Extremes in Economics and the Economics of Extremes*

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1 About the title

Within econometrics, probability theory and statistics, an enormous literature exists on the topic of Extremes in Economics. See for instance Mikosch [47] and Klüppelberg [36] in this volume, and the numerous references listed in those contributions. For a long time, econometric research has shown that, for instance logarithmic returns of financial data are non–normal. Extremal moves up or down do occur much more regularly than standard (normal based) models make us believe. Extreme Value Theory (EVT) has become a standard toolkit within quantitative finance useful for describing these non–normal phenomena. Statistically exploring and stochastically modelling such extremes in financial data is however a rather different task from answering the question: "Given a financial market where such extremes occur, how are they to be handled from an economic point of view?". Perhaps the most

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striking answer to the latter question came in the late eighties, early nineties through the emergence of risk management (RM) regulations. In this paper, I want to highlight some of these developments, stress the interplay between EVT and RM and hint at possible areas of research where the focus is more on the second part of the title "The Economics of Extremes". Several references will guide the reader to related publications.

2 On the history of financial risk management

In his excellent text Steinherr [51], the author states "Risk Management: one of the most important innovations of the 20th century". This statement summarises the revolution we witnessed on financial markets during the second half of the 20th century. Some key dates that emphasise this revolution are (without striving for completeness):

— 1933 (4 years after the 1929 crash): The Glass–Steagall Act was passed in the USA in the aftermath of the Depression prohibiting commercial banks from underwriting insurance and most kind of securities. From that Act emerged a new trend of financial institutions: the investment bank. Many of the limitations embedded in the Glass–Steagall Act were gradually softened, leading to its abolishment and reformulation through the 1999 Financial Services Act repealing many key provisions of Glass–Steagall. As a consequence, bank holding companies will continue to expand the range of their financial services, and further convergence of finance and insurance is likely. For a critical discussion on the latter, see Cummins [14].

— Around the early fifties, the foundation of modern portfolio theory was laid, for instance through the seminal work of Harry Markowitz. Risk (measured through standard deviation) entered as an extra dimension next to (excess) return. The risk–return diagram with its efficient frontier became the bread and butter of any portfolio manager.

— In 1970, the Bretton–Woods system of fixed exchange rates was abolished, leading overnight to increased exchange rate volatility. The 70's also saw several oil crises making energy an unpredictable, highly volatile market component. Investors were looking for instruments that would enable them to hedge this increased riskiness on financial markets.

— Through the work of Fisher Black, Myron Scholes and Robert Merton (1972), this search for hedging instruments got a scientific response in that financial derivatives could be rationally priced and hedged. Advanced mathematics and finance joint forces in order to come up with, what we now call the Black–Scholes pricing formula (framework) for options; see Black and Scholes [7]. The floadgates opened starting with the opening in 1973 of the Chicago Board of Options Exchange (CBOE). In those days, it seemed that the sky was the limit.

— An enormous growth in both volume and complexity of instruments traded on financial markets resulted. For example, on the New York Stock Exchange, the 3.5 million shares traded daily in 1970 grew to a volume of 40 million in 1990. The nominal value of the so–called Over The Counter (OTC) derivatives increased over the period from 1995 till 1998 from $13 trillion to $18 trillion for forex contracts, $26 to $50 trillion for interest rate contracts and across all types from $47 to $80 trillion. To be sure, $1 trillion $= 1 \times 10^{12}$; an enormous figure indeed! For details on these statistics, and much more, see Crouchy et al. [12]. At this point, it needs stressing that all these developments were made possible by an unprecedented growth in IT technology.

— In the late eighties and early nineties, first attempts were made, both industry–internally as well as from a regulatory point of view, to get these so–called off–balance instruments (derivatives) under control.
Again, Crouchy et al. [12] has the full story. For the purpose of this paper, it suffices to recall the work of the Basel Committee on Banking Supervision. No doubt this regulatory framework came very much into being due to some spectacular losses as there are Orange County, Met-

allgesellschaft, Barings, and indeed more recently LTCM and Enron. A lot has been written on these spectacular losses, so much that Professor Stephen Ross (MIT) has been going around starting his talk with "I am like a financial pathologist, I disect financial corpses". Under the title "Disasters: Divine Results Rocked by Human Recklessness", Boyle and Boyle [9] have written an excellent account on Orange County, Barings and LTCM. See also Jorion [34] for an interesting discussion on LTCM. Specific contributions on RM for alternative investments (hedge funds, private equity, alternative risk transfer, ...) are Jaeger [32] and Lane [38].

It is essentially the regulatory framework that contributes strongly to laying down the rules for the Economics of Extremes, hence in the next section, we will have a closer look at these rules.

3 Basel I and II

When I refer to Extremes in Economics, I refer to the modelling and analysis of extremes in econometric data. As an example, look at the recent paper by Longin [43], one of the pioneers of EVT in finance. In this paper, the author for instance estimates the probability of exceedance and waiting time period for the ten largest daily return price movements in the US equity market (S&P 500) over the period July 1962 – December 1999. This "hitparade" ranges from -18.35% on October 19, 1987 to -3.29% on October 9, 1979. Other, perhaps less well known applied econometric work on Extremes in Economics concerns spill–over events; see for instance Hartmann et al. [31]. In this case, EVT in a more–dimensional set–up appears.

The second part of the title, The Economics of Extremes concentrates on the crucial question: given the econometric evidence on quantifiable extremal events in finance (and insurance, say), how can we handle these extremes from an economic point of view. Some concrete questions could be:

— How can one device prudent regulatory rules aiming at market stability? Here the Basel Committee enters; see below for more details.

— Measure and (more importantly) price the time–dimension of system–wide risk. For these questions, see for instance Crockett [11] and Borio et al. [8]. An interesting review on systemic risk, an area where EVT as a quantitative tool has a lot to offer, is De Brandt and Hartmann [17]. Important in these problems is finding, typically macro–economic structures which help the economy/market to dampen (hopefully avoid) the more negative consequences of extremal events.

For most of the more mathematically minded extreme value theorist, working in risk management is equivalent to estimating Value–at–Risk (VaR) for ever more complicated stochastic models. In their (our) terminology, VaR is "just" a quantile of some underlying process or distribution. However, VaR is to finance what body temperature is to a patient; an indicator of bad health but not an instrument telling us what is wrong and far less a clue on how to get the patient (system) healthy again. Let us look at some of the main issues in Risk Management (RM) from the perspective of the regulator as personified by the famous Basel Committee. Details underlying the summary below are to be found on the homepage www.bis.org of the Bank of International Settlements in Basel.

The Basel Committee was established by the Central–Bank Governors of the Group of Ten at the end of 1974. The Committee does not posess any formal supranational supervisory authority, and hence its conclusions do not have legal force. Rather, it formulates broad supervisory standards and guidelines and recommends statements of best practice in the expectation that individual authorities will take steps to implement them through detailed arrangements – statutory or otherwise – which are best suited to their own national system. In 1988, the Committee introduced a capital measurement system, commonly referred to as the Basel Capital Accord (also called Basel I). This system provided for the implementation of a Credit Risk measurement framework with a minimum capital standard of 8% (a so–called haircut) by end–92. From the start, banks criticised the lack of risk sensitivity in this approach. On the Credit Risk side, this led to the New Capital Adequacy (so–called Basel II) framework of June 1999. The latter is now under discussion with the industry and is planned to become operational by the beginning of 2005. Besides these key developments within the Credit Risk area, already around 1994 we saw various amendments to Basel I catering for Market Risk, in particular for derivative positions. The 1996 report on the Amendment to the Capital Accord to Incorporate Market Risks opened the floodgates for the VaR–modellers. Through this Amendment, a direct link between the quantitative VaR measure for Market Risk and Regulatory Capital was established. The exact form of the link very much depends on the statistical qualities of the underlying market risk models through backtesting. For banks opting for the so–called internal modelling approach, the following formula yields the capital charge C_t at time t:

$$C_t = \max\left\{ \mathrm{VaR}_{t-1} + d_t\, \mathrm{ASR}^{\mathrm{VaR}}_{t-1}\,, M_t \frac{1}{60}\sum_{j=1}^{60} \mathrm{VaR}_{t-j} + d_t \frac{1}{60}\sum_{j=1}^{60} \mathrm{ASR}_{t-j} \right\}$$

where

— VaR_{t-i} is the 99%, 10–day VaR at day $t-i$;

— M_t is the multiplyer for day t, $M_t \geq 3$, mainly depending on the statistical qualities of the model, in particular, depending on backtesting results;

— $\mathrm{ASR}^{\mathrm{VaR}}$ is the extra VaR–based charge derived from specific portfolio risk for equity and interest rate instruments (using the CAPM language), and

— d_t is a $\{0,1\}$–indicator function which for day t possibly includes specific risk.

For details on the formula, and its related economic interpretation, see for instance Jovic [35]. There are also various regulatory rules on the allowable size of C_t in function to the banks's so–called Tier 1 and 2 capital, as well as bank internal allocation rules of C_t to subunits and the safeguarding of limits spoken based on VaR–measures. Moreover, the independent calculation/supervision/verification of VaR and $\mathrm{ASR}^{\mathrm{VaR}}$ poses a major problem implying that there is much, much more to the calculation of VaR than just saying that “we are estimating a quantile”. I find it very important that

EVT specialists, especially those participants to this SemStat meeting with an interest in finance, take a deeper interest in these underlying economic and more detailed computational issues. As already stated above, the BIS website is a good place to start. J.P. Morgan's RiskMetrics is a further source of more applied reading. Especially its more recent updated technical document, Mina and Xiao [48], makes a nice link between current EVT research and its impact on Market RM.

Though above I already used various examples of risk classes, in order to move more in detail to Basel II, it may be useful to give a brief classification of financial risks as referred to in the Basel documents:

— Market Risk (MR): the risk associated with fluctuations in the value of traded assets.

— Credit Risk (CR): the risk associated with the uncertainty that debtors will honour their financial obligations.

— Operational Risk (OR): the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events.

— Liquidity Risk (LR): the risk that positions cannot be unwound quickly enough at critical times.

— Other Risks: Business, Reputational, ...

A modern financial institution will have to map the above zoology of risks with indications of relevance, size, organisational issues, qualitative and quantitative assessment. See for instance Litterman [41], [42] for a discussion of some of these issues. An important point concerns aggregation across risk classes and allocation of resulting risk capital to the various relevant layers within the institution. In all these fundamental steps, mathematical techniques enter at more or less prominent levels.

The key improvement within Basel II concerns an increased risk sensitivity for Credit Risk internal models. This can be achieved through various analytical models that go under the names of contingent claim, actuarial and reduced form approaches; see Crouchy et al. [12] for details. Gordy [29] gives an excellent review, Frey and McNeil [27] unify the above models from a statistical (latent variable) point of view, whereas a definitive textbook may well become Duffie and Singleton [21]. See also Arvanitis and Gregory [5] for a guide to pricing, hedging and risk management of credit positions. Within CR management extremes play a role through the typical skewness of the loss distributions, but more importantly through the non–Gaussian dependence between credit loss events. As shown by Frey and McNeil [27], the EVT based modelling of default correlation is of key importance to any well functioning CR management system. See also the latter paper for further references on this. A critical discussion on the use of EVT to CR modelling is Lucas et al. [44].

An important, economic consequence of the risk sensitiveness improvements made for CR within Basel II is an anticipated reduction in total regulatory capital. At the same time however, the Basel Committee introduced within Basel II Operational Risk (OR) as a new risk class. Although the consultation with industry is still ongoing, it is to be expected that the decrease in CR capital charge will be (approximately) offset by the new OR charge. Given that there will be a new OR capital charge forthcoming, EVT in combination with standard actuarial modelling will be called for in a fundamental way. In order to see this, consider the following, OR setup. A stylised OR data base will look as follows:

$$\left\{Y_k^{i,j}\,,\, i, j, k\right\}$$

where

$$\begin{array}{ll} i = 1, 2, \ldots, T & \text{(years, say, e.g. } T = 10\text{);} \\ j = 1, 2, \ldots, s & \text{(\# claim types, e.g. } s = 6\text{), and} \\ k = 1, 2, \ldots, N^{i,j} & \text{(\# claims of type } j \text{ in year } i\text{).} \end{array}$$

Note that typically $Y_k^{i,j}$ is censored from below, i.e.

$$Y_k^{i,j} = \left(\widetilde{Y}_k^{i,j} - d^{i,j}\right)^+$$

for the full (ground up) claims $\left(\widetilde{Y}_k^{i,j}\right)$ and some company specific lower thresholds $(d^{i,j})$. As a result, the total yearly OR loss amounts across all

s types are

$$\left\{ \sum_{j=1}^{s} \sum_{k=1}^{N_{i,j}} Y_k^{i,j} , \quad i = 1, \ldots, T \right\} .$$

Because of Basel II, banks using an internal OR modelling approach will have to come up with an estimate of the $100(1 - \alpha)\%$ quantile (OR-VaR) with α small ($\alpha = 0.0005$, say) of the distribution function of next year's total loss

$$\sum_{j=1}^{s} \sum_{k=1}^{N_{T+1,j}} Y_k^{T+1,j} .$$

Of the few facts available for real OR losses, one is very clear: losses are heavy tailed. Hence from an Extremes in Economics point of view, actuarial total loss modelling under a heavy tailed regime is natural; for some publications along these lines, see Medova [46] and Cruz [13]. Embrechts and Samorodnitsky [26] contains some advanced ruin theoretic results motivated by OR. At present, the more important issue falls under the Economics of Extremes heading: why introduce an OR capital charge in the first place? The already quoted BIS website (www.bis.org) contains under "Basel Committee: Comments Received" several discussion papers on this topic. As an example, see Daníelsson et al. [16] where some of the more fundamental economic issues underlying quantitative risk management regulations à la Basel II are critically assessed.

In the next section, I summarise some current mathematical research originating from the above discussions on risk management in general and Basel I and II more in particular. The choice of topics made is rather subjective, I have however tried (mainly from an Economics of Extremes point of view) to complement other EVT applications within finance and insurance discussed elsewhere in this volume.

4 Some current research

4.1 Coherent risk measurement

In a sequel of fundamental papers, Artzner, Delbaen, Eber and Heath ([3], [4], [18], [19]) posed and answered the following questions:

Q1. What economic properties ought a "good" risk measure have?

Q2. Characterise all "good" risk measures.

Q3. Is VaR "good"?

Q4. If the answer to Q3. is no, suggest improvements.

In a one–period setup, a risk X is a bounded random variable ($X \in L^o(\Omega, \mathcal{F}, P)$) denoting the profit–and–loss of a financial position which we hold today for a fixed future period, 10 days, say. Suppose the risk free interest over this one period is $r \geq 1$. In the above publications, a "good" risk measure

$$\rho : L^o(\Omega, \mathcal{F}, P) \to \mathbb{R}$$

is termed coherent and has to satisfy the following axioms:

(C1) (Translation Invariance)
$\forall X \in L^o$, $\alpha \in \mathbb{R} : \rho(X + \alpha r) = \rho(X) - \alpha$.

(C2) (Subadditivity)
$\forall X, Y \in L^o : \rho(X + Y) \leq \rho(X) + \rho(Y)$.

(C3) (Positive Homogeneity)
$\forall X \in L^o$, $\lambda \geq 0 : \rho(\lambda X) = \lambda \rho(X)$.

(C4) (Monotinicity)
$\forall X, Y \in L^o$, $X \leq Y$, we have $\rho(X) \geq \rho(Y)$.

In Artzner et al. [4], the link to economics is made through the notion of acceptance set associated with a coherent risk measure ρ:

$$\mathcal{A}_\rho = \{X \in L^o\,, \quad \rho(X) \leq 0\}\ .$$

Hence $\mathcal{A}_\rho$ contains those financial positions for which, using the risk measure ρ, no further capital charge ($\rho(X) = 0$) is necessary, or even ($\rho(X) < 0$)

capital can be redrawn. A further result from this paper is the following representation theorem: a risk measure ρ is coherent if and only if there exists a set $\mathcal{P}$ of probability measures on $(\Omega, \mathcal{F})$, such that

$$\rho(X) = \sup \left\{ E^Q \left(-X/r\right) \,, \quad Q \in \mathcal{P} \right\} \,,$$

i.e. ρ is a so–called generalised scenario through which Q2. is answered. It is not difficult to see that in general VaR is not coherent. Indeed typically for non–elliptically distributed portfolios, VaR fails to satisfy the for economic purposes important subadditive property (C2). The following, easy example goes back to Claudio Albanese, a similar example is to be found in Artzner et al. [3].

Example 1 (VaR is not necessarily coherent)
Suppose $X_1, \ldots, X_{100}$ correspond to the profit–and–loss (P&L) positions of 100 defaultable (one year) bonds, each with face value \$100, default probability 1% and 2% yearly coupon. Hence, for $i = 1, \ldots, 100$,

$$X_i = \begin{cases} 2 & \text{with probability } 99\% \\ -100 & \text{with probability } 1\% \,. \end{cases}$$

Surely, the "more diversified" position $\sum_{i=1}^{100} X_i$ should have a lower capital charge as the "all eggs in one basket" position $100X_1$. When we take $\rho = \mathrm{VaR}_{95\%}$ however, it is easy to check that

$$\rho\left(100X_1\right) = \sum_{i=1}^{100} \rho\left(X_i\right) < 0 < \rho\left(\sum_{i=1}^{100} X_i\right) \,.$$

The sign convention, $\mathrm{VaR}_{95\%}$ = minus the 5% left quantile of the P&L distribution, corresponds to usage in practice to report VaR positively and the definition of coherence used above where losses are in the left tail of the P&L distribution. Indeed (C3 and C4) yield that a risky position ($X \leq 0$) becomes a positive net regulatory capital charge ($\rho(X) \geq 0$). This convention is not material for the example. ■

The main reason that VaR fails the subadditivity property is the high skewness of the positions X_i; these so–called "spike the firm"–positions (terminology coined by Dilip Madan) do however occur in practice, especially in markets where high severity – low probability events occur. Embrechts et al. [25] discuss the relevance of the above for portfolio theory and stress that one other reason why VaR may lack subadditivity in more general situations is the typical non–Gaussian dependence structure of financial data. I shall come back to this point in Section 4.2.

VaR has a further, very obvious shortcoming in that it only yields a frequency estimate of a high loss, it does not give information on the severity for when that (rare) loss happens. For instance, saying that a 99%, 10 day VaR equals \$1 Mio means that with probability 1%, by the end of a 10 day period, our present portfolio (held fixed) will incur a loss of \$1 Mio or more. VaR does not give any information on this crucial "or more". Going now to Q4., an obvious risk measure stressing the "or more" would be the conditional VaR measure

$$\rho_{\mathrm{CV}}(X) = E(-X/r \mid -X/r > \mathrm{VaR}(X))$$

for some given VaR. Under weak conditions, see Delbaen [19] and Acerbi and Tasche [1], ρ_{CV} is coherent. Clearly $\rho_{\mathrm{CV}} \geq \mathrm{VaR}$ and in some extreme (though realistic) cases $\rho_{\mathrm{CV}} \gg \mathrm{VaR}$. When we would move to ρ_{CV} as a measure determining regulatory capital, the immediate economic question arises of how to handle in practice (given the present regulatory environment) the difference $\rho_{\mathrm{CV}} - \mathrm{VaR}$. One would have to come up with a fully, economically sound regulatory capital framework based on ρ_{CV}; this task is definitely doable but needs combined input from academia, regulators and industry. I refer to Daníelsson et al. [16] for a discussion of the relevant economic pitfalls underlying such a task. At this point, I would like to stress that so far we do not have a full theory for coherent multiperiod risk measurement. First attempts to come up with such a theory, already showing that ρ_{CV} is also problematic, are under discussion (private communication with Freddy Delbaen and Philippe Artzner).

4.2 Allocation and aggregation of risk

As soon as one has reached a concensus between regulators and industry on how to quantify and measure risk, one immediately faces the question: how to use this technology to improve capital allocation. As in the previous section, it would be useful to come up with an axiomatic definition of (risk) capital allocation. Denault [20] has worked out the details of such a coherent risk capital allocation, based on the work on coherent risk measures (Section 4.1), game theory and related results from the actuarial literature. As before, after understanding what rules are scientifically sound, the next step is to work out their actual implementation in practice. On the latter, Matten [45] yields a good introduction.

A related (in some sense, reverse) question concerns the aggregation of risk measures. One often faces the problem that risk measures $\rho(X_i)$, $i = 1, \ldots, d$ have been calculated for separate risk classes; how can we estimate the risk measure $\rho\left(\Psi\left(\underset{\sim}{X}\right)\right)$ of a global position $\Psi\left(\underset{\sim}{X}\right)$ on $\underset{\sim}{X} = (X_1, \ldots, X_d)'$. Typical examples for $X_1, \ldots, X_d$ are one–period risks within certain risk classes (market, credit, operational), but also across different classes. At the highest level, one could think of X_1 standing for market, X_2 for credit and X_3 for operational risk of a particular financial institution over a comparable fixed time period. Another example would correspond to d lines of business in a multiline insurance contract. Depending on the context, for Ψ one could think of examples like:

$$\Psi\left(\underset{\sim}{X}\right) = \sum_{i=1}^{d} X_i = S_d$$

$$\Psi\left(\underset{\sim}{X}\right) = \max_{i=1,\ldots,d} X_i = M_d$$

$$\Psi\left(\underset{\sim}{X}\right) = \sum_{i=1}^{d} (X_i - k)^+$$

$$\Psi\left(\underset{\sim}{X}\right) = \left(\sum_{i=1}^{d} X_i - k\right)^+$$

$$\Psi\left(\underset{\sim}{X}\right) = M_d I_{\{S_d > q_\alpha\}}\,, \quad \text{etc.}$$

For the risk measure ρ one could restrict attention to the class of coherent risk measures. The more interesting case however is that of non–coherence like VaR. In general, one could even take $\rho\left(\Psi\left(\underset{\sim}{X}\right)\right) = F_{\Psi\left(\underset{\sim}{X}\right)}$, the distribution of the financial position $\Psi\left(\underset{\sim}{X}\right)$ or some functional of $\Psi\left(\underset{\sim}{X}\right)$ as for instance moments $E\left(\Psi\left(\underset{\sim}{X}\right)^k\right)$. Often in practice one is given only the marginal distribution functions $F_1, \ldots, F_d$ of $X_1, \ldots, X_d$ respectively, together with some notion of dependence between $X_1, \ldots, X_d$. The crucial point is that most often one does not have full (even usable statistical) information on $F_{\underset{\sim}{X}}$. How can one construct optimal bounds

$$\rho_L\left(\Psi\left(\underset{\sim}{X}\right)\right) \leq \rho\left(\Psi\left(\underset{\sim}{X}\right)\right) \leq \rho_U\left(\Psi\left(\underset{\sim}{X}\right)\right) \tag{4.1}$$

in agreement with the above assumptions. A full discussion of this problem, with several examples, is to be found in Embrechts et al. [23]. The notion of dependence is defined using the language of copulas; suppose $F_1, \ldots, F_d$ are continuous, then there exists a unique function

$$C : [0,1]^d \to [0,1]$$

which is a distribution function with standard uniform marginals so that

$$P\left(X_1 \leq x_1, \ldots, X_d \leq x_d\right) = C\left(F_1\left(x_1\right), \ldots, F_d\left(x_d\right)\right).$$

The function C is called copula as it couples the marginal laws $F_1, \ldots, F_d$ to the joint distribution of $\underset{\sim}{X}$. A typical dependence condition on the unknown copula C of $\underset{\sim}{X}$ could be $C \geq C_o$ for some known copula C_o, e.g. the independence copula $C_o\left(\underset{\sim}{u}\right) = \prod_{i=1}^d u_i$. From Embrechts et al. [23] take for instance $d = 2$, $\Psi\left(\underset{\sim}{X}\right) = X_1 + X_2$ and $\rho = \mathrm{VaR}_{95\%}$ (here we only look at the right tail of the distribution function so that $\mathrm{VaR}_{95\%}$ corresponds to the 95th percentile). If we assume for example that $F_i = \Gamma(3,1)$, $i = 1, 2$, then $\rho\left(X_i\right) = 6.3$, $i = 1, 2$. The unconstrained range of possible values for $\rho\left(X_1 + X_2\right)$ in (4.1) is $[6.47, 14.44]$. If we assume X_1 and X_2 to be independent, then $\rho\left(X_1 + X_2\right) = 10.52$. Whenever X_1 and X_2 are comonotone, i.e. there exist increasing functions f_1, f_2 and a random variable Z so that $X_i = f_i(Z)$, $i = 1, 2$, then ρ becomes additive so that $\rho\left(X_1 + X_2\right) = \rho\left(X_1\right) + \rho\left(X_2\right) = 12.60$. In case $C \geq C_o$, the independent copula, then the possible range becomes $[8.17, 14.41]$. The crucial observation stems from a comparison of the (attainable) upper bound for the unconstrained case (14.44) and the value of $\rho\left(X_1 + X_2\right) = \rho\left(X_1\right) + \rho\left(X_2\right)$ for comonotonic risks (12.60). The gap $[12.60, 14.44]$ corresponds to dependence structures (copulas) on (F_1, F_2) which yield, for the corresponding bivariate model for (X_1, X_2), a non–subadditive risk measure $\rho = \mathrm{VaR}_{95\%}$. The key issue here is not the shape of F_i (we could also have taken $F_i = N(0,1)$) but rather the "damage" non–Gaussian dependence structures can cause on risk management systems. These issues, and their economic implications, need further investigation. See Embrechts et al. [25] for a start.

4.3 Portfolio management under general constraints

Using the one–period setup so far, given $X_o, X_1, \ldots, X_d$ where X_o corresponds to a riskless investment and $X_1, \ldots, X_d$ correspond to risky positions, the basic problem of portfolio analysis concerns the following. Given some risk measure ρ, find the portfolio weights $a_0^*, a_1^*, \ldots, a_d^*$ so that

$$\underset{\sim}{a}^* = \underset{\underset{\sim}{a} \in \mathbb{R}^{d+1}}{\arg\min}\, \rho\left(\underset{\sim}{a}' \underset{\sim}{X}\right)$$

$$\text{so that } r\left(\underset{\sim}{a}' \underset{\sim}{X}\right) = r_0, \text{ fixed.}$$

Here, $r(Y)$ stands for the one–period excess return on the investment Y. The case $\rho = \sigma$ (standard deviation) corresponds to the classical Markowitz problem, leading to the notion of efficient frontier/portfolios. Numerous authors have considered this problem for a variety of risk measures ρ. For instance, going from $\rho = \sigma$ to $\rho = \mathrm{VaR}$ seems a very natural thing to do; however, from an economic (stability) point of view, such optimisation can readily lead to dangerous situations and should be treated with care. For this particular case, a detailed discussion is to be found in Basak and Shapiro [6]. See also Krokhmal et al. [37] and Rockafeller and Uryasev [49]. I also would like to stress that already in the early days of portfolio theory, optimisation with respect to alternative risk measures was considered; see for example Lemus et al. [39] for a review.

4.4 Dynamic models catering for extremes

Most of the work within the realm of Extremes in Economics has centered around either static or discrete time modelling. However, given that extremal moves do occur and are important, one has to take the logical step following on from this and come up with dynamic models for derivative pricing and hedging replacing the Black–Scholes–Merton framework based on geometric Brownian motion:

$$dS(t) = S(t)(\mu\, dt + \sigma\, dW(t))\,.$$

One of the key models already in use in practice is the one where in the above SDE, standard Brownian motion is replaced by a more general Lévy process $\{L(t), t \geq 0\}$. Replacing $(W(t))$ in such a way immediately leads to an incomplete market where there is no unique pricing martingale measure. Fairly recently, several authors have worked out a possible framework. Readers interested in this area of research could for instance consult Eberlein [22], Carr et al. [10], Geman et al. [28] and Levin and Tchernitser [40]. The latter paper can be downloaded via www.gloriamundi.org/var/wps.html, a website containing numerous working papers on VaR.

5 Final comments

In the above discussion, I have tried to stress the need for more economic thinking/modelling in the interplay between EVT and risk management. Especially now when, through Basel II, new basic guidelines for quantitative risk management are under discussion, extreme value theorists with an interest in applying their techniques to finance have to take a closer look at the underlying economic fundamentals. That extremes in finance matter is clear. Looking back at the LTCM case in 1998 where extreme market movements resulted from the Russian moratorium on government bonds, it is interesting to see that the key player in LTCM's up and down, John Meriwether on 21/8/2000 (The Wall Street Journal), talking about his new business JWM Partners, was quoted as follows: "With globalisation increasing, you'll see more crises. Our whole focus is on the extremes now – what's the worst that can happen to you in any situation – because we never want to go through that again". Already in the introduction to our book Embrechts et al. [24], we stated that "Though not providing a risk manager in a bank with the final product he or she can use for monitoring financial risk on a global scale, we (i.e. EVT models) will provide that manager with stochastic methodology needed for the construction of various components of such a global tool". By now, EVT has provided RM for banking and insurance with a useful set of techniques for looking more realistically at extremes. The main emphasis of the present paper is for EVT researchers to take the step beyond and look at the economic implications of their research. The following papers may offer some further guidance along this road:

— Daníelsson [15] offers a critical assessment of the use of statistical, EVT based techniques in RM. With respect to VaR based RM, the author states that "For regulatory use, the VaR measure may give misleading information about risk, and in some cases may actually increase both idiosyncratic and systemic risk". This paper also formed the basis of the earlier quoted Basel II response Daníelsson et al. [16]. Also the following papers offer a useful introduction to the main issues at hand: Jorgensen et al. [33] and Zigrand and Daníelsson [52].

— In [50], Myron Scholes rediscusses some of the basic issues underlying the collapse of LTCM, stressing the crucial importance of market liquidity. Concerning VaR, Scholes concludes the following: "Over the last number of years, regulators have encouraged financial entities to use portfolio theory to produce dynamic measures of risk. VaR, the product of portfolio theory, is used for short–run day–to–day profit and loss exposures. Now is the time to encourage the BIS and other regulatory bodies to support studies on stress test and concentration methodologies. Planning for crises is more important than VaR analysis. And such new methodologies are the correct response to recent crises in the financial industry."

— In the discussions above, I have mainly restricted myself to EVT issues in finance. Similarly, I could have discussed more specifically examples in insurance. At present one witnesses an increasing collaboration between insurance and banking regulators. On the website of the Canadian Institute of Actuaries (www.actuaries.ca), one finds the following Vision Statement: "For actuaries to be recognized as the leading professionals in the financial modelling and management of risk and contingent events". On that same website, a presentation by Allen Brender on Capital Requirements and Stochastic Methods can be found where he concludes "We are only in the early days of a new actuarial age". The publication Hancock et al. [30] gives a very readable introduction to this "new actuarial age".

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