Insurance Risk Management in the Light of Basel II*

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*also in the light of Solvency II.

This talk is based on the following:

- Hans Peter Boller: Solvabilität im Wandel. Internationale Überlegungen und Umsetzung in der Schweiz. BPV Seminar, ETH Zürich, November 3, 2003
- Report of Solvency Working Party, Prepared for IAA Insurance Regulation Committee, February 2002
- Swiss Solvency Test (SST), BPV Bern, December 12, 2003
- D. Filipović and P. Keller: Zielkapital für Lebensversicherer. Swiss Solvency Test. BPV Bern, March 16, 2004
- **Remark**: Don't shoot me, I am only the piano player!

Background and Goals

- Worldwide search for the assessment of solvency for insurance companies

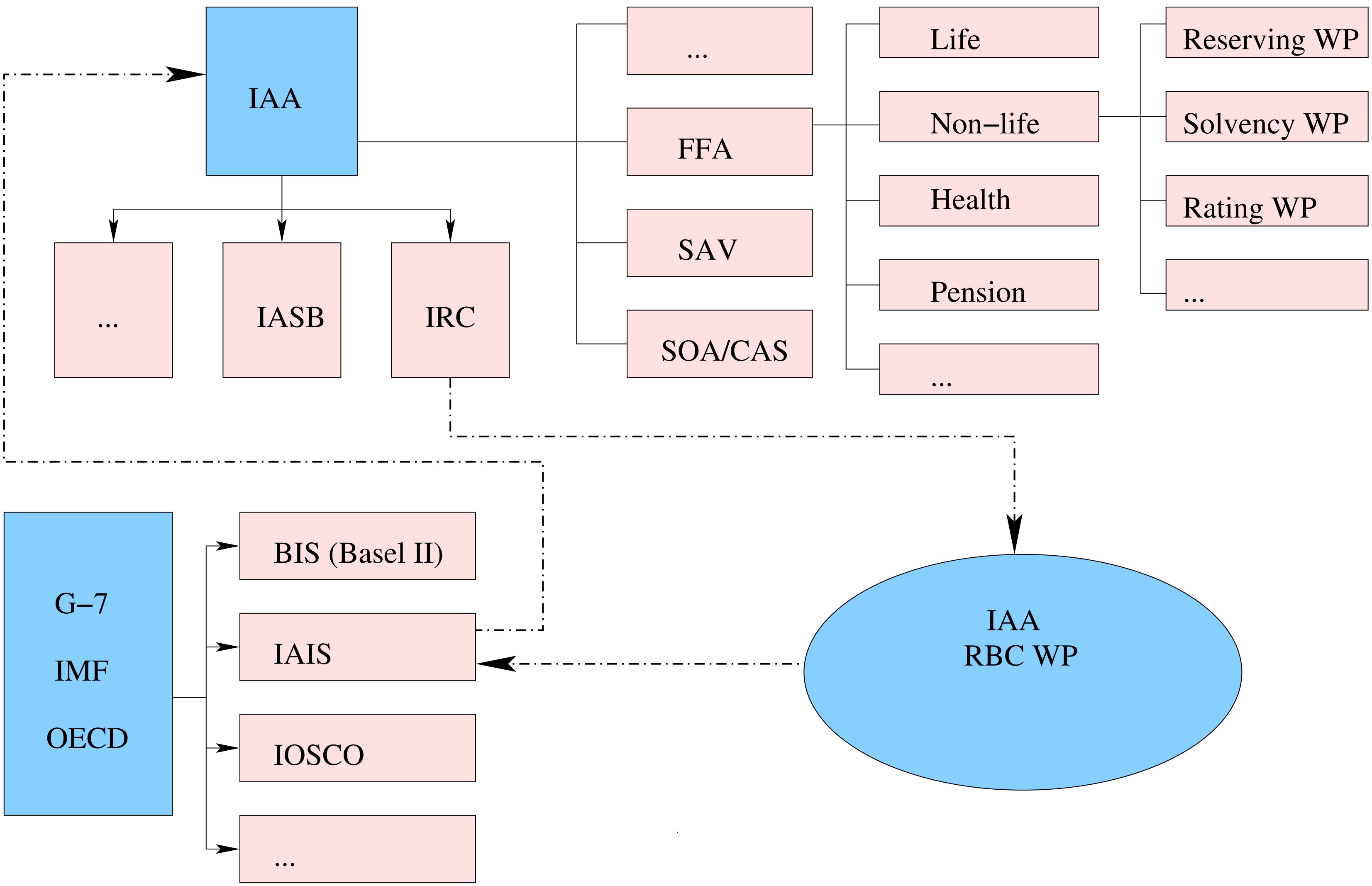
- Who is looking? ➜ Regulators

- Ansätze:
  - Solvency is linked to risk
  - Traditional systems/tools/rules often failed
  - Insufficient early warning from rating agencies

- **Ansatz**: Actuaries should be key players

- As a consequence, regulators look for actuarial support leading to:
  IAA Risk Based Capital Solvency Structure Working Party

Regulation on International Level



Tasks of the IAA WP

IAA forms "Risk Based Capital Solvency Structure Working Party" in the spring of 2002, as requested by the IAIS.

- **Tasks**:
  - Design of a globally applicable risk-based solvency framework for the calculation of capital requirement for non-life, life and health
  - Formulation of methods and principles towards the quantification of solvency
  - Identification of adequate techniques towards the quantification of the risk potentials and the independence between risks
  - Focus on praxis-relevant riskmeasures and internal risk models

IAA WP: Proposed Framework

- Three pillar approach for insurance supervision (à la Basel II):

  ❶ Pillar 1: minimal capital requirements

  ❷ Pillar 2: a supervisory review process

  ❸ Pillar 3: measures to foster market discipline

- **Remark**: rules for capital adequacy (pillar 1) are necessary but not sufficient towards a solvency judgement

- Which risks are to analysed/included?

  - all risks relevant for an insurance company

Proposed Framework (cont'd)

- Principle -versus rules- based approach
  - principle based (CH, UK, NL, "CA",. . .): "do the right thing"-oriented, based on common thrust and risk based regulatory framework
  - rules based ("EU", USA,. . .): objective and easy but probably not capable of mapping all relevant risk factors facing an insurance company
    ➜ could give rise to regulatory arbitrage
- Separate the issues of accounting from the questions of solvency:
  - **accounting** determines the financial progress from period to period (going-concern principle), as such it gives greater emphasis to the annual P&L statement than does
  - prudential **regulation** which focusses on the total balance-sheet under a system that depends upon realistic rules and does not generate hidden surplus

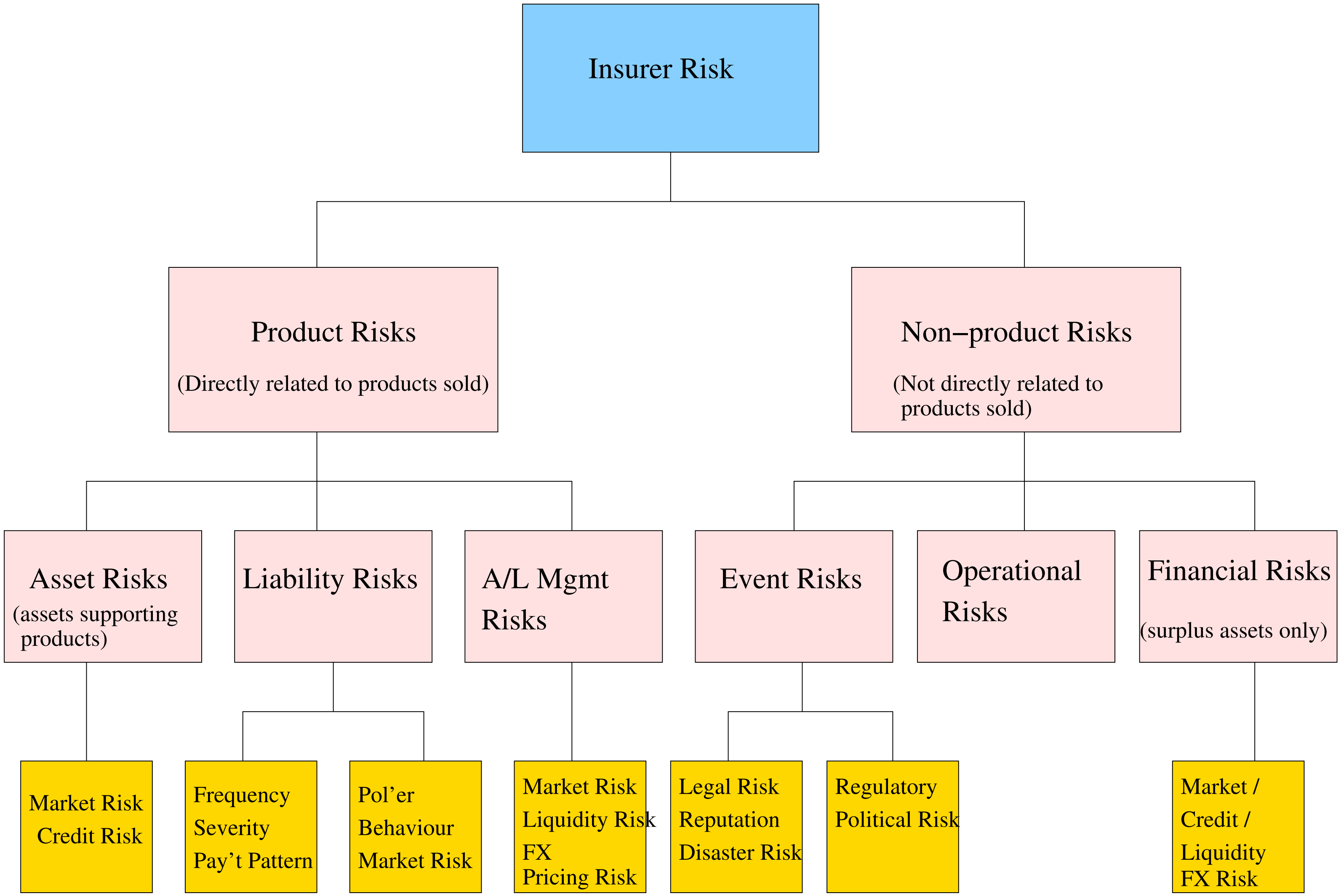
Proposed Framework (cont'd)

- Prudential regulation measures the capacity of insurers to meet their obligations to pay the present and future claims to policy holders (wind-up, run-off, temporarily going-concern,. . .)

- Solvency capital requirement =

$$\text{(total balance sheet requirement)} - \text{(liability requirement)}$$

A general view of insurer risks



WP chosen risk categories

- Underwriting
- Credit
- Market
- Operational (internal OpRisk Basel II)
- Liquidity (linked to market risk)
- Event (external OpRisk Basel II)

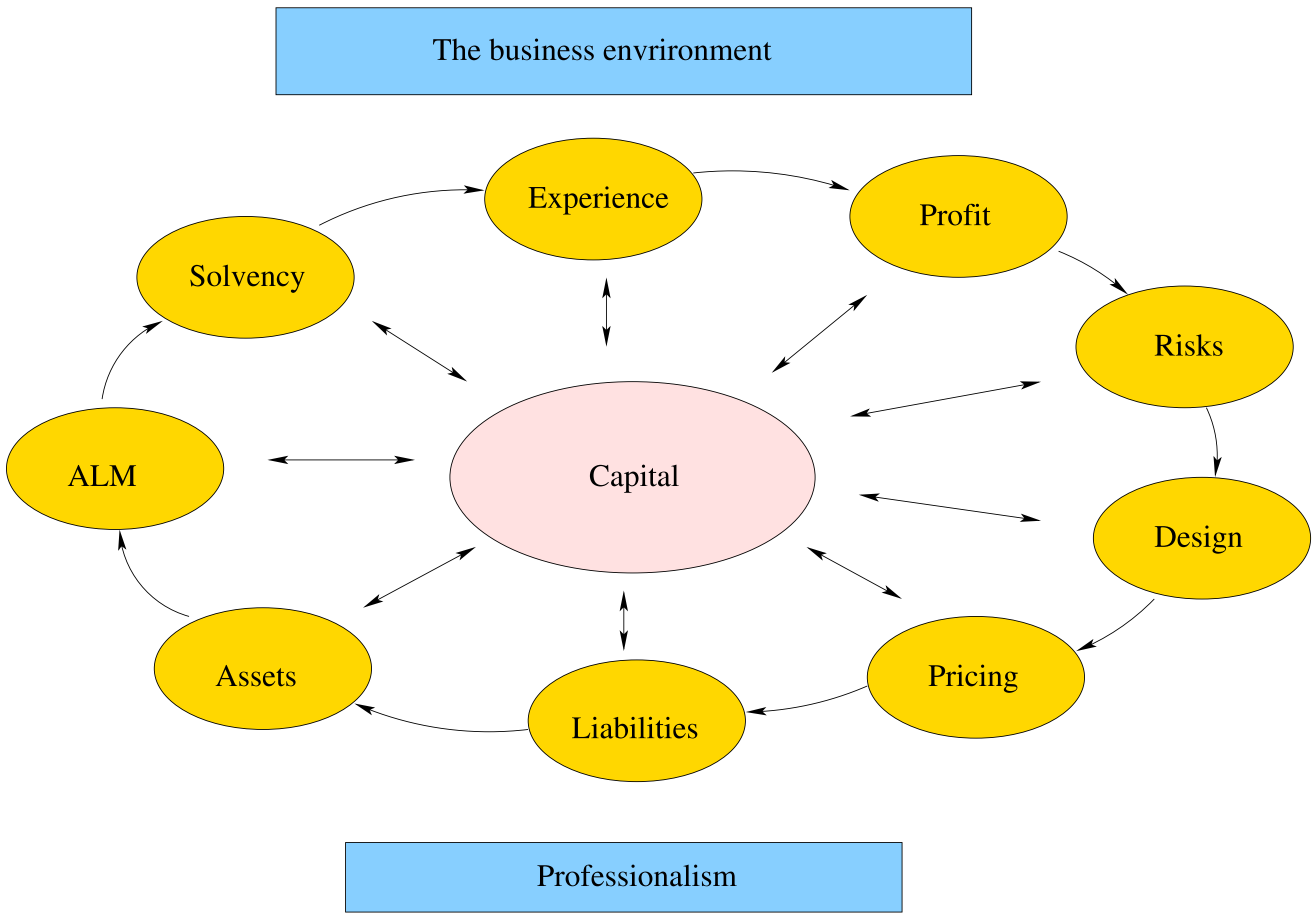
Also note that there are clear interdependencies which need modeling

Overall management of risk by an insurer

- **Ingredients**:
  - design, pricing and underwriting of its insurance policies
  - selection of assets backing the policies
  - estimation of the size and volatility of the liabilities associated with those policies
  - determination of the insurer's capital needs
  - claims management
  - updating of all these elements over time as more data and other information become available or because the underlying risk processes change
  - adequate/sound disclosure/communication process to key stakeholders, e.g. management, investors, policyholders and regulators
  - periodic financial condition analysis, providing a prospective view of the company as a whole

The actuarial control cycle

(Australian Institute of Actuaries)



- A multitude of actuarial tools enter
  - collective risk model
  - aggregate risk models
  - diffusion models and other stochastic processes
  - multi-state models
  - cashflow models

- Specific concerns are:
  - time horizon (long-term RM*)
  - combining risks (copulae*)
  - appropriate risk measures (coherence*)
  - extreme event risk (EVT*)
  - and other . . .

- But at the end of the day, the proof of the pudding is in the eating!

*RiskLab projects/contributions: www.risklab.ch

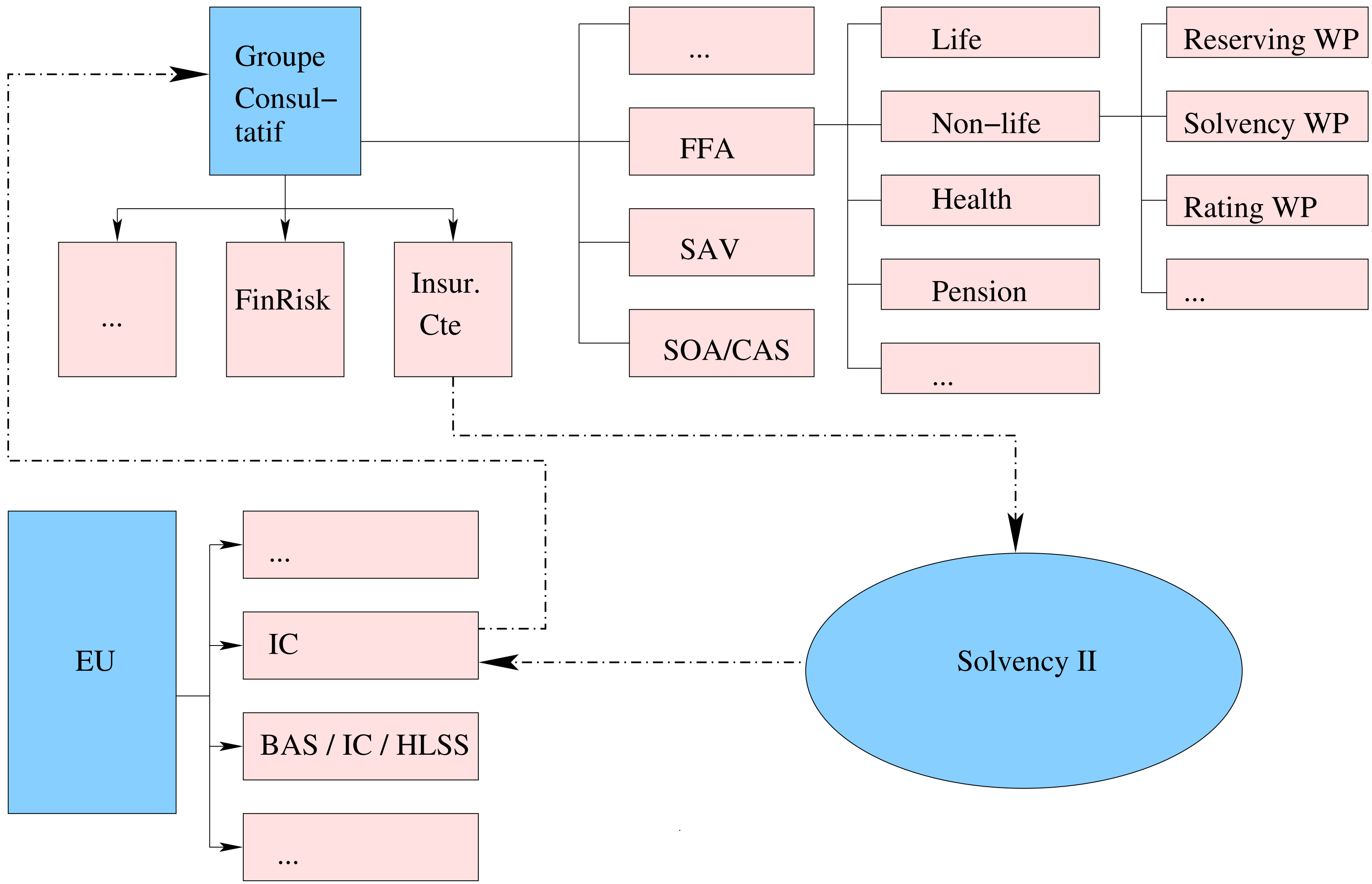
Key components of the new regulatory framework

- Summary:
  - principle-based solvency requirement
  - risk-based
  - expected shortfall as risk measure (beyond VaR!)
  - modeling of dependencies through "increased correlations" or copula based approach, stress scenarios (EVT)
  - transition from conservative standard-factor-model to a full internal model Ansatz (allowing for intermediate levels)
  - potential issues:
    - time-horizon (life versus non-life)
    - confidence level (per risk category, overall level)

Introduction of such a regulatory framework: when?

- EU evaluates the applicability of the framework for
  - solvency 2 (200x?)
  - reinsurance-guidelines (now: 2004)
- new Swiss supervisory body (2004/5)
- consequences for Australia or Canada?
- revision of the US-RBC model in USA?

Regulation in Europe



Solvency in Europe (EU and CH)

http://europa.eu.int/comm/internal_market/insurance/solvency_en.htm

- **1970s**: first EU non-life and life directives on solvency margins (= extra capital as a buffer against unforeseen events such as higher than expected claims levels or unfavourable investment results)
- **1997**: Müller Report ("Solvency of insurance undertakings")
  ➜ review of solvency rules ➜ Solvency I project initiated
- **2001**: Solvency II initiated ➜ Sharma Report
- **2002**: Solvency I completed (in force by 2004)
- **2003**: Solvency II: end of phase 1 (design of the system)
- **2004**: Solvency II: phase 2 (detailed technical rules)

Solvency I

- **Aim**: revising and updating current EU solvency regime
- Minimum guarantee fund (minimal capital required) = 3 Mio EUR
- Solvency margin = 16% – 18% of premium (non-life), 4% of technical provisions (life)
- **Pros**
  - simple, robust
  - easy to understand and use
  - inexpensive to administer
- **Cons**: volume-based, **not explicitly** risk-based (e.g. no difference between investment mixes or asset/liability profiles . . .)
  ➜ Solvency II

Solvency II: general principles

- risk-oriented assessment of overall solvency
- Basel II type 3 pillar structure as starting point
- encourage insurers to measure and manage their risks
- consistency between financial sectors (banking & insurance)
- efficient supervision of insurance groups and financial conglomerates
- increased harmonisation of supervisory methods between legislations
- compatible with international developments (IAIS, IAA, IASB)

Solvency II: 3 pillars

- **Pillar 1**: capital requirements
- **Pillar 2**: supervisory review process
  - internal control and risk management, ALM rules, reinsurance
  - harmonization at EU level
  - minimum criteria for on-site inspections
  - peer reviews of supervisory practice by national authorities
- **Pillar 3**: market transparency and disclosure
  - co-ordination of reporting requirements

Solvency II: Pillar 1

- Technical provisions (policy liabilities)
- Appropriate assets supporting those obligations
- **Capital requirements**: two levels
  - minimum capital level (safety net): simple, no internal model, court actions
  - target capital: riskmeasure-based, internal models allowed

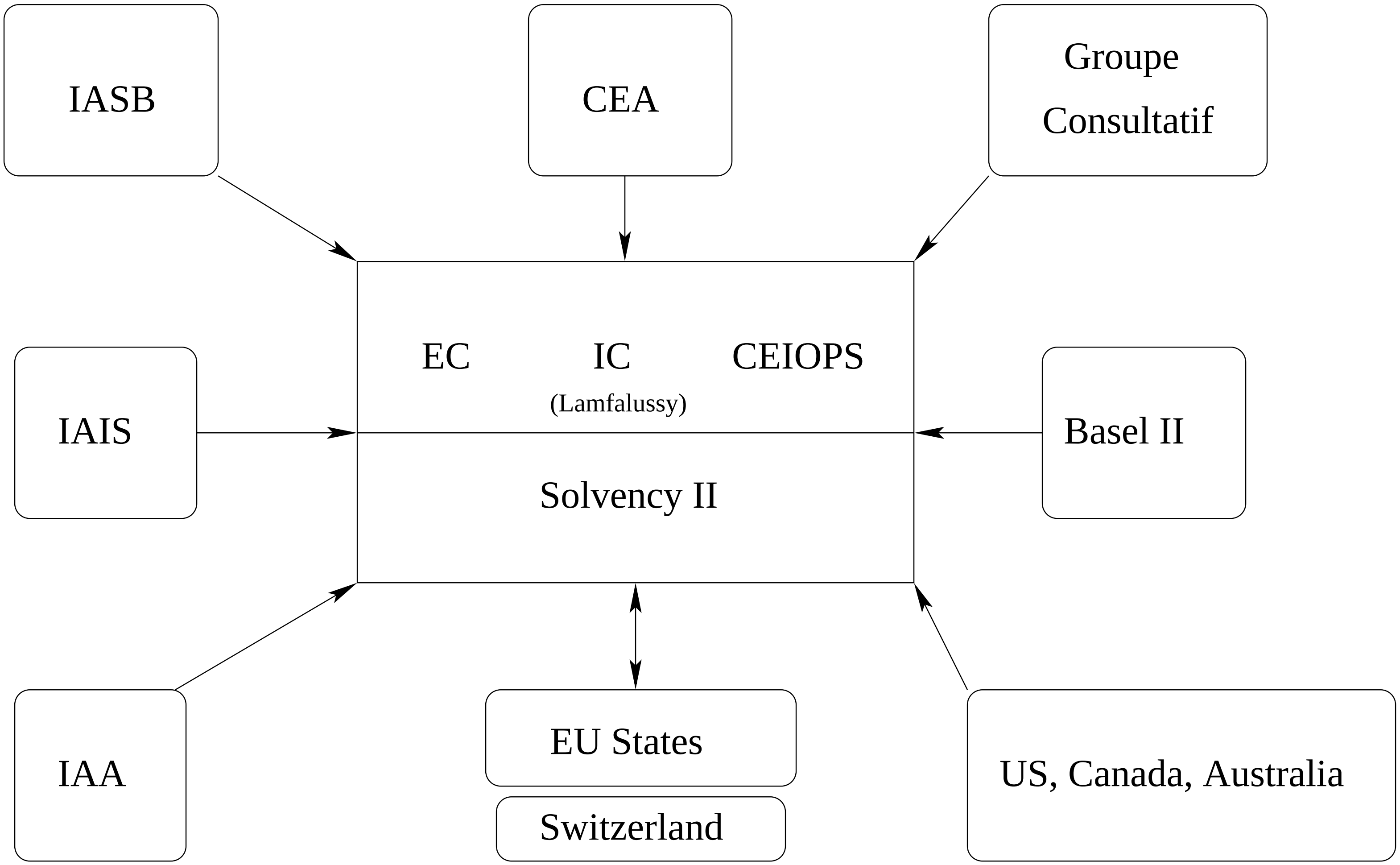
Basel II $\neq$ Solvency II

- "The difference between the two prudential regimes goes further in that their actual objectives differ. The prudential objective of the Basel Accord is to reinforce the soundness and stability of the international banking system. To that end, the initial Basel Accord and the draft New Accord are directed primarily at banks that are internationally active. The draft New Accord attaches particular importance to the self-regulating mechanisms of a market where practitioners are dependent on one another. In the insurance sector, the purpose of prudential supervision is to protect policyholders against the risk of (isolated) bankruptcy facing every insurance company. The systematic risk, assuming that it exists in the insurance sector, has not been deemed to be of sufficient concern to warrant minimum harmonisation of prudential supervisory regimes at international level; nor has it been the driving force behind European harmonisation in this field"

  (EU Insurance Solvency Sub-Committee (2001))

- Also: continued convergence in the supervision of the financial services sector

Solvency II: organisation/compatibility



Solvency II: (actuarial) work in progress

- technical provisions (vs. regulatory capital)
- market-based (financial) valuation methods
  - stochastic cashflows and discounting
  - interest guarantees, surrender and other options
- target capital: development, calibration, field-testing
- internal models: validation, data, methods, parameters

($\longrightarrow$ Vesa Ronkainen (EC), Nov 2003)

A concrete set-up: The Swiss Solvency Test (SST)

- Fall 2002: Herbert Lüthy new director of FOPI (Federal Office of Private Insurance)
- Reorientation of FOPI to increased prudential supervision
- new draft Insurance Supervision Act (to be in force by 1 Jan 2005 (!)) specifies solvency to be risk-based
- May 2003: start of Swiss Solvency Test (SST) project
- Dec 2003: first conceptual phase accomplished
  ➜ "SST: Project Status Dec 2003"
  `www.bpv.admin.ch/en/aktuell/dossiers.htm`

- Standard Setting Board (steering committee) founded
- current: specifics, standard framework for non-life, life and health
- May 2004: start of test run
- SST working groups in cooperation with industry (all major Swiss insurers and reinsurers participate)
- SAV (Swiss Actuarial Association) and SVV (Swiss Insurance Association) support SST project
- **Aim**: to develop in a short time a flexible, EU (Solvency II)-compatible solvency framework

Requirements for SST

- risk-based
- consistency in asset and liability valuations
- compatibility with EU (Solvency II)
- compatibility with regulatory demands on other market players (Basel II)
- principles-based
- **Aims**:
  - regulatory density should remain reasonable
  - cooperation with SST project should add value to companies
  - SST should enable companies to better assess and manage risks
  - SST should give incentive for use of internal models

Risks

- Which risks should be covered by target capital?

- Choice is not unique, as some risks

  - can be quantified (Pillar 1):

    insurance, market, credit risk

  - should be qualitatively assessed (Pillar 2):

    liquidity risk, operational risk (?)

  - are *not* quantifiable:

    management, strategy, operational risk (?)

  - should not give rise to capital requirements (Pillar 2):

    very rare events

Market-consistent valuation

Assets: 3 classes

- Marking-to-market if liquid market exists: government bonds, liquid shares,. . .
- Derived from prices of similar, traded instruments (mix of marking-to-market and marking-to-model): illiquid bonds, real estate,. . .
- Marking-to-model: participations, private equity,. . .

Liabilities:

- Risk-neutral expectation of future cash flows discounted with risk free interest rate
- Reinsurance can be fully taken into account
- Market value margin (loading) defined as cost of regulatory capital

Minimum capital and target capital

- Solvency II: minimum capital and target capital
- Minimum capital level is last line before default: simple and objectively calculable
- Target capital is more risk- and company specific but less objective
- SST has a two-level approach:
  - minimum capital level as in Solvency I, based on statutory values
  - target capital based on market-consistent valuation
- **Advantage**: new system is at least as safe as old one (Solvency I) and takes account of (economically correct) market-consistent valuation

Mathematical setup

- $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$, $\mathbb{P} =$ real-world measure
- Final time horizon $T \in \mathbb{N}$
- Financial instruments $S(t) = (S_0(t), S_1(t), \dots, S_m(t))$,
- $S_0 =$ numéraire, disounting $\widetilde{S}(t) := \frac{S(t)}{S_0(t)}$
- Insurance policies with initial premiums $P_1, \dots, P_n$
- Policy P_i receives cashflow $Z_i(1), Z_i(2), \dots, Z_i(T)$
  (no new business after $t = 1$)

A simple arbitrage result

- **Assumptions**:
  - can reinsure any fraction $b_i \in [0,1]$ of risk P_i at $t=0$
  - self-financing predictable trading strategy $a(t)$ in $S(t)$

- Final result for insurer (discounted!)

$$\widetilde{G}(T) := \sum_{0<t\leq T} \Big(a(t)\cdot \Delta\widetilde{S}(t) - (1-b)\cdot \widetilde{Z}(t) \Big)$$

- **Definition**: (a,b) is **(negative) arbitrage** if

$$\widetilde{G}(T) \overset{(\leq)}{\geq} 0 \quad \text{and} \quad \mathbb{P}\left[\widetilde{G}(T) \overset{(<)}{>} 0 \right] > 0.$$

A simple arbitrage result (cont'd)

- **Theorem**: There is no (negative) arbitrage iff $\exists\ \mathbb{Q}_u \sim \mathbb{P}$ $(\mathbb{Q}_l \sim \mathbb{P})$ such that

$$P_i(0) \overset{(\geq)}{\leq} \mathbb{E}_{\mathbb{Q}_{u(l)}}\Big[\sum_{0<t\leq T} \widetilde{Z}_i(t)\Big]$$

and $\widetilde{S}$ is a $\mathbb{Q}_u$-martingale ($\mathbb{Q}_l$-martingale).

- **Convention**: from now on $b = 0$, and write $Z(t) := \sum_i Z_i(t)$

Assumption: special information structure

- $(\mathcal{G}_t)$ and $(\mathcal{H}_t)$ $\mathbb{P}$-independent filtrations with $\mathcal{F}_t = \mathcal{G}_t \vee \mathcal{H}_t$.
- $S(t)$ is $\mathcal{G}_t$-measurable ("financial risk")
- $Z_i(t)$ is $\mathcal{H}_t$-measurable ("insurance risk")
- pricing measure $\mathbb{Q} \sim \mathbb{P}$ with $\frac{d\mathbb{Q}}{d\mathbb{P}}\big|_{\mathcal{F}_t} = \frac{d\mathbb{Q}}{d\mathbb{P}}\big|_{\mathcal{G}_t} \times \frac{d\mathbb{Q}}{d\mathbb{P}}\big|_{\mathcal{H}_t}$ such that
  - $\widetilde{S}$ is a $\mathbb{Q}$-martingale
  - $\mathbb{Q}|_{\mathcal{H}_t} = \mathbb{P}|_{\mathcal{H}_t}$: no risk premium in $\mathbb{Q}$ for insurance risk (LLN)

➜ $(\mathcal{G}_t)$ and $(\mathcal{H}_t)$ also $\mathbb{Q}$-independent

➜ no (negative) arbitrage

Market-consistent valuation

- Market-consistent $=$ risk-neutral valuation of cashflows
- $\widetilde{S}(t) = \mathbb{E}_{\mathbb{Q}}\Big[\widetilde{S}(T) \mid \mathcal{F}_t\Big]$
- **Define** liabilities value $L_i(t)$ (=best estimate) by

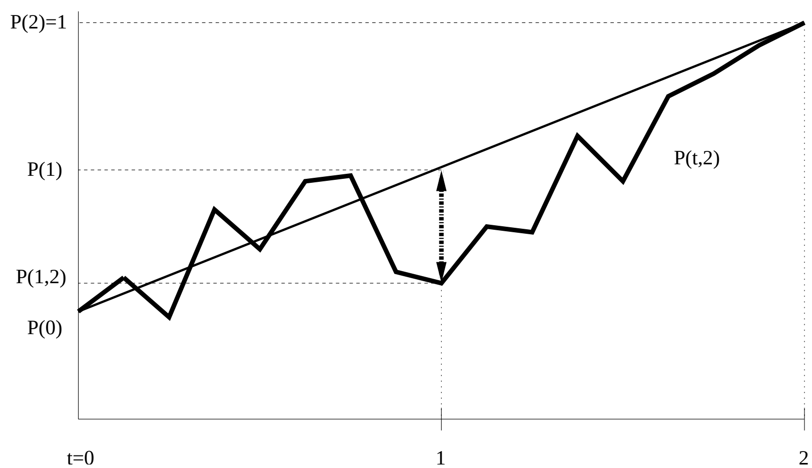
$$L_i(t) := S_0(t)\mathbb{E}_{\mathbb{Q}}\Big[\sum_{t<u\leq T} \widetilde{Z}_i(u)\Big|\mathcal{F}_t\Big] = \sum_{t<u\leq T} P(t,u)\mathbb{E}\big[Z_i(u) \mid \mathcal{F}_t\big]$$

with $P(t,u) := \mathbb{E}_{\mathbb{Q}}\left[\frac{S_0(t)}{S_0(u)} \mid \mathcal{F}_t\right]$ = zero-coupon bond price

- $L_i(t)$ is not a prudential provision (LLN, no safety loading yet)

Options are taken into account. A simple example

- pure endowment with term $T = 2$
- surrender option at $t = 1$ for statutory reserve $P(1)$
- initial reserve (= premium) $P(0)$, final reserve (= payoff) $P(2) = 1$
- τ = life time of insured ($\mathcal{H}_t$-stopping time)



- **Cashflow**:

$$Z(1) = P(1)1_{\{P(1)>P(1,2)\}}1_{\{\tau>1\}}$$
$$Z(2) = (1 - 1_{\{P(1)>P(1,2)\}})1_{\{\tau>2\}}$$

- **Liability at $t = 0$**:

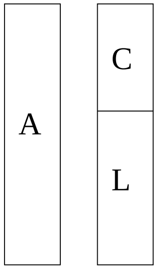
$$\begin{aligned}
L(0) &= \mathbb{E}_{\mathbb{Q}}[\widetilde{Z}(1) + \widetilde{Z}(2)] = \mathbb{E}_{\mathbb{Q}}\big[\widetilde{Z}(1) + \mathbb{E}_{\mathbb{Q}}[\widetilde{Z}(2) \mid \mathcal{F}_1]\big] \\
&= \cdots \\
&= \mathbb{E}_{\mathbb{Q}}\left[\frac{(P(1) - P(1,2))^+}{S_0(1)}\right]\mathbb{P}[\tau > 1] && \text{(caplet)} \\
&\quad + \mathbb{E}_{\mathbb{Q}}\left[\widetilde{P}(1,2)1_{\{P(1)>P(1,2)\}}\right](\mathbb{P}[\tau > 1] - \mathbb{P}[\tau > 2]) && \text{(correction (small))} \\
&\quad + P(0,2)\mathbb{P}[\tau > 2] && \text{(terminal cashflow)}
\end{aligned}$$

- $\mathbb{P}[\tau > 1]$, $\mathbb{P}[\tau > 2]$: survival probabilities

Risk-bearing capital = assets − liabilities

- **Value of the assets**:

$$\begin{aligned} \text{at } t-1: \quad & A(t-1) = a(t) \cdot S(t-1) \\ \text{at } t: \quad & A(t) = a(t) \cdot S(t) - Z(t) \\ a \text{ self-financing}: \quad & A(t) = a(t+1) \cdot S(t) \end{aligned}$$



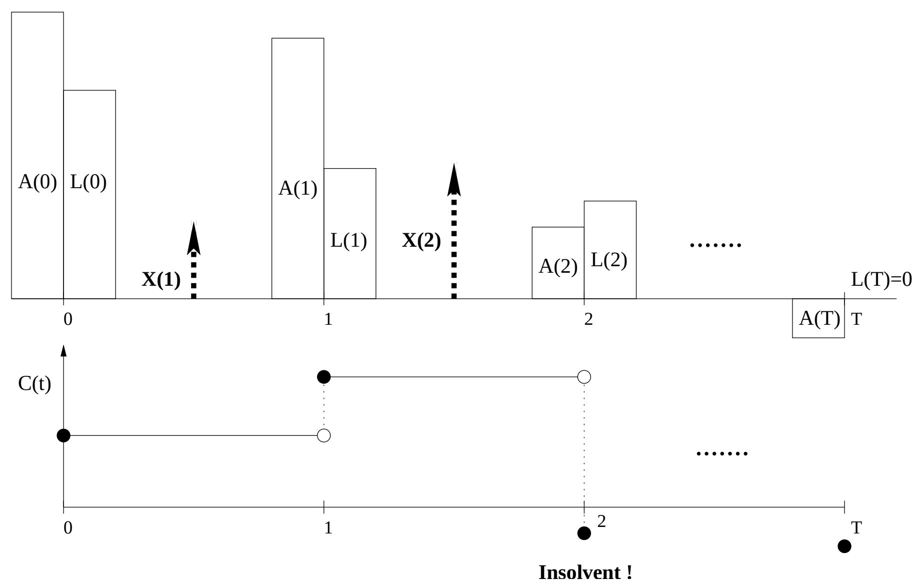
- **Risk-bearing capital**: $\qquad C(t) := A(t) - L(t)$

- **Annual result** (discounted)

$$\begin{aligned} \Delta\widetilde{C}(t) &= \Delta\widetilde{A}(t) - \Delta\widetilde{L}(t) \\ &= \underbrace{a(t) \cdot \Delta\widetilde{S}(t)}_{\text{financial gain}} + \underbrace{\widetilde{L}(t-1) - \widetilde{Z}(t) - \widetilde{L}(t)}_{\text{technical result}} \end{aligned}$$

- **Theorem**: $\tilde{C}$ is a $\mathbb{Q}$-martingale (Hattendorf et al).
- $\widetilde{A}(t) = \widetilde{C}(t) + \widetilde{L}(t)$ is a $\mathbb{Q}$-supermartingale
- $L(T) = 0 \Rightarrow A(T) = C(T)$
- $\widetilde{C}(t) = \mathbb{E}_{\mathbb{Q}}\left[\widetilde{A}(T) \mid \mathcal{F}_t\right]$ functional of $\widetilde{A}$
- $\widetilde{L}(t) = \mathbb{E}_{\mathbb{Q}}\left[\widetilde{A}(t) - \widetilde{A}(T) \mid \mathcal{F}_t\right]$ functional of $\widetilde{A}$

- **Solvency condition**: $A(t) \geq 0$ for all $t \leq T$ with "sufficient certainty"



➜ multiperiod risk measure ρ

➜ target capital at inception $t = 0$:

$$TC(0) = C(0) + \rho\left(\widetilde{A}(0), \ldots, \widetilde{A}(T)\right)$$

Coherent multiperiod risk measures (Artzner et al.)

- $\mathcal{G} = \{X \text{ bounded adapted processes}\} = L^\infty(\Omega', \mathcal{F}', \mathbb{P}')$ with
  - $\Omega' = \Omega \times \{0, 1, 2 \ldots, T\}$
  - $\mathcal{F}' = \sigma\{B_t \times \{t\} \mid B_t \in \mathcal{F}_t\}$
  - $\mathbb{P}'\left[\bigcup_{0 \le t \le T} B_t \times \{t\}\right] = \sum_{0 \le t \le T} \mu_t \mathbb{P}[B_t], \quad \mu_t \ge 0, \quad \sum_{0 \le t \le T} \mu_t = 1$
- **Representation result**: ρ coherent and satisfies Fatou property

$$\rightarrow \quad \rho(X) = \sup_{f \in \mathcal{P}} \sum_{0 \le t \le T} \mu_t \mathbb{E}[-f(t) X(t)]$$

for $\mathcal{P} \subset L^1_+(\mathbb{P}')$ closed convex with $\sum_{0 \le t \le T} \mu_t \mathbb{E}[f(t)] = 1$ for all $f \in \mathcal{P}$

Coherent multiperiod risk measures (cont'd)

- **Example**: multiperiod expected shortfall at level α

$$ES_\alpha[X] = \frac{1}{\alpha} \sum_{0 \le t \le T} \mu_t \mathbb{E}\left[(q_\alpha(X) - X(t))^+\right] - q_\alpha(X)$$

where $q_\alpha(X)$ is an α-quantile of X

$$\mathbb{P}'[X < q_\alpha(X)] \le \alpha \le \mathbb{P}'[X \le q_\alpha(X)]$$

Optimization over strategies (Föllmer–Schied 2002)

- $\rho(X) = \sup_{f \in \mathcal{P}} \sum_{0 \le t \le T} \mu_t \mathbb{E}[-f(t)X(t)]$ coherent risk measure

- $X(t) = \widetilde{Z}(1) + \cdots + \widetilde{Z}(t)$ "risk process" $(X(0) := 0)$

- Optimize

$$\pi(X) := \inf_{a \in \mathcal{A}} \rho(X + V(a))$$

where $V(a,t) := \sum_{0 < u \le t} a(u) \cdot \Delta \widetilde{S}(u) =$ value process,
$\mathcal{A} = \{a$ bounded predictable processes$\}$

- **Minimax Theorem**: if $\mathcal{P}$ locally compact in $L^1(\mathbb{P}')$ then

$$\pi(X) = \inf_{a\in\mathcal{A}} \sup_{f\in\mathcal{P}} \sum_{0\le t\le T} \mu_t \mathbb{E}[-f(t)(X(t)+V(a,t))]$$
$$= \sup_{f\in\mathcal{P}} \inf_{a\in\mathcal{A}} \sum_{0\le t\le T} \mu_t \mathbb{E}[-f(t)(X(t)+V(a,t))]$$

- **Polar Theorem**:

$$\Rightarrow \quad \pi(X) = \sup_{f\in\mathcal{P}\cap\mathcal{M}} \sum_{0\le t\le T} \mu_t \mathbb{E}[-f(t)X(t)]$$

where $f \in \mathcal{M} \Leftrightarrow f > 0$ and $f\widetilde{S}$ are martingales (f = density process of EMM)

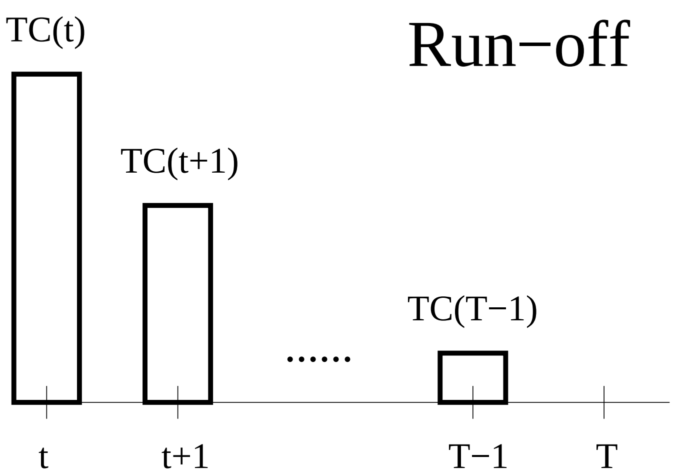
- **Corollary**: if $\mathcal{P} \subset \mathcal{M}$ then ρ is already optimal w.r.t. investment

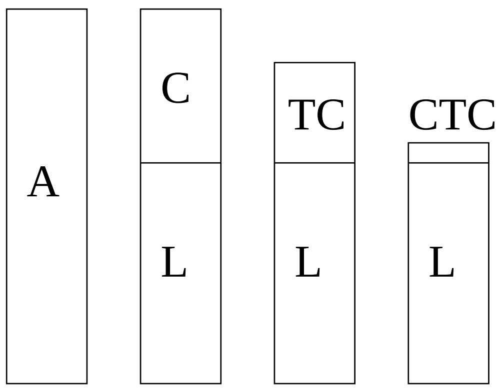
Required value of tied assets

- $L(t) = S_0(t)\mathbb{E}_{\mathbb{Q}}\left[\sum_{t<u\leq T} \widetilde{Z}(u) \mid \mathcal{F}_t\right]$ is not a prudential provision

- To guarantee that an insurer B can take over the portfolio from A for run-off at t (B has to provide target capital $TC(u)$ at $u = t, t+1, \ldots, T$) we require that the cost of target capital $CTC(t)$ in addition to $L(t)$ be covered by A at any time:

$$A(t) \geq L(t) + CTC(t) =: \text{required value of tied assets}$$





CTC = safety loading

- $CTC(t)$: cost of capital factor s (e.g. spread on subordinated bonds)

$$\Rightarrow \widetilde{CTC}(t) = \mathbb{E}_{\mathbb{Q}}\Big[s \sum_{t \le u < T} \frac{TC(u)}{S_0(u+1)} \mid \mathcal{F}_t\Big]$$

- **CTC = safety loading**: Cost of target capital $CTC(t)$ can be viewed as safety loading for liabilities
  - Economically based argument
  - Market view of riskyness of insurance companies goes into calculation
  - Depends on whole duration of run-off
  - Less arbitrary than a quantile approach
  - Compatible with Fair Value methodology (IFRS)

Risk measure of SST

- Expected shortfall on a one-year time horizon:

$$TC(0) = C(0) + ES_\alpha\left[\widetilde{C}(1) - \widetilde{CTC}(1)\right]$$

- Pragmatic approach: $\dfrac{\mathbb{E}_{\mathbb{Q}}\big[\widetilde{TC}(t)\big]}{\mathbb{E}_{\mathbb{Q}}\big[\widetilde{L}(t)\big]} = \dfrac{TC(0)}{L(0)}, \qquad t = 1, \ldots, T$ and

$$\widetilde{CTC}(1) \approx s\mathbb{E}_{\mathbb{Q}}\Big[\sum_{1 \le t < T} \widetilde{TC}(t)\Big] = s\,\frac{TC(0)}{L(0)} \sum_{1 \le t < T} E_{\mathbb{Q}}\big[\widetilde{L}(t)\big]$$

$$\rightarrow \quad TC(0) = s\,\frac{TC(0)}{L(0)} \sum_{1 \le t < T} E_{\mathbb{Q}}\big[\widetilde{L}(t)\big] + ES_\alpha\big[\widetilde{C}(1) - C(0)\big]$$

$$= \frac{L(0)}{L(0) - s\sum_{1 \le t < T} E_{\mathbb{Q}}\left[\widetilde{L}(t)\right]} ES_\alpha\left[\widetilde{C}(1) - C(0)\right]$$

- Remark: $TC(0)$ is of the form $C(0) + \rho\left(\widetilde{A}(0), \ldots, \widetilde{A}(T)\right)$.

Aggregation with extreme scenarios

- additional state variable $\Theta = \begin{cases} \theta_0 & \text{normal year} \\ \theta_1 & \text{extreme year} \end{cases}$, $\qquad \mathbb{P}[\Theta = \theta_1] =: p$
- model distribution of $\widetilde{C}(1)$ w.r.t. $\mathbb{P}_{\theta_0} := \mathbb{P}[\cdot \mid \Theta = \theta_0]$
- approximate distribution of $\widetilde{C}(1)$ w.r.t. $\mathbb{P}_{\theta_0} := \mathbb{P}[\cdot \mid \Theta = \theta_0]$ by scenario evaluation
- extreme scenario = event $\mathcal{S} \subset \{\Theta = \theta_1\}$ ($\Rightarrow \mathbb{P}[\mathcal{S}] < p$)
- Example: $\mathcal{S} =$"Spanish influenza 1918", expected every 80 years $\Rightarrow \mathbb{P}[\mathcal{S}] = \frac{1}{80}$
- chose and evaluate n scenarios $\mathcal{S}_1, \ldots, \mathcal{S}_n$

$$c_i := \mathbb{E}_{\theta_1}\left[\widetilde{C}(1) \mid \mathcal{S}_i\right] = \mathbb{E}\left[\widetilde{C}(1) \mid \mathcal{S}_i\right] \qquad (\mathcal{S}_i \subset \{\Theta = \theta_1\})$$

and set

$$\mathbb{P}_{\theta_1}[\widetilde{C}(1) \leq x] \approx \frac{\#\{i \mid c_i \leq x\}}{n}$$

Aggregation with extreme scenarios (cont'd)

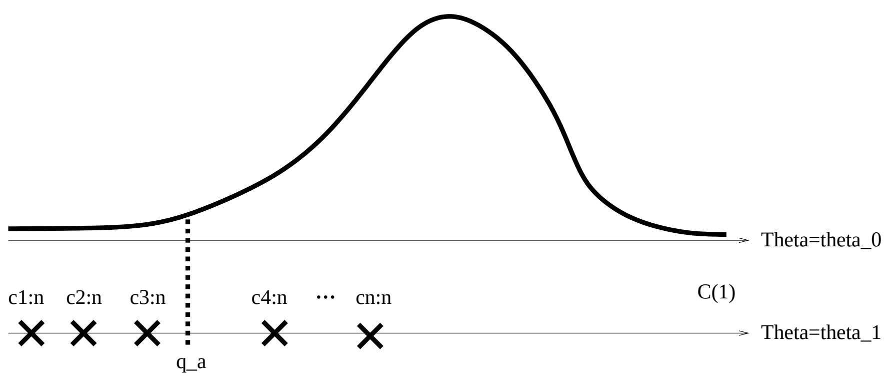
- find α-quantile q_α (suppose $q_\alpha \neq c_i$ and $\mathbb{P}_{\theta_0}[\widetilde{C}(1) < q_\alpha] = \mathbb{P}_{\theta_0}[\widetilde{C}(1) \leq q_\alpha]$)

$$p\,\frac{\#\{i \mid c_i < q_\alpha\}}{n} + (1-p)\mathbb{P}_{\theta_0}[\widetilde{C}(1) < q_\alpha] = \alpha$$

- Expected Shortfall:

$$ES_\alpha[\widetilde{C}(1)] = -\frac{p}{\alpha}\underbrace{\frac{\sum_i c_i 1_{\{c_i < q_\alpha\}}}{n}}_{\approx \mathbb{E}_{\theta_1}\left[\widetilde{C}(1)1_{\{\widetilde{C}(1)<q_\alpha\}}\right]} - \frac{1-p}{\alpha}\mathbb{E}_{\theta_0}\left[\widetilde{C}(1)1_{\{\widetilde{C}(1)<q_\alpha\}}\right]$$

- If $\mathbb{E}_{\theta_0}\left[\widetilde{C}(1)1_{\{\widetilde{C}(1)<q_\alpha\}}\right] \approx 0$ then extreme scenarios dominate (➜ $p \geq \alpha$)



Normal regime: factor structure

- **Aim**: model $\widetilde{C}(1)$ under $\mathbb{P}_{\theta_0}$ (normal regime)

- Identify risk factors $X(t) = (X_0(t) = t, X_1(t), \ldots, X_d(t))$

  financial instruments: $S(t) = f(X(t))$; liabilities: $L(t) = g(X(t))$

- X_i: financial risk factors (interest rates, stock market index, FX, real estate index, options implied volatility), insurance risk factors (mortality, lapse rate, disability, cost, claim size distribution parameters, etc.)

- $C(1) = a(1) \cdot f(X(1)) - g(X(1)) - Z(1) =: F(X(1)) - Z(1)$

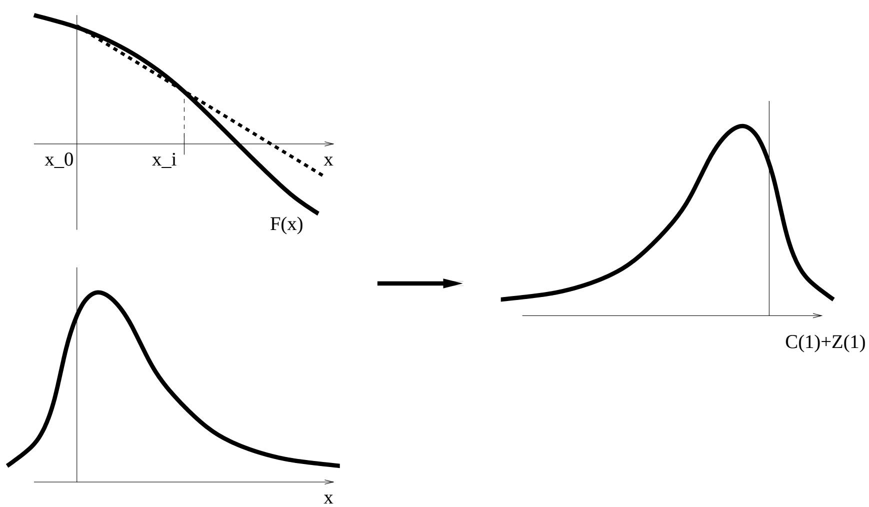
- Today we know

$$C(0) = F(X(0)).$$

Normal regime: factor structure (cont'd)

- **Sensitivity analysis**: insurers evaluate $F(x_1), \ldots, F(x_N)$
- Approximation: find $\widehat{F}$ (e.g. piecewise linear) such that $\widehat{F}(x_i) = F(x_i)$
- Model $X(1)$ and $Z(1)$ under $\mathbb{P}_{\theta_0}$ (normal regime)
- Approximation of risk-bearing capital:

$$\widehat{C}(1) = \widehat{F}(X(1)) - Z(1)$$



Scenarios

$\Theta = \theta_1$: extreme year with one or more scenarios of the following categories

- Hail
- Windstorm
- Flood
- Electricity blackout
- Traffic accident
- Industry (explosion, gas, etc.)
- Aviation
- Pandemics (e.g. "Spanish influenza 1918")
- Market crash
- Default of reinsurer
- ...

Some aspects on OpRisk

- **Advanced Measurement Approaches (AMA)**: Allows banks to use their own methods for assessing their exposure to OpRisk

- Preconditions: Bank must meet

  - qualitative and
  - quantitative

  standards before using the AMA

- Risk mitigation via insurance allowed

Quantitative standards

- The Committee does not stipulate which (analytical) approach banks must use
- However, banks must demonstrate that their approach captures *severe tail loss events*, e.g 99.9% VaR
- In essence,

$$\text{MRC} = \text{EL} + \text{UL}$$

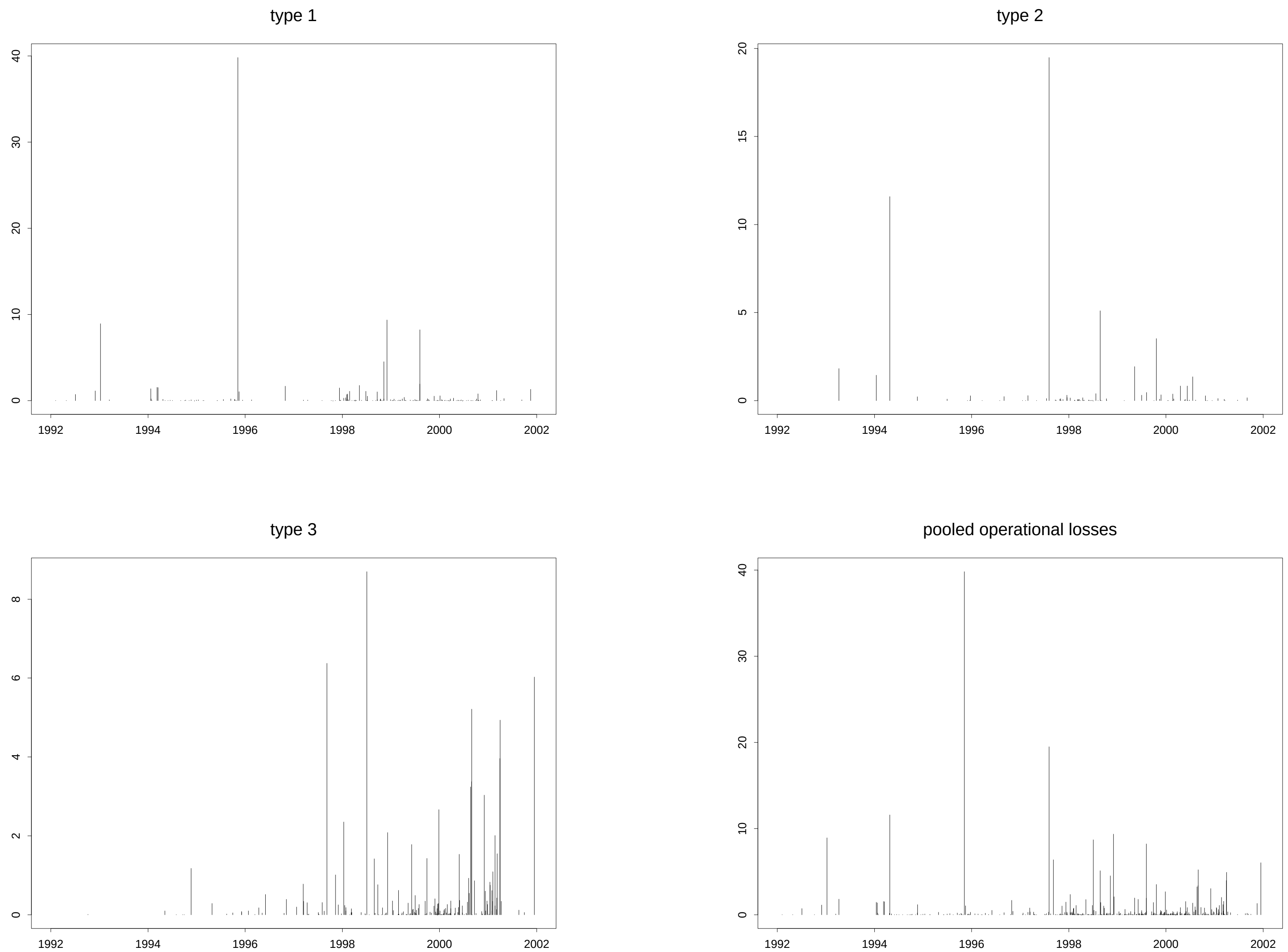
If ELs are already captured in a bank's internal business practices, then

$$\text{MRC} = \text{UL}$$

Quantitative standards (cont'd)

- **Internal data**: Banks must record internal loss data
- Internally generated OpRisk measures must be based on a five-year observation period
- **External data**: A bank's OpRisk measurement system must also include external data

Some internal data



OpRisk modeling issues

- Stylized facts about OP risk losses
  - Loss occurrence times are irregularly spaced in time
    (selection bias, economic cycles, regulation, management interactions,. . .)
  - Loss amounts show extremes

- Large losses are of main concern!

- Repetitive vs non-repetitive losses

- Red flag: Are observations in line with modeling assumptions?

- Example: "iid" assumption implies
  - NO structural changes in the data as time evolves
  - Irrelevance of which loss is denoted X_1, which one X_2,. . .

A mathematical (actuarial) model

- OpRisk loss database

$$\Big\{X_\ell^{(t,k)},\ t \in \{T-n+1,\ldots,T-1,T\}\ \text{(years)},\\ k \in \{1,2,\ldots,7\}\ \text{(loss event type)},\\ \ell \in \{1,2,\ldots,N^{(t,k)}\}\ \text{(number of losses)}\Big\}$$

- Loss event types:
  - Internal fraud
  - External fraud
  - Employment practices and workplace safety
  - Clients, products & business practices
  - Damage to physical assets
  - Business disruption and system failures
  - Execution, delivery & process management

A mathematical (actuarial) model (cont'd)

- Truncation and (random) censoring

$$X_\ell^{(t,k)} = X_\ell^{(t,k)} 1_{\left\{X_\ell^{(t,k)} > d^{(t,k)}\right\}}$$

- A further index j, $j \in \{1, \ldots, 8\}$ indicating business line can be introduced (suppressed for this talk)

- Estimate a risk measure for $F_{L^{(T+1,k)}}(x) = \mathbb{P}\left[L^{(T+1,k)} \le x\right]$ like

  - Op-VaR$_\alpha^{(T+1,k)} = F^{\leftarrow}_{L^{(T+1,k)}}(\alpha)$

  - Op-ES$_\alpha^{T+1} = \mathbb{E}\left[L^{(T+1,k)} | L^{(T+1,k)} > \text{Op-VaR}_\alpha^{(T+1,k)}\right]$

  where

- $L^{(T+1,k)} = \sum_{\ell=1}^{N^{(T+1,k)}} X_\ell^{(T+1,k)}$: OpRisk loss for loss event type k over the period $[T, T+1]$.

A mathematical (actuarial) model (cont'd)

- **Discussion**: Recall the stylized facts
  - X's are heavy-tailed
  - N shows non-stationarity
- **Conclusions**:
  - $F_X(x)$ and $F_{L(t,k)}(x)$ difficult to estimate
  - In-sample estimation of VaR$_\alpha$ (α large) almost impossible!
  - actuarial tools may be useful:
    - ∗ Approximation (translated gamma/lognormal)
    - ∗ Inversion methods (FFT)
    - ∗ Recursive methods (Panjer)
    - ∗ Simulation
    - ∗ Extreme Value Theory (EVT)

How accurate are VaR-estimates?

- Make inference about the tail decay of the aggregate loss $L^{(t,k)}$ via the tail decay of the individual losses $X^{(t,k)}$:

$$L = \sum_{\ell=1}^{N} X_\ell \,, \qquad 1 - F_X(x) \sim x^{-\alpha} h(x) \;, \quad x \to \infty$$

$$\Rightarrow \quad 1 - F_L(x) \sim \mathbb{E}[N]\, x^{-\alpha} h(x) \,, \quad x \to \infty.$$

- **Assumptions**: (X_m) iid $\sim F$ and for some ξ, β and u large

$$F_u(x) := \mathbb{P}[X - u \le x | X > u] = G_{\xi,\beta(u)}(x)$$

where

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\beta(u)}\right)^{-1/\xi} , & \xi \neq 0, \\ 1 - e^{-x/\beta} , & \xi = 0. \end{cases}$$

How accurate are VaR-estimates? (cont'd)

- Tail- and quantile estimate:

$$\boxed{\begin{aligned} 1-\hat{F}_X(x) &= \frac{N_u}{n}\Big(1+\hat{\xi}\,\frac{x-u}{\hat{\beta}}\Big)^{-1/\hat{\xi}}, \quad x>u. \\ \widehat{\mathrm{VaR}}_\alpha = \hat{q}_\alpha &= u-\frac{\hat{\beta}}{\hat{\xi}}\bigg(1-\Big(\frac{N_u}{n(1-\alpha)}\Big)^{\hat{\xi}}\bigg) \end{aligned}} \tag{1}$$

- **Idea**: Comparison of estimated quantiles with the corresponding theoretical ones by means of a simulation study (McNeil and Saladin 1997).

How accurate are VaR-estimates? (cont'd)

- **Simulation procedure**:

  ❶ Choose F and fix $\alpha_0 < \alpha < 1$, $N_u \in \{25, 50, 100, 200\}$ (N_u: $\#$ of data points above u)

  ❷ Calculate $u = q_{\alpha_0}$ and the true value of the quantile q_α

  ❸ Sample N_u independent points of F above u by the rejection method. Record the total number n of sampled points this requires

  ❹ Estimate ξ, β by fitting the GPD to the N_u exceedances over u by means of MLE.

  ❺ Determine $\hat{q}_\alpha$ according to (1)

  ❻ Repeat N times the above to arrive at estimates of Bias$(\hat{q}_\alpha)$ and SE$(\hat{q}_\alpha)$

How accurate are VaR-estimates? (cont'd)

- **Accuracy** of the quantile estimate expressed in terms of bias and standard error:

$$\text{Bias}(\hat{q}_\alpha) = \mathbb{E}[\hat{q}_\alpha - q_\alpha], \qquad \text{SE}(\hat{q}_\alpha) = \mathbb{E}\Big[(\hat{q}_\alpha - q_\alpha)^2\Big]^{1/2}$$

$$\widehat{\text{Bias}(\hat{q}_\alpha)} = \frac{1}{N}\sum_{j=1}^{N} \hat{q}_\alpha^j - q_\alpha\,, \quad \widehat{\text{SE}(\hat{q}_\alpha)} = \Big(\frac{1}{N}\sum_{j=1}^{N}(\hat{q}_\alpha^j - q_\alpha)^2\Big)^{1/2}$$

- For comparison purposes (different distributions) introduce

$$\text{Percentage Bias} := \frac{\widehat{\text{Bias}(\hat{q}_\alpha)}}{q_\alpha}, \qquad \text{Percentage SE} := \frac{\widehat{\text{SE}(\hat{q}_\alpha)}}{q_\alpha}$$

How accurate are VaR-estimates? (cont'd)

- **Criterion** for a "good estimate": Percentage Bias **and** Percentage SE should be **small**, e.g.

	$\alpha = 0.99$	$\alpha = 0.999$
Percentage Bias	≤ 0.05	≤ 0.10
Percentage SE	≤ 0.30	≤ 0.60

Example: Pareto distribution $1 - F_X(x) = x^{-\theta}$, $\theta = 2$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	A minimum number of 100 exceedances (corresponding to **333 observations**) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of 200 exceedances (corresponding to **667 observations**) is required to ensure accuracy wrt bias and standard error.
$q = 0.9$	0.99	Full accuracy can be achieved with the minimum number 25 of exceedances (corresponding to **250 observations**).
	0.999	A minimum number of 100 exceedances (corresponding to **1000 observations**) is required to ensure accuracy wrt bias and standard error.

Example: Pareto distribution $1 - F_X(x) = x^{-\theta}$, $\theta = 1$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	For all number of exceedances up to 200 (corresponding to a minimum of **667 observations**) the VaR estimates fail to meet the accuracy criteria.
	0.999	For all number of exceedances up to 200 (corresponding to a minimum of **667 observations**) the VaR estimates fail to meet the accuracy criteria.
$q = 0.9$	0.99	A minimum number of 100 exceedances (corresponding to **1000 observations**) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of 200 exceedances (corresponding to **2000 observations**) is required to ensure accuracy wrt bias and standard error.

How accurate are VaR-estimates? (cont'd)

- Large number of observations necessary to achieve targeted accuracy.
- Minimum number of observations increases as the tails become thicker (McNeil and Saladin 1997 [8]).
- **Remember**: The simulation study was done under idealistic assumptions. OpRisk losses, however, typically do NOT fulfil these assumptions.

Pricing risk under incomplete information

- **Recall**: $L^{(T+1,k)}$: OpRisk loss for loss event type k over the period $[T, T+1]$

$$L^{(T+1,k)} = \sum_{\ell=1}^{N^{(T+1,k)}} X_\ell^{(T+1,k)}$$

- **Question**: Suppose we have calculated risk measures $\rho_\alpha^{(T+1,k)}$, $k \in \{1, \ldots, 7\}$ for each loss event type. When can we consider

$$\sum_{k=1}^{7} \rho_\alpha^{(T+1,k)}$$

a "good" risk measure for the total loss $L^{T+1} = \sum_{k=1}^{7} L^{(T+1,k)}$?

Pricing risk under incomplete information (cont'd)

- **Answer**: Ingredients
  - (non-) coherence of risk measures (Artzner, Delbaen, Eber, Heath framework)
  - optimization problem: given $\rho_\alpha^{(T+1,k)}$, $k \in \{1,\ldots,7\}$, what is the worst case for the overall risk for L^{T+1}?
    Solution: using copulas in [4] and references therein.
  - aggregation of banking risks [1]

A ruin-theoretic problem motivated by OpRisk

- **OpRisk process**:

$$V_t^{(k)} = u^{(k)} + p^{(k)}(t) - L_t^{(k)}, \qquad t \geq 0$$

for some initial capital $u^{(k)}$ and a premium function $p^{(k)}(t)$ satisfying $\mathbb{P}[L_t^{(k)} - p^{(k)}(t) \to -\infty] = 1$.

- Given $\varepsilon > 0$, calculate $u^{(k)}(\varepsilon)$ such that

$$\mathbb{P}\Big[\inf_{T \leq t \leq T+1} \Big(u^{(k)}(\varepsilon) + p^{(k)}(t) - L_t^{(k)}\Big) < 0\Big] \leq \varepsilon \qquad (2)$$

$u^{(k)}(\varepsilon)$ is a risk capital charge (internal)

Ruin-theoretic problem (cont'd)

- Solving for (2) is difficult
  - complicated loss process $L_t^{(k)}$
  - heavy-tailed case
  - finite time horizon $[T, T+1]$

- Recall for the classical Cramer-Lundberg model

$$Y(t) = u + ct - \sum_{k=1}^{N(t)} X_k = u + ct - S_N^X(t)$$

$$\Psi(u) = \mathbb{P}\left[\sup_{t \geq 0}\left(S_N^X(t) - ct\right) > u\right]$$

Ruin-theoretic problem (cont'd)

- In the heavy-tailed case: $\mathbb{P}[X_1 > x] \sim x^{-(\beta+1)} L(x)$, L slowly varying, implies that

$$\Psi(u) \sim \kappa u^{-\beta} L(u)\,, \quad u \to \infty.$$

- Important: Net-profit condition

$$\mathbb{P}\Big[\lim_{t\to\infty}\Big(S_N^X(t) - ct\Big) = -\infty\Big] = 1$$

- Now assume for some general loss process $(S(t))$

$$\mathbb{P}\Big[\lim_{t\to\infty}(S(t) - ct) = -\infty\Big] = 1$$

$$\Psi_1(u) = \mathbb{P}\Big[\sup_{t\geq 0}\Big(S(t) - ct\Big) > u\Big] \sim u^{-\beta} L(u)\,, \quad u \to \infty. \qquad (3)$$

Ruin-theoretic problem (cont'd)

- **Question**: How much can we change S keeping (3)?

- **Solution**: use time change $S_\Delta(t) = S(\Delta(t))$

$$\Psi_\Delta(u) = \mathbb{P}\left[\sup_{t\geq 0}\big(S_\Delta(t) - ct\big) > u\right]$$

Under some technical conditions on Δ and S, general models are given so that

$$\lim_{u\to\infty} \frac{\Psi_\Delta(u)}{\Psi_1(u)} = 1$$

i.e. ultimate ruin behaves similarly under the time change

Ruin-theoretic problem (cont'd)

- **Example**:

  - start from the homogeneous Poisson case (classical Cramer-Lundberg, heavy-tailed case)

  - use Δ to transform to changes in intensities motivated by operational risk, see [6].

Conclusions

- Concretization of minimal solvency rules
- Principle-based thinking
- Driven by Solvency 2 and in line with Basel II
- From collaboration to joint supervision
- Many technical an organisatorial problems remain
- Insurance analytics

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