronized rises and falls in financial markets occur infrequently but they do occur. The problem with the models is that they did not assign a high enough chance of occurrence to the scenario in which **many things go wrong at the same time** - the “perfect storm” scenario”

  (Business Week, September 1998)

- “Regulators have criticised LTCM and banks for not “stress-testing” risk models against extreme market movements. . . The markets have been through the financial equivalent of several Hurricane Andrews hitting Florida all at once. Is the appropriate response to accept that it was mere bad luck to run into such a **rare event** - or to get new forecasting models that assume more storms in the future?”

  (The Economist, October 1998, after the LTCM rescue)

Some statements on extremes and correlation

- "... The trading floor is quiet. But this masks their attempt at picking up the pieces with a new fund, JWM Partners. Now, Mr. Meriwether is preaching new gospel: World financial markets are bound to hit **extreme turbulences** again... Mr. Meriwether's crew, once bitten, also is betting on more liquid securities: "With globalisation increasing, you'll see more crises," he says. "Our whole focus is on the extremes now - **what's the worst that can happen to you in any situation** - because we never want to go through that again."

  (John Meriwether, The Wall Street Journal, 21/8/2000)

Some statements on extremes and correlation

- "Over the last number of years, regulators have encouraged financial entities to use portfolio theory to produce dynamic measures of risk. VaR, the product of portfolio theory, is used for short-run day-to-day profit and loss exposures. Now is the time to encourage the BIS and other regulatory bodies to support studies on stress test and concentration methodologies. Planning for crises is more important than VaR analysis. And such **new methodologies** are the correct response to recent crises in the financial industry"

  (Myron Scholes, American Economic Review, May 2000)

- "Someone told me that the bell curve is wrong"

  (Banker, private communication, 1999)

Correlation confusion: in words

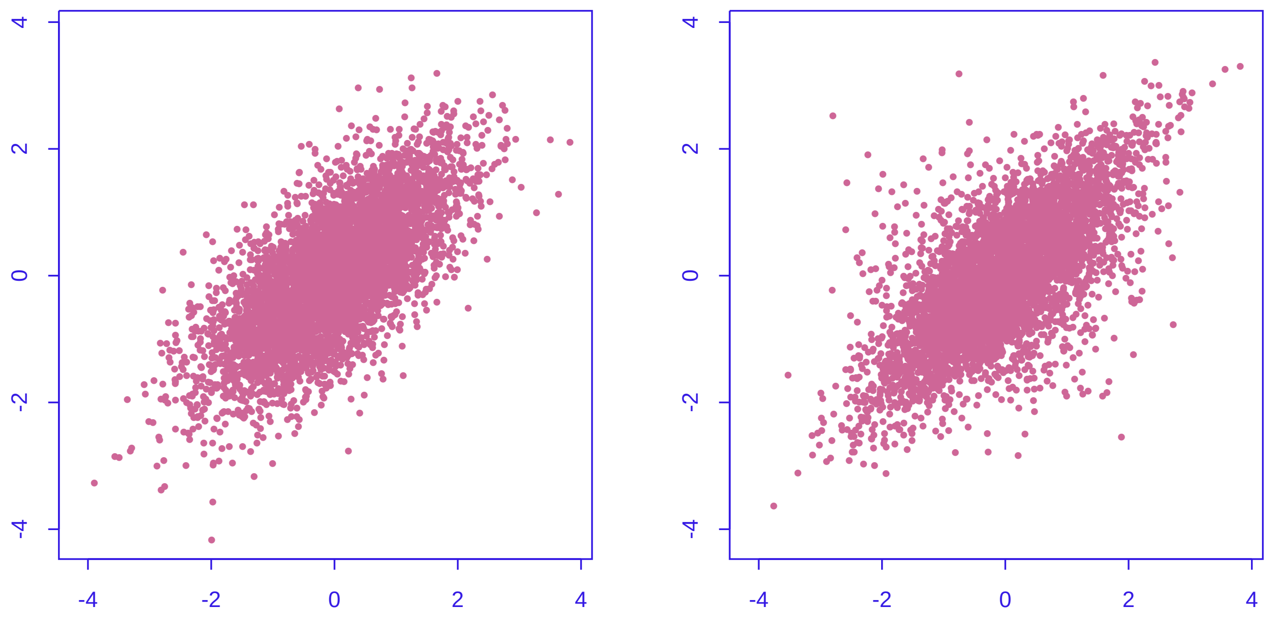
- "Among nine big economies, stock market correlations have averaged around 0.5 since the 1960s. In other words, for every 1 per cent rise (or fall) in, say, American share prices, share prices in the other markets will typically rise (fall) by 0.5 per cent"

  (The Economist, 8th November 1997)

- "A correlation of 0.5 does not indicate that a return from stock-market A will be 50% of stockmarket B's return, or vice-versa. . . A correlation of 0.5 shows that 50% of the time the return of stockmarket A will be positively correlated with the return of stockmarket B, and 50% of the time it will not"

  (The Economist (letter), 22nd November 1997)

Correlation confusion: in a picture



Messages from the methodological frontier

- **Static case** (time fixed)
  - ✤ $d = 1$: Classical Extreme Value Theory (EVT)
    Peaks-over-threshold method (POT)
  - ✤ $d \geq 2$: Multivariate Extreme Value Theory (MEVT)
    Copulae

- **Dynamic case**
  - ✤ Extremes of stochastic processes in $d > 1$, only in rather special cases (Gaussian, Markov, ...)
  - ✤ Non-BSM models: Lévy driven price processes, incompleteness

Some examples

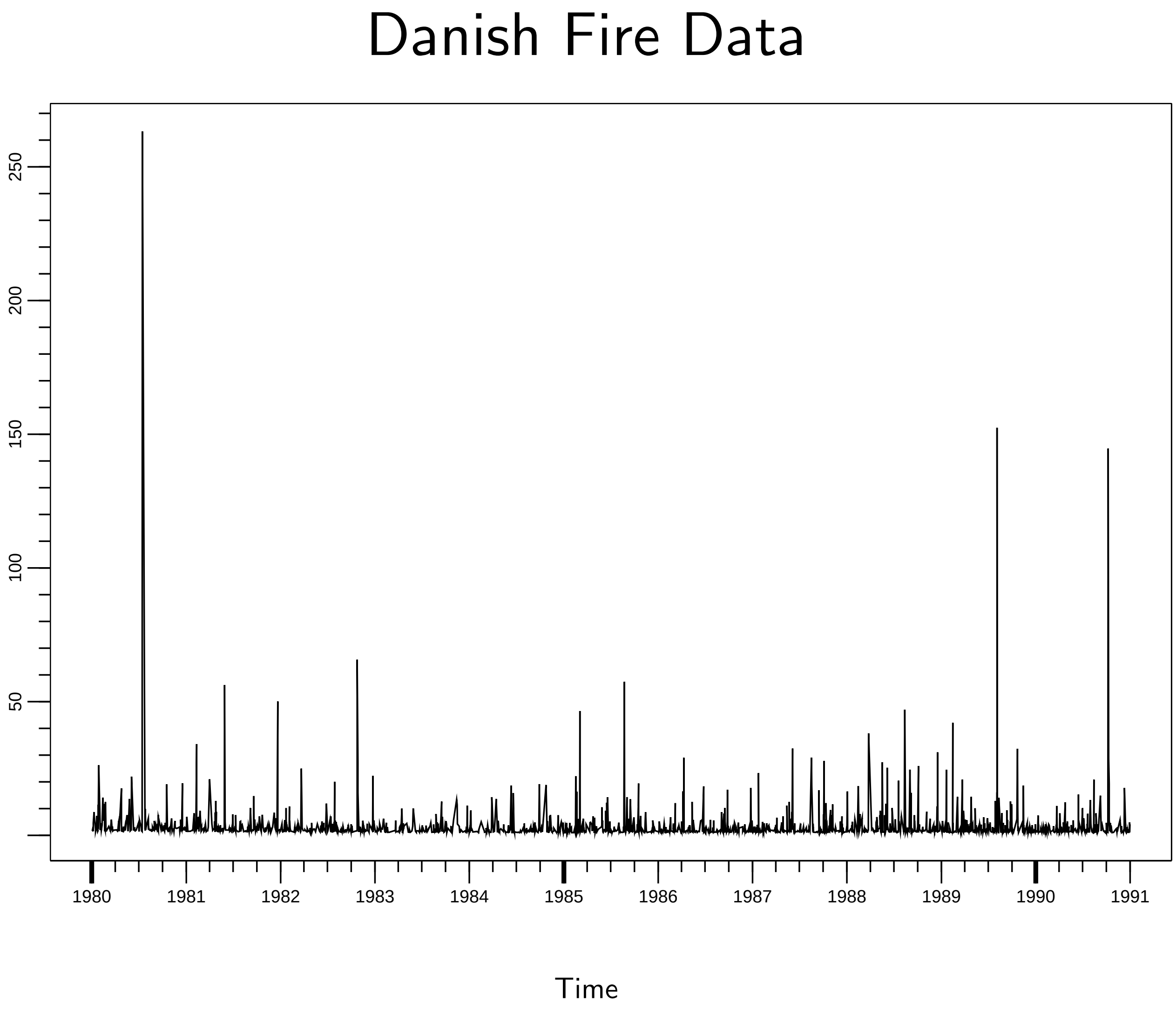
❶ EVT - POT

❷ MEVT

❸ Copulae

and their risk management consequences

Example 1: EVT - POT



Example 1: EVT - POT

- **Notation:** $M_n = \max(X_1, \ldots, X_n)$, $X_i \sim F$,

$$F_u(x) = \mathbb{P}(X - u \leq x | X > u)$$

- **Fisher-Tippet theorem:** Let (X_n) be a sequence of iid rvs. If there exist norming constants $c_n > 0$, $d_n \in \mathbb{R}$ and some non-degenerate df H such that $c_n^{-1}(M_n - d_n) \xrightarrow{d} H$, then H belongs to the Fréchet $(\xi > 0)$, Weibull $(\xi < 0)$ or Gumbel $(\xi = 0)$ type of distributions

- **Balkema-de Haan-Pickands result:** For every $\xi \in \mathbb{R}$, $F \in$ MDA(H_ξ) if and only if

$$\lim_{u \uparrow x_F} \sup_{0 < x < x_F - u} |F_u(x) - G_{\xi,\beta(u)}(x)| = 0$$

  for some positive function β and where $G_{\xi,\beta(u)}$ is the generalized Pareto distribution (GPD)

- **Tail estimation:** $\bar{F}(x) = \mathbb{P}(X > x) \approx \frac{N_u}{n} \overline{G}_{\hat{\xi},\hat{\beta}}(x - u), \quad x \geq u\,.$

Example 1: EVT - POT

- For $(\xi > 0)$,

$$F \in \mathrm{MDA}(H_\xi) \Longleftrightarrow \overline{F}(x) = x^{-1/\xi} L(x)$$

with L slowly varying. This means that for $x > 0$,

$$\frac{\overline{F}(tx)}{\overline{F}(t)} = \frac{\mathbb{P}(X > tx)}{\mathbb{P}(X > t)} \longrightarrow x^{-1/\xi}, \quad t \to \infty\,.$$

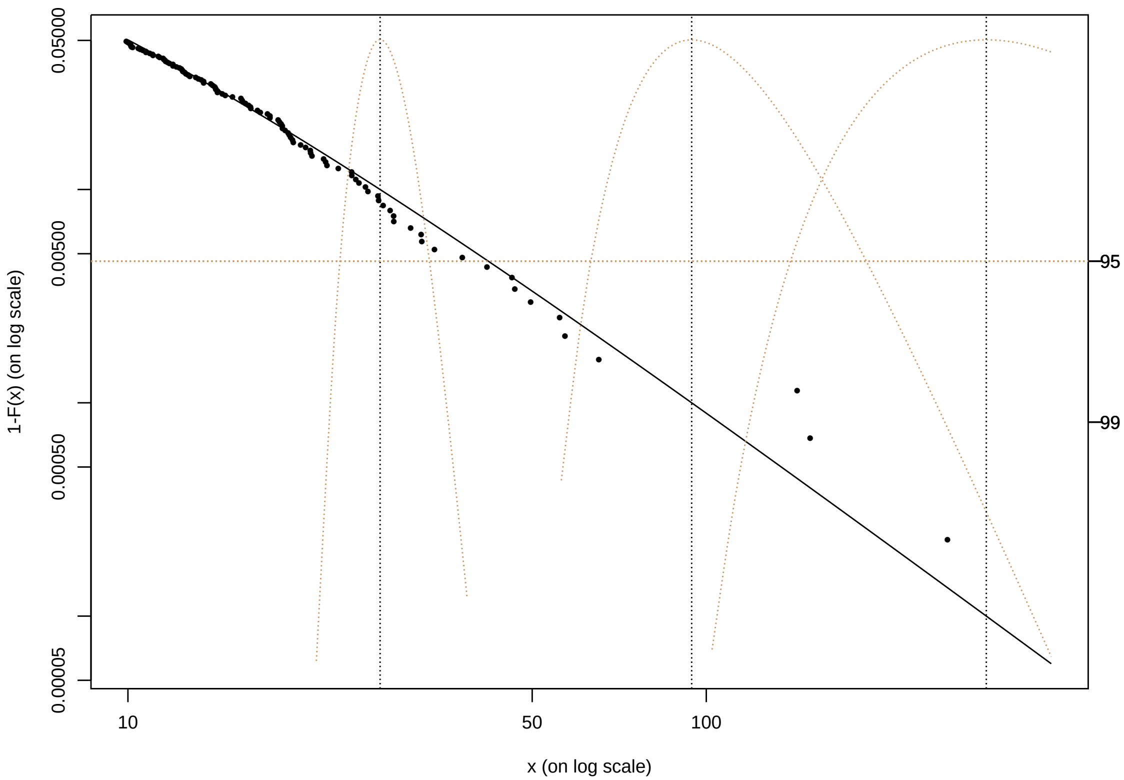
- A graphical device for checking the above condition:

$$\textbf{plot} \quad \log(\overline{F}_n(x)) \quad \textbf{versus} \quad \log(x)$$

where F_n is the empirical distribution function of $(X_1, \ldots, X_n)$ and check for

(ultimate) linearity

Example 1: EVT - POT

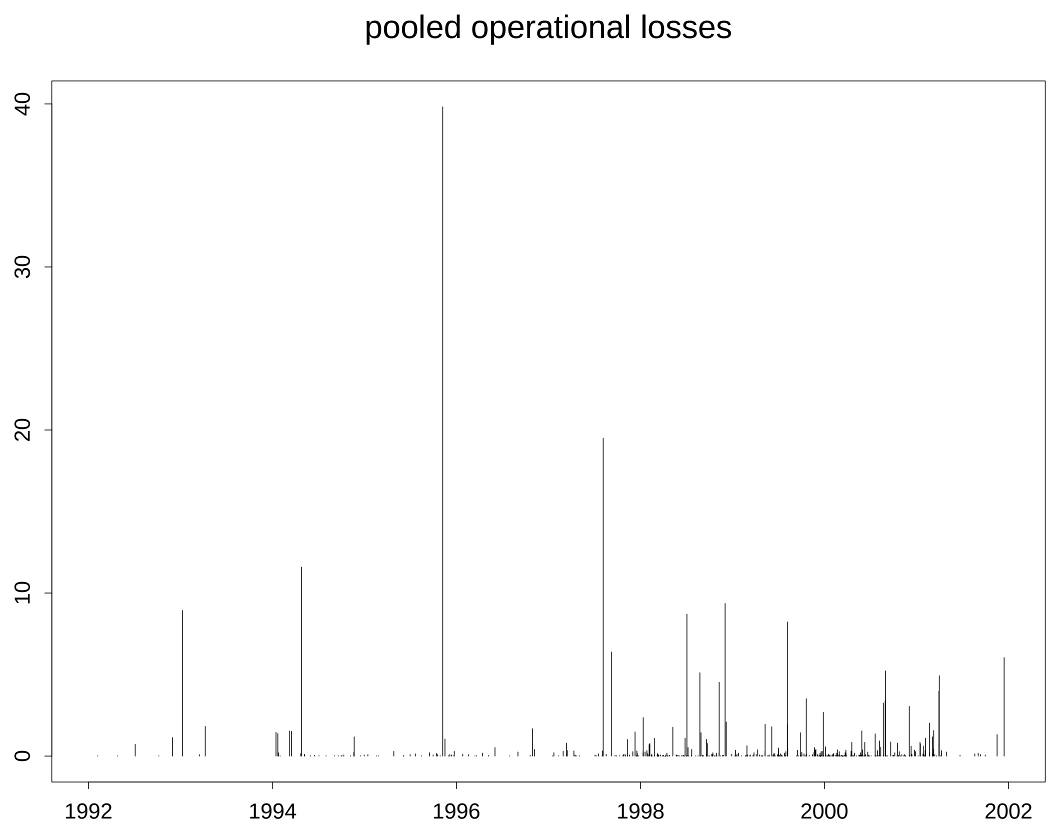


p	Quantile	ES
99.00%	27.28	58.24
99.90%	94.33	191.53
99.99%	304.90	610.13

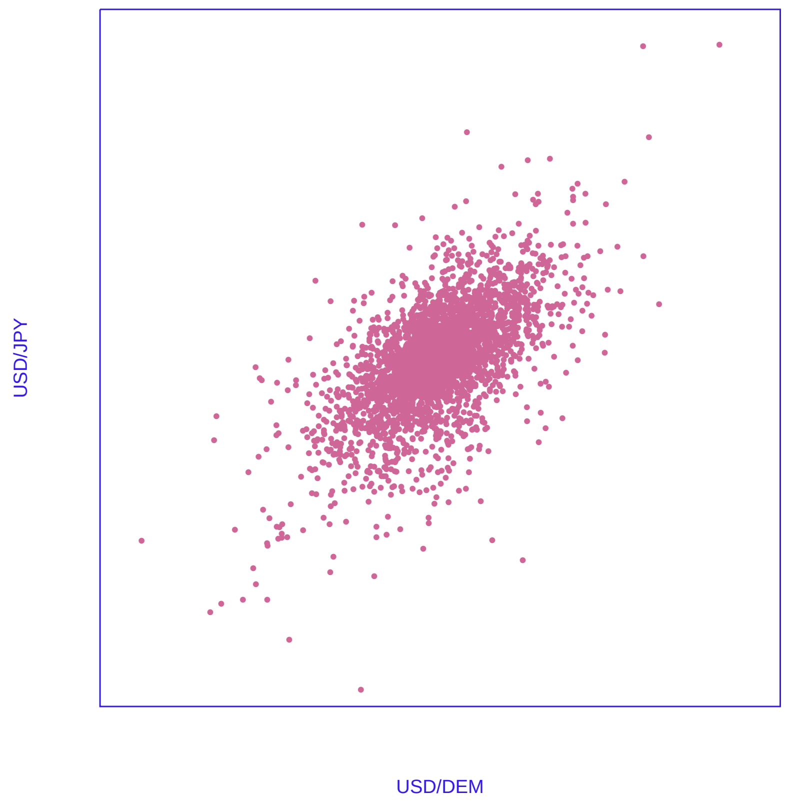
EVT software: EVIS (`www.math.ethz.ch/~mcneil`)

Example 1: EVT - POT

- Modeling of Operational Risk (Basel II)



Example 2: MEVT



Bivariate daily returns of DEM and JPY FX rates quoted against US Dollar from February 1986 up to the end of December 1998

Example 2: MEVT

- Suppose that the d-dimensional random vector $\mathbf{X}$ has a regularly varying tail distribution, i.e., the tail behaviour of $\mathbf{X}$ is characterised by a tail index α and the limit

$$\frac{\mathbb{P}(\,\|\mathbf{X}\| > tx, \mathbf{X}/\|\mathbf{X}\| \in \cdot)}{\mathbb{P}(\|\mathbf{X}\| > t)} \quad \xrightarrow{v} \quad x^{-\alpha}\,\mathbb{P}(\boldsymbol{\Theta} \in \cdot),$$

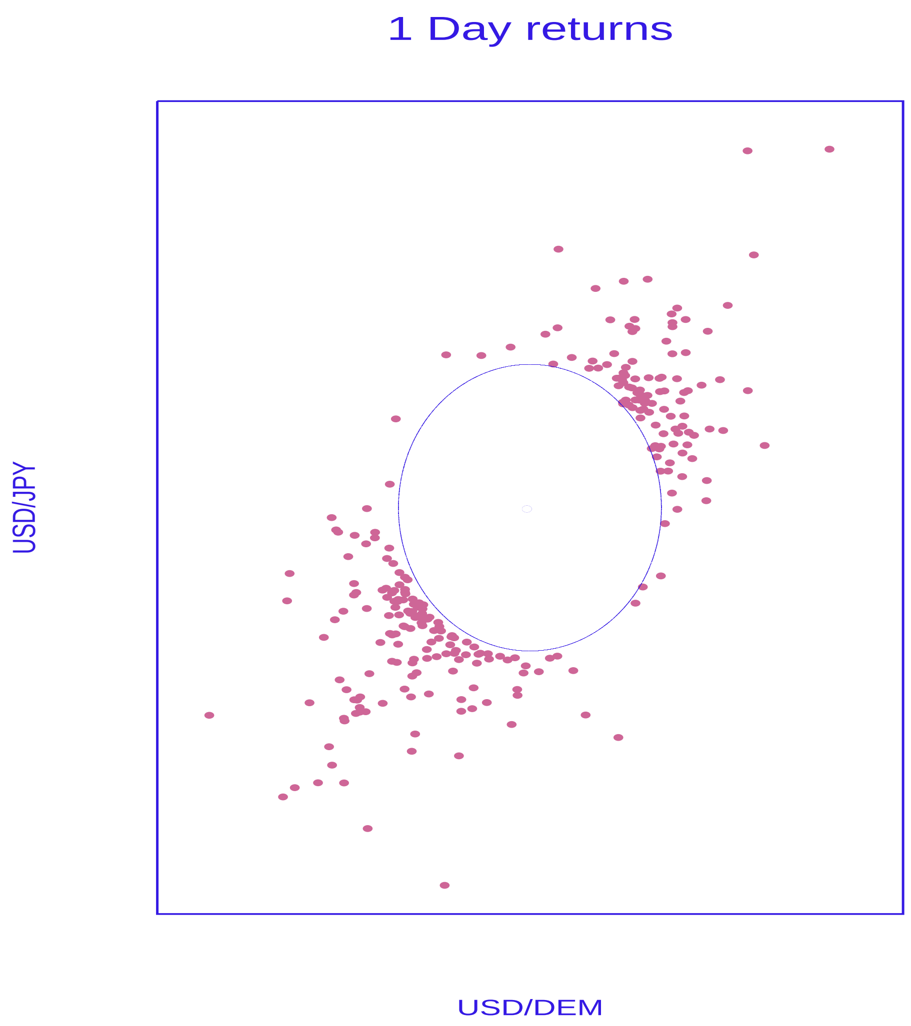
  where $x > 0$, $t \to \infty$, exists. The distribution function of Θ is the spectral distribution of $\mathbf{X}$

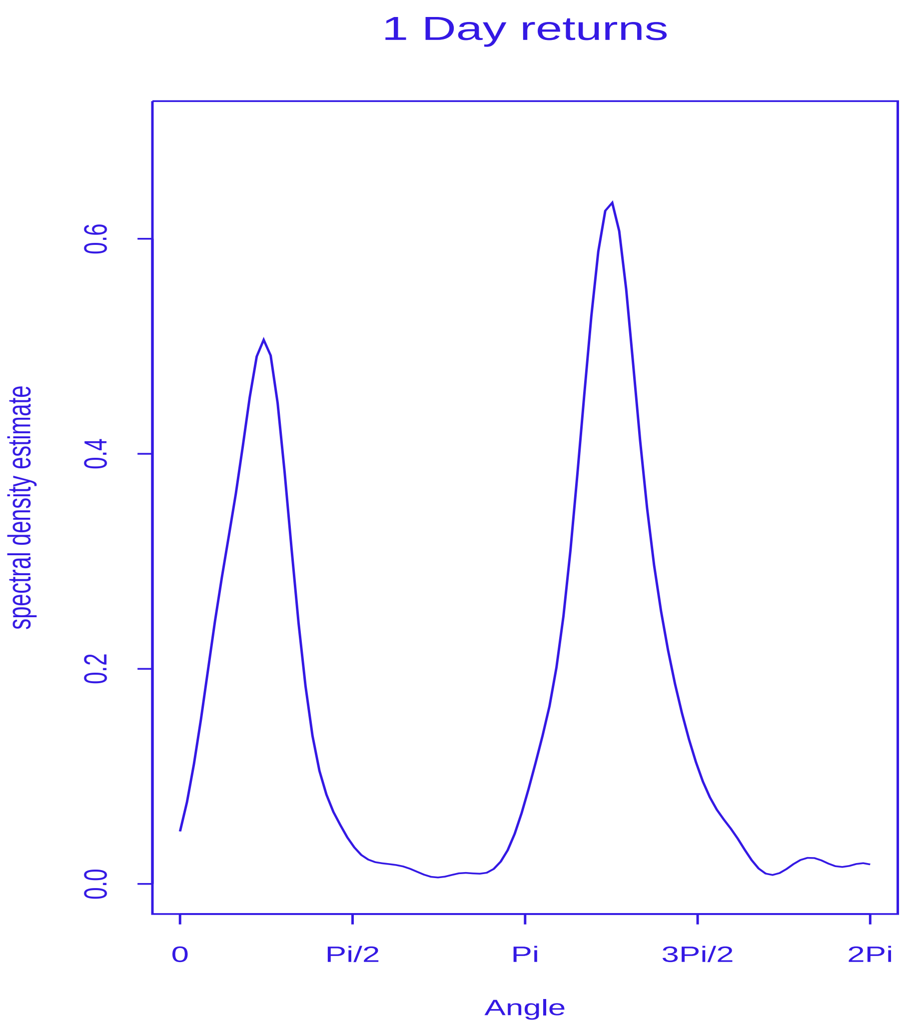
- **Estimator:**

$$\hat{\mathbb{P}}(\boldsymbol{\Theta} \in S) = \frac{1}{k_n} \sum_{i=1}^{n} \epsilon_{\mathbf{x}_i/\|\mathbf{x}\|_{k_n,n}}(V(S))$$

  where $V(S) = \{\mathbf{x} \in \mathbb{S}_{+}^{d-1} : \mathbf{x}/\|\mathbf{x}\| \in S\}$.

Example 2: MEVT





Example 3: COPULAE

d risks $X_1, \ldots, X_d$ with continuous marginal distribution functions $F_1, \ldots, F_d$

- Given the joint law $F_{\mathbf{X}}(\mathbf{x}) = \mathbb{P}(X_1 \leq x_1, \ldots, X_d \leq x_d)$, there exists a unique function C on $[0,1]^d$ so that

$$F_{\mathbf{X}}(\mathbf{x}) = C(F_1(x_1), \ldots, F_d(x_d))$$

(a copula C is a df on $[0,1]^d$ with uniform margins)

- Given only $F_1, \ldots, F_d$ and any copula C,

$$C(F_1(x_1), \ldots, F_d(x_d))$$

yields a joint model with the prescribed margins

Why are copulae useful

- **Pedagogical**: "Thinking beyond linear correlation"

- **Stress testing**: dependence: joint extremes, spillover, contagion, . . .

- **Worst case analysis** under incomplete information:

given: • $X_i \sim F_i$, $i = 1, \ldots, d$, marginal 1-period risks
• $\Psi(\mathbf{X})$: a financial position
• Δ: a one-period risk or pricing measure

task: find $\min \Delta(\Psi(\mathbf{X}))$ and $\max \Delta(\Psi(\mathbf{X}))$ under the above constraints

Simulation study

We consider $m = 10000$ companies. All losses given default are one unit; total loss is number of defaulting companies. Set $\pi = 0.005$ and $\rho = 0.038$, these being values corresponding to a homogeneous group of "medium" credit quality in the KMV/CreditMetrics Gaussian approach. We set $\nu = 10$ in the t-model and perform 100000 simulations to determine the loss distribution.

The risk is compared by comparing high quantiles of the loss distributions (the so-called Value-at-Risk approach to measuring risk)

Results	Min	25%	Med	Mean	75%	90%	95%	Max
Gauss	1	28	43	49.8	64	90	109	331
t	0	1	9	49.9	42	132	235	3238

Many questions remain to be studied

- statistical fitting
- high-dimensional copulae
- dynamic modeling
- stylized facts on copulae

Risk management consequences

- **First Fundamental Theorem of Integrated Risk Management (1FTIRM):** For elliptically distributed risk vectors, classical IRM tools like VaR, Markowitz portfolio approach, work fine

  Recall:

  - $\mathbf{Y}$ in $\mathbb{R}^d$ is spherical if $\mathbf{Y} \stackrel{d}{=} U\mathbf{Y}$ for all orthogonal matrices U
  - $\mathbf{X} = A\mathbf{Y} + \mathbf{b}$, $A \in \mathbb{R}^{d \times d}$, $\mathbf{b} \in \mathbb{R}^d$ is called elliptical
  - Let $\mathbf{Z} \sim \mathrm{N}_d(\mathbf{0}, \mathbf{\Sigma})$, $W \geq 0$, independent of $\mathbf{Z}$, then

$$\mathbf{X} = \boldsymbol{\mu} + W\mathbf{Z}$$

  is elliptical (multivariate normal variance-mixtures)

Risk management consequences

- If one takes

  $W = \sqrt{\nu/V}$, $V \sim \chi^2_\nu$, then $\mathbf{X}$ is multivariate t_ν
  W normal inverse Gaussian, then $\mathbf{X}$ is generalized hyperbolic

- **2FTIRM:** (much more important!)
  For non-elliptically distributed risk vectors, 1FTIRM breaks down:

  - VaR is typically non-subadditive
  - risk capital allocation is non-consistent
  - portfolio optimization is risk-measure dependent
  - correlation based methods are insufficient

- A(n early) stylized fact:
  In practice, portfolio risk factors typically are non-elliptical
  Question: are these deviations relevant, important?

Some common fallacies

(often appearing in disguise!)

- **Fallacy 1:** Marginal distributions and their correlation matrix uniquely determine the joint distribution
  True for elliptical families, wrong in general (copulae)

- **Fallacy 2:** Given two one-period risks X_1, X_2, VaR$(X_1 + X_2)$ is maximal for the case where the correlation $\rho(X_1, X_2)$ is maximal
  True for elliptical families, **wrong** in general (non-coherence of VaR)

- **Fallacy 3:** Small correlation $\rho(X_1, X_2)$ implies that X_1 and X_2 are close to being independent

An example concerning fallacy 3

- Two country risks X_1 and X_2
  - $Z \sim \mathsf{N}(0,1)$, $U \sim \mathsf{UNIF}(\{-1,+1\})$, $\mathbb{P}(U=-1) = 1/2 = P(U=1)$
    U stands for an economic stress generator, independent of Z
  - As a consequence:

$$X_1 = Z \sim \mathsf{N}(0,1)\,, \qquad X_2 = UZ \sim \mathsf{N}(0,1)$$

  - Moreover:

$$\mathbb{C}\mathsf{ov}(X_1, X_2) = \mathbb{E}(X_1 X_2) = \mathbb{E}(UZ^2) = \mathbb{E}(U)E(Z^2) = 0$$

    Hence $\rho(X_1, X_2) = 0$.
  - However, X_1 and X_2 are strongly dependent: with 50% probability comonotone, with 50% countermonotone
  - Also note that $X_1 + X_2 = Z(1+U)$ is **not** normally distributed

- This Example can be made much more realistic

C. Quantifying Regulatory Capital for Operational Risk

The New Accord (Basel II)

- **1988**: Basel Accord (Basel I): minimum capital requirements against credit risk. One standardised approach

- **1996**: Amendment to Basel I: market risk.

- **1999**: First Consultative Paper on the New Accord (Basel II).

- **to date**: CP3: Third Consultative Paper on the New Basel Capital Accord. (`www.bis.org/bcbs/bcbscp3.htmcp3`)

- **end of 2003 (?)**: Revision of CP3

- **end of 2006 (?)**: full implementation of Basel II ([10])

What's new?

- **Rationale** for the New Accord: More flexibility and risk sensitivity
- **Structure** of the New Accord: Three-pillar framework:

❶ Pillar 1: minimal capital requirements (risk measurement)

❷ Pillar 2: supervisory review of capital adequacy

❸ Pillar 3: public disclosure

What's new? (cont'd)

- Two options for the measurement of credit risk:
  - Standard approach
  - Internal rating based approach (IRB)

- Pillar 1 sets out the minimum capital requirements:

$$\frac{\text{total amount of capital}}{\text{risk-weighted assets}} \geq 8\%$$

- MRC (minimum regulatory capital) $\stackrel{\text{def}}{=} 8\%$ of risk-weighted assets

- Explicit treatment of operational risk (*the risk of losses resulting from inadequate or failed internal processes, people and systems, or external events*)

What's new? (cont'd)

- **Notation**: C_{OP}: capital charge for operational risk
- **Target**: $C_{\mathsf{OP}} \approx 12\%$ of MRC
- **Estimated total losses** in the US (2001): $50b
- **Some examples**
  - 1977: Credit Suisse Chiasso-affair
  - 1995: Nick Leeson/Barings Bank, £1.3b
  - 2001: Enron (largest US bankruptcy so far)
  - 2003: Banque Cantonale de Vaudoise, KBV Winterthur

Risk measurement methods for OP risks

Pillar 1 regulatory minimal capital requirements for operational risk:
Three distinct approaches:

❶ Basic Indicator Approach

❷ Standardised Approach

❸ Advanced Measurement Approaches (AMA)

Basic Indicator Approach

- Capital charge:

$$\boxed{C_{\mathsf{OP}}^{\mathsf{BIA}} = \alpha \times GI}$$

- $C_{\mathsf{OP}}^{\mathsf{BIA}}$: capital charge under the Basic Indicator Approach

- GI: average annual gross income over the previous three years

- $\alpha = 15\%$ (set by the Committee)

Standardised Approach

- Similar to the BIA, but on the level of each business line:

$$C_{\text{OP}}^{\text{SA}} = \sum_{i=1}^{8} \beta_i \times GI_i$$

$\beta_i \in [12\%, 18\%]$, $i = 1, 2, \ldots, 8.$

- 8 business lines:

Corporate finance	Payment & Settlement
Trading & sales	Agency Services
Retail banking	Asset management
Commercial banking	Retail brokerage

Advanced Measurement Approaches (AMA)

- Allows banks to use their internally generated risk estimates
- Preconditions: Bank must meet qualitative and quantitative standards before using the AMA
- Risk mitigation via insurance allowed
- **AMA1**: Internal measurement approach
- **AMA2**: Loss distribution approach

Internal Measurement Approach

- Capital charge:

$$C_{\mathrm{OP}}^{\mathrm{IMA}} = \sum_{i=1}^{8} \sum_{k=1}^{7} \gamma_{ik}\, e_{ik}$$

e_{ik}: expected loss for business line i, risk type k

γ_{ik}: scaling factor

- 7 loss types: Internal fraud
  External fraud
  Employment practices and workplace safety
  Clients, products & business practices
  Damage to physical assets
  Business disruption and system failures
  Execution, delivery & process management

Loss Distribution Approach

- For each business line/risk type cell (i, k) one models

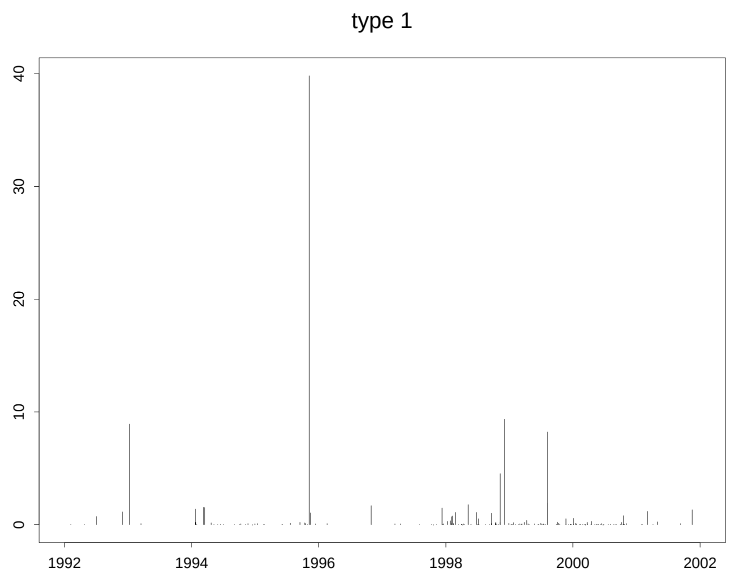
$L_{i,k}^{T+1}$: OP risk loss for business line/risk type cell (i, k) over the period $[T, T+1]$.

$$L_{i,k}^{t} = \sum_{\ell=1}^{N_{i,k}^{t}} X_{i,k}^{\ell}$$

$$C_{i,k}^{\text{OP}} = g(L_{i,k}^{t+1}) = \begin{cases} F_{L^{t+1}}^{\leftarrow}(\alpha) = \text{VaR}_{\alpha}(L^{t+1}) \\ \text{ES}_{\alpha}(L^{t+1}) = \mathbb{E}[L^{t+1} | L^{t+1} > \text{VaR}_{\alpha}(L^{t+1})] \end{cases}$$

$$C_{\text{OP}} = \sum_{i,k} g(L_{i,k}^{t+1}) \qquad \text{(perfect correlation)}$$

Some data


type 1
0
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1992
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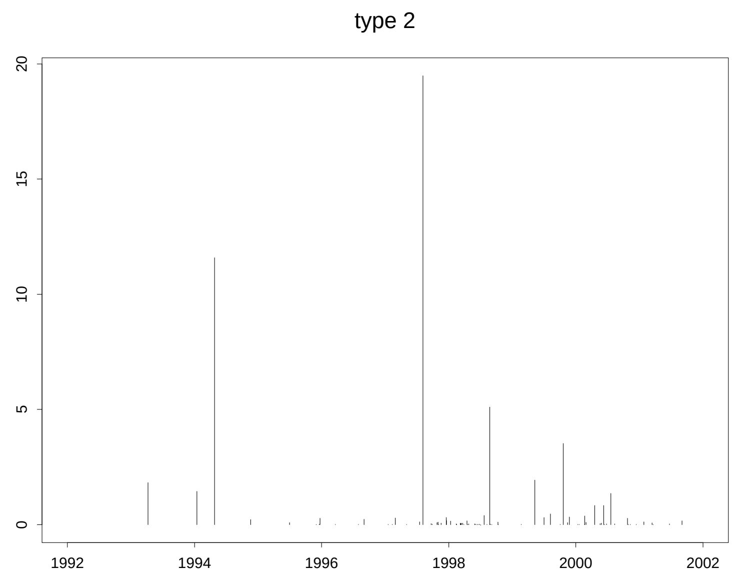
type 2
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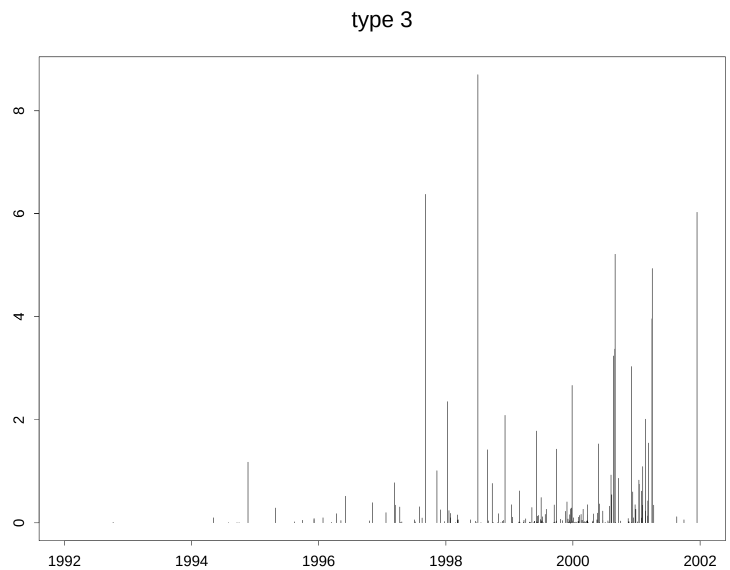
type 3
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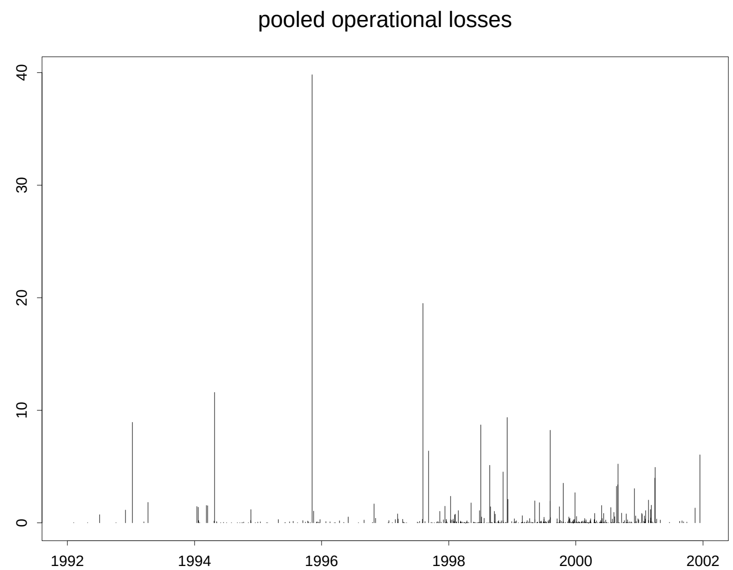
2002


pooled operational losses
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20
30
40
1992
1994
1996
1998
2000
2002

Modelling issues

- Stylized facts about OP risk losses
  - Loss occurrence times are irregularly spaced in time
    (selection bias, economic cycles, regulation, management interactions,. . .)
  - Loss amounts show extremes
- Large losses are of main concern!
- Repetitive vs non-repetitive losses
- **Warning flag**: Are observations in line with modeling assumptions?
- Example: "iid" assumption implies
  - NO structural changes in the data as time evolves
  - Irrelevance of which loss is denoted X_1, which one X_2,. . .

The Problem

- In-sample estimation of $\text{VaR}_\alpha(L^{t+1})$ (α large) impossible!

- Estimation of the (far-) tail of L_t via subcategories:

$$L = \sum_{\ell=1}^{N} Y_\ell \,, \qquad 1 - F_Y(x) \sim x^{-\alpha} h(x) \ , \qquad x \to \infty$$

$$\Rightarrow \quad 1 - F_L(x) \sim \mathbb{E}[N]\, x^{-\alpha} h(x) \ , \qquad x \to \infty$$

- Standard actuarial techniques:
  - Approximation (translated gamma/lognormal)
  - Inversion methods (FFT)
  - Recursive methods (Panjer)
  - Simulation

How accurate are VaR-estimates?

- **Assumptions**:
  - (L_m) iid $\sim F$
  - For some ξ, β and u large ($G_{\xi,\beta}$: GPD):

$$F_u(x) := \mathbb{P}(L - u \leq x | L > u) = G_{\xi,\beta(u)}(x)$$

- Tail- and quantile estimate:

$$\boxed{\begin{aligned} 1 - \hat{F}_L(x) &= \frac{N_u}{n}\Big(1 + \hat{\xi}\,\frac{x-u}{\hat{\beta}}\Big)^{-1/\hat{\xi}}, \quad x > u. \\ \widehat{\text{VaR}}_\alpha &= \hat{q}_\alpha = u - \frac{\hat{\beta}}{\hat{\xi}}\Big(1 - \Big(\frac{N_u}{n(1-\alpha)}\Big)^{\hat{\xi}}\Big) \end{aligned}} \qquad (1)$$

How accurate are VaR-estimates? (cont'd)

- **Idea**: Comparison of estimated quantiles with the corresponding theoretical ones by means of a simulation study ([9]).

- **Simulation procedure**:

  ❶ Choose F and fix $\alpha_0 < \alpha < 1$, N_u (# of data points above u)

  ❷ Calculate $u = q_{\alpha_0}$ and the true value of the quantile q_α

  ❸ Sample N_u independent points of F above u by the rejection method. Record the total number n of sampled points this requires

  ❹ Estimate ξ, β by fitting the GPD to the N_u exceedances over u by means of MLE.

  ❺ Determine $\hat{q}_\alpha$ according to (1)

  ❻ Repeat N times the above to arrive at estimates of Bias$(\hat{q}_\alpha)$ and SE$(\hat{q}_\alpha)$

How accurate are VaR-estimates? (cont'd)

- **Accuracy** of the quantile estimate expressed in terms of bias and standard error:

$$\mathsf{Bias}(\hat{q}_\alpha) = \mathbb{E}[\hat{q}_\alpha - q_\alpha], \qquad \mathsf{SE}(\hat{q}_\alpha) = \mathbb{E}\big[(\hat{q}_\alpha - q_\alpha)^2\big]^{1/2}$$

$$\widehat{\mathsf{Bias}} = \frac{1}{N}\sum_{j=1}^{N} \hat{q}_\alpha^j - q_\alpha \qquad \widehat{\mathsf{SE}} = \Big(\frac{1}{N}\sum_{j=1}^{N}(\hat{q}_\alpha^j - q_\alpha)^2\Big)^{1/2}$$

- **Ideally**, $\widehat{\mathsf{Bias}}$ AND $\widehat{\mathsf{SE}}$ small

Example: Pareto distribution with $\theta = 2$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	A minimum number of **100 exceedances** (corresponding to 333 observations) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of **200 exceedances** (corresponding to 667 observations) is required to ensure accuracy wrt bias and standard error.
$q = 0.9$	0.99	Full accuracy can be achieved with the minimum number **25** of **exceedances** (corresponding to 250 observations).
	0.999	A minimum number of **100 exceedances** (corresponding to 1000 observations) is required to ensure accuracy wrt bias and standard error.

Example: Pareto distribution with $\theta = 1$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	For all number of exceedances up to **200** (corresponding to a minimum of 667 observations) the VaR estimates fail to meet the accuracy criteria.
	0.999	For all number of exceedances up to **200** (corresponding to a minimum of 667 observations) the VaR estimates fail to meet the accuracy criteria.
$q = 0.9$	0.99	A minimum number of **100 exceedances** (corresponding to 1000 observations) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of **200 exceedances** (corresponding to 2000 observations) is required to ensure accuracy wrt bias and standard error.

How accurate are VaR-estimates? (cont'd)

- Minimum number of observations increases as the tails become thicker ([9]).
- Large number of observations necessary to achieve targeted accuracy.
- **Remember**: The simulation study was done under idealistic assumptions. OP risk losses, however, typically do NOT fulfil these assumptions.

Conclusions

- OP risk $\neq$market risk, credit risk
- "Multiplicative structure" of OP risk losses ([8])
  $S \times T \times M$ (Selection-Training-Monitoring)
- Actuarial methods (including EVT) aiming to derive capital charges are of limited use due to
  - lack of data
  - inconsistency of the data with the modeling assumptions
- OP risk loss databases must grow
- Sharing/pooling internal operational risk data?

Conclusions (cont'd)

- Choice of risk measure?
- Heavy-tailed ruin estimation for general risk processes ([7])
- Alternatives?
  - Insurance. Example: FIORI, Swiss Re (Financial Institution Operating Risk Insurance)
  - Securitization / Capital market products
- OP risk charges can not be based on statistical modeling alone

▶ Pillar 2 (overall OP risk management such as analysis of causes, prevention, . . .) more important than Pillar 1

References

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D. Conclusions

- Questions in Quantitative Risk Management are directly concerned with extremes and dependence
- EVT (stochastic theory of extremal events) and Copulae (joint distributions) are canonical tools
- These tools should not be used blindly
- Always check necessary conditions
- They are essential Risk Management tools