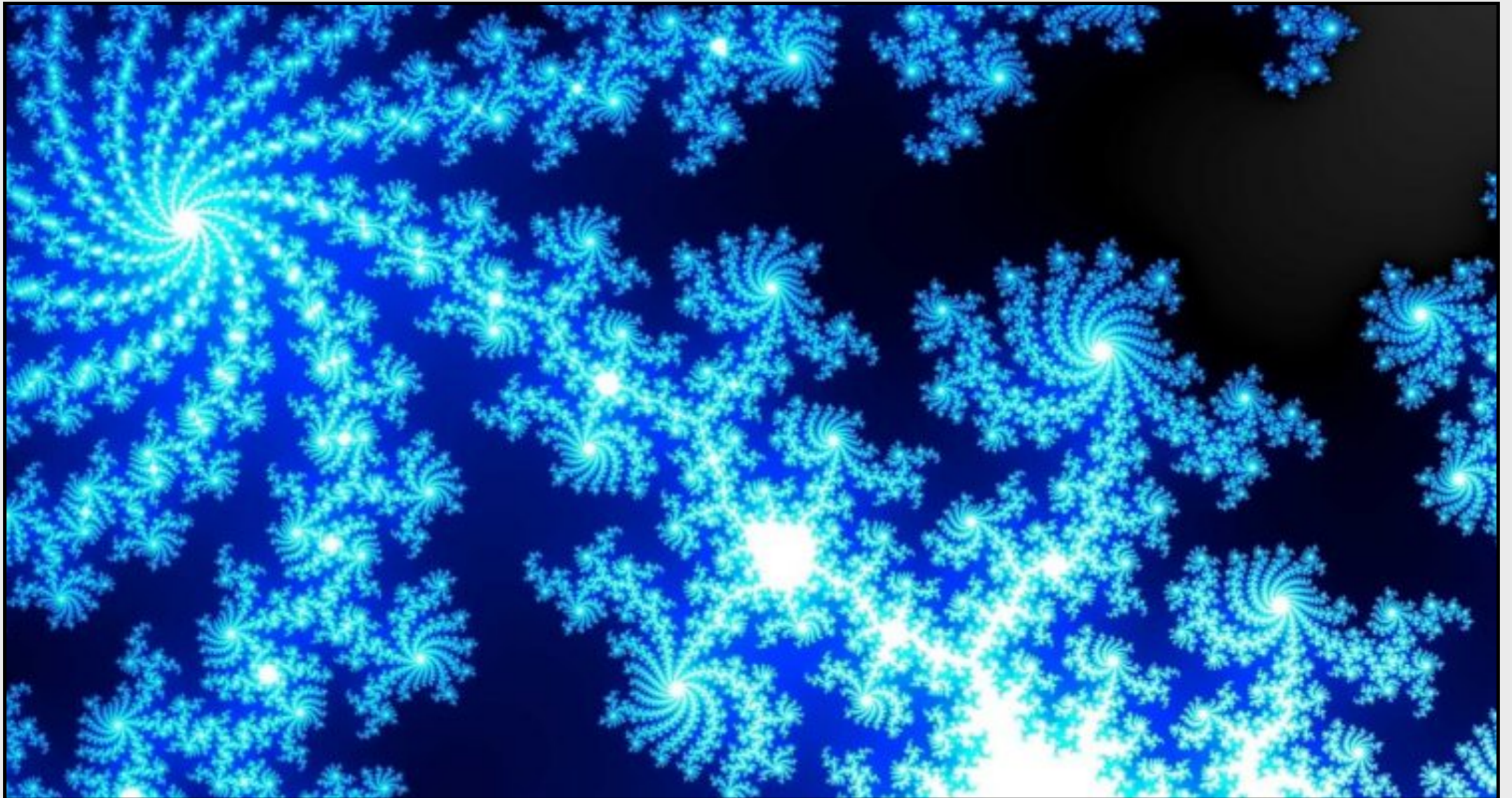


***WELCOME EVERYBODY!***  
***ALLE HERZLICH WILLKOMMEN!***



# MASS UND INTEGRAL, D-MATH

## 401-2284-00L, SS2020

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# ADMINISTRATIVE INFORMATION

Course Webpage

<https://metaphor.ethz.ch/x/2020/fs/401-2284-00L/>

My Webpage (for Lecture Notes, Class Content and other Material)

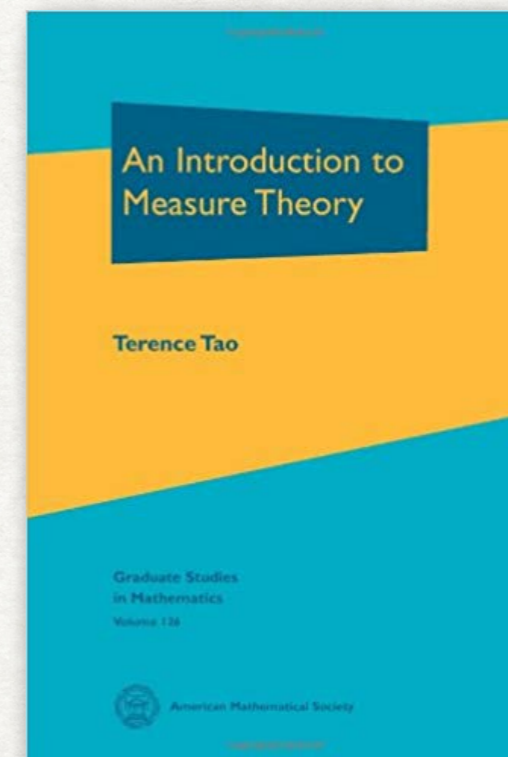
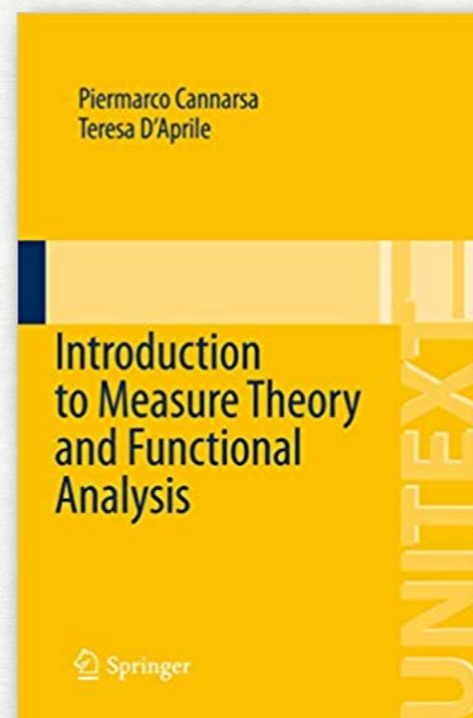
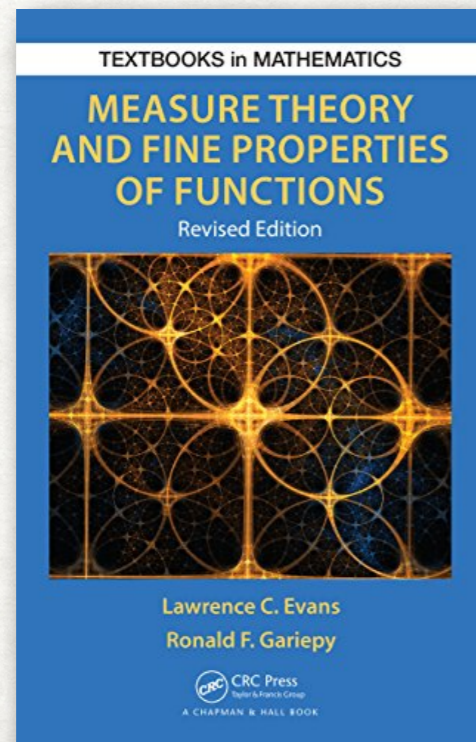
<https://people.math.ethz.ch/~fdalio/MASSundINTEGRAL>

AND

<http://www.vorlesungsverzeichnis.ethz.ch>

# TEXTBOOKS

- My Lecture Notes (which will be continuously updated according also to your remarks and comments) (in **English**)
- **Michael Struwe's Notes: Analysis III, Mass und Integral** (in **German**)
- **Additional recommended bibliography:**



- ★ **Evaluation:** There will be a 20 minute oral exam :it will consist in **two questions** where you will have to prove two results (sometime if I am not satisfied I ask a 3rd question).
- ★ **Weakly homeworks:** I really encourage active and regular participation to our weekly problem sessions: they will give you the opportunity to review the topics in smaller groups, to discuss problems and see some of them solved in great detail. I advise you to work in a timely manner.  
Studying Mathematics is effective if it is a regular activity. Studying right before the exam does not give in general good results.
- ★ I advise to attend the lectures: they aim at guiding you in understanding the key concepts in each chapter
- ★ I encourage you to ask me questions during the lecture. Do not be scared to stop me during the lecture!

# THIS COURSE:

The goal of this course is to provide notions of abstract measure and integral which are more general and robust than the notion of [Jordan measure](#) and [Riemann integral](#) (for a nice presentation of Jordan measure and Riemann integral look for instance at the notes of Analysis 1 & 2 by Michael Struwe or the book by Terence Tao).

## Why do we need a finer concept of measure than the one we already have with the Jordan's measure?

1. From the **point of view of geometry**, we may be interested in being able to “measure” as many quantities as possible in a natural way. For this we need a measure with which we can also measure countable unions of measurable quantities. The Jordan measure cannot do this, as some examples show.
2. From the **point of view of the analysis** we need a theory of integration which extends Riemann's theory and concerns with a more general class of functions, not necessarily continuous or piecewise continuous (the so-called Borel or measurable functions).
3. Finally, abstract measure theory is also of fundamental importance for the **field of stochastics**, since calculating with probabilities is only possible in the language of measure theory.

# PRELIMINARY PROGRAM

- Measure Spaces (Lebesgue Measure, Hausdorff Measure, Radon Measure)
- Measurable Functions: definition and properties
- Integration: definition, properties, theorems of convergence, Lebesgue  $L^p$  spaces
- Product Measures and Multiple Integrals. Fubini and Tonelli Theorems, Convolutions
- Differentiation of measures