WELCOME TO EVERYBODY!

Analysis III - D-MAVT, D-MATL
401-0363-10, Fall 2017

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ADMINISTRATIVE INFORMATION

Course Website: https://moodle-app2.let.ethz.ch/course/view.php?id=3375

Look also at my Webpage:

https://people.math.ethz.ch/~fdalio/ANALYSISIIIDMAVTDMATL

and

http://www.vvz.ethz.ch/Vorlesungsverzeichnis/

TEXTBOOKS

Alessandra Iozzi's Lecture Notes:

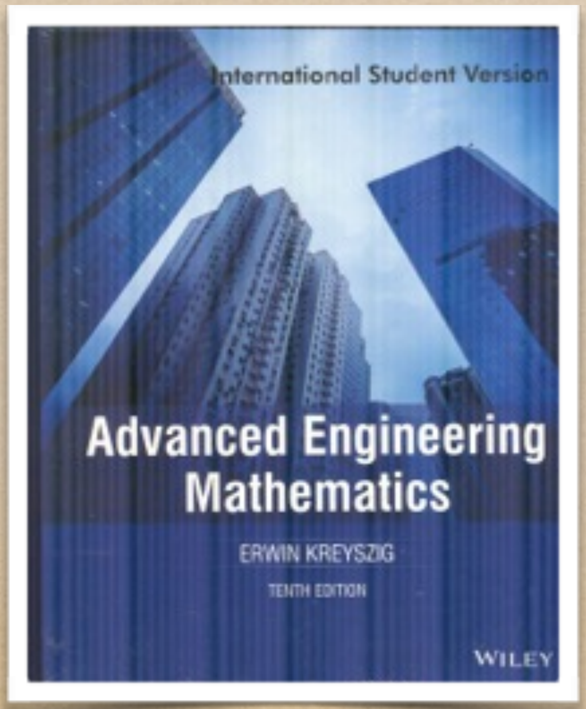
https://people.math.ethz.ch/~fdalio/AnalysisIII-MAVTMATL7.pdf

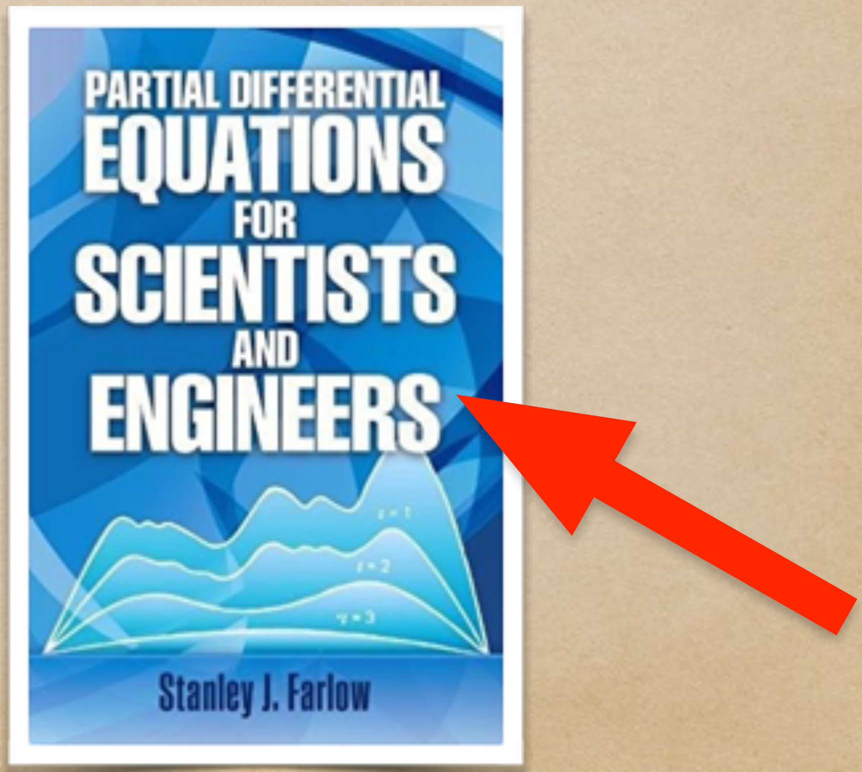
and (partially)

Martin Larsson'a Class Notes

(they will be uploaded in moodle)

Additional Recommended Bibliography:





- **Evaluation**: There will be a 2 hour written final examination. More information will be provided towards the end of the semester.
- **Weekly homeworks:** Please do work on them in a timely manner and do not accumulate and study them later.
- **Lectures:** Thursday, 13 - 15, HG F 7.
- **Problem Sections:** Thursday, 15-16, Friday, 15-16.
- **EduApp:** to install https://www.ethz.ch/content/associates/services/de/lehre/lehrbetrieb/it-services-lehrbetrieb/lehrunterstuetzende-applikationen/eduapp-service.html

CONTENT

The main goal is to study basic linear partial differential equations used widely in engineering applications.
We develop solution techniques through transformations and study their analytical properties:

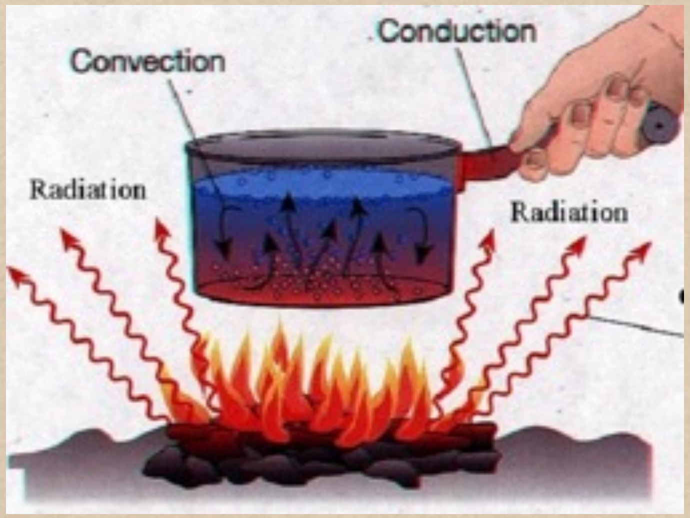
- Laplace Tranformation
- Fourier Analysis
  - → Fourier Series
  - → Fourier Tranformation

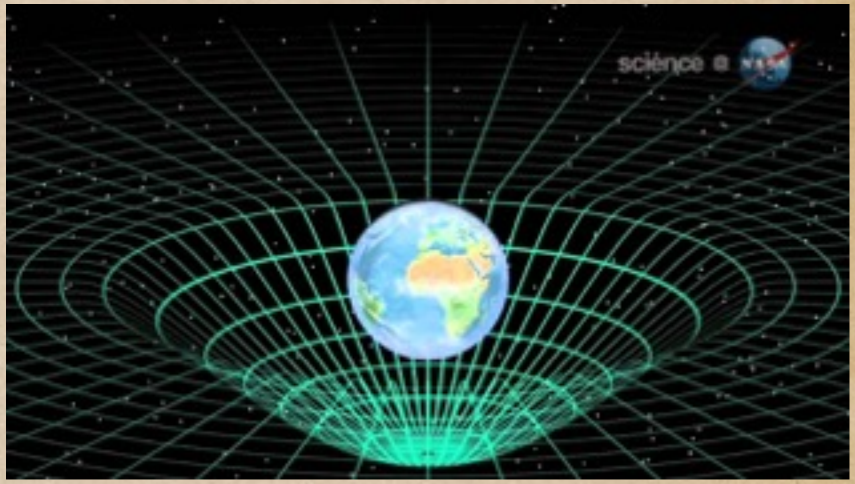
What is a PDE?

- It relates state variables like mass, velocity, energy….to their variations with respect to space,time…..
- It provides an important modeling tool for the physical sciences, theoretical chemistry, biology, socio-economics sciences, engineering sciences….
- It governs many phenomena occurring in the nature around us
- PDE involve deep and beautiful mathematics

Partial Differential Equations
(PDE)

<u>MAIN EXAMPLES</u>





Wave Equation

$$u_{tt} - c^2 u_{xx} = 0$$

Heat Equation

$$u_t - k^2 u_{xx} = 0$$

Laplace's Equation

$$u_{tt} + u_{xx} = 0$$

Other Examples

Navier-Stokes Equations

The Clay Mathematics Institute, a private American foundation, proposed a list of seven problems, the resolution of which will be rewarded with a prize of one million dollars for each. Most of these challenges are of great abstraction and may seem remote from any physical reality. By contrast, the problem of Navier-Stokes equations appears quite concrete in this list: these equations are supposed to describe the flows of ordinary fluids (liquids or gases) (http://images.math.cnrs.fr/Turbulences-sur-les-equations-des.html?lang=fr)

- Maxwell's Equation: description of electromagnetic phenomena

- Schrödinger's Equation: mathematical formulation for studying quantum mechanical systems

$$i\hbar u_t = -\frac{\hbar}{2m}\Delta u + Vu.$$

V is a known function (potential), m is the particle's mass and $\hbar$ is the Planck's constant

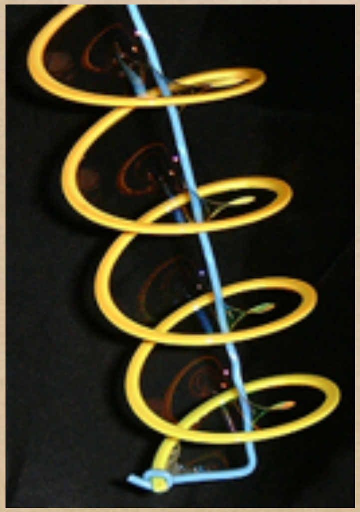
- Reaction diffusion equations

$$u_t(x,t) - \kappa^2 u_{xx}(x,t) = F(u(x,t))$$

They arise in a variety of biological applications and in modeling certain chemical reactions.

- Minimal Surface Equation

$$(1+u_x^2)u_{yy} - 2u_x u_y u_{xy} + (1+u_y^2)u_{xx} = 0$$



A helicoid minimal surface formed by a soap film on a helical frame

https://en.wikipedia.org/wiki/Minimal_surface

How do you solve a PDE?

1. Separation of Variables: This technique reduces a PDE in n variables to a n ODEs.

2. Integral Transforms: It reduces a PDE in n independent variables to one in n-1 variables.

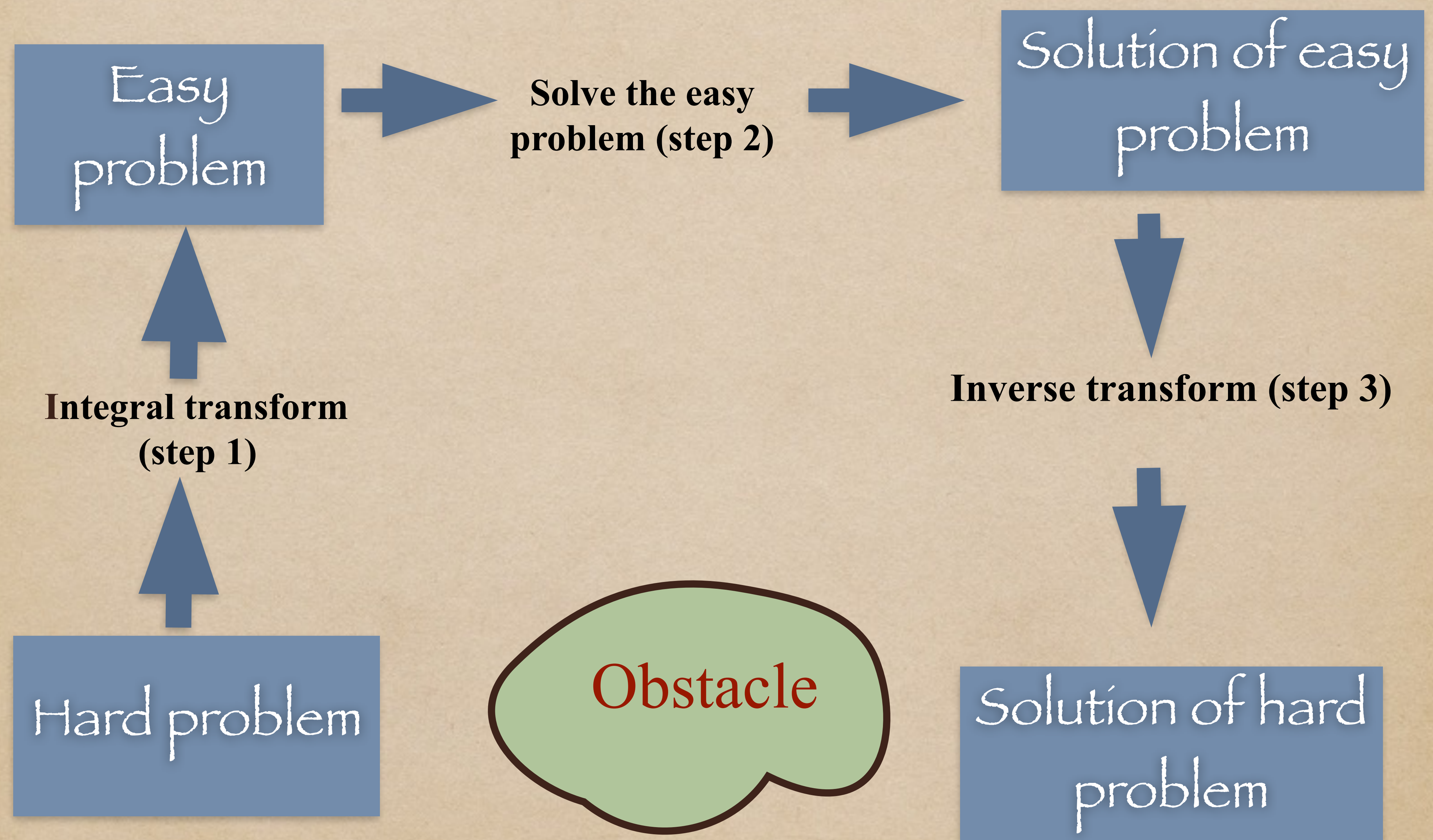
3. Change of Coordinates: It changes the original PDE to an ODE or an easier PDE by changing the coordinates.

4. Numerical Methods: They change a PDE to a system of differential equations than can be solved by means of iterative techniques.

5. ....

Chapter 1: The Laplace Transform

Transforms: basic idea in solving mathematical problems



The Laplace transform uses this philosophy

$$f'(t) = f(t), \; f(0) = 1$$

Laplace Transform

$$-1 + sF(s) = F(s)$$

solve the algebraic equation

$$F(s) = \frac{1}{s-1}$$

inverse Laplace Transform

$$f(t) = e^t$$

What is not revealed by this diagram is how to pass from the original equation

$$f'(t) = f(t), \ f(0) = 1$$

to the algebraic equation involving the Laplace transform F(s) and then back from the solution F(s) of the algebraic equation to the solution f(t) of the original equation.

That's our next task: to develop the theory of Laplace transform

Thanks for the attention