Presentation of 401-2283-00L Analysis III (Masstheorie)

Francesca Da Lio

Fall Semester 2022

WELCOME EVERYBODY

HERZLICH WILLKOMMEN!

TEAM

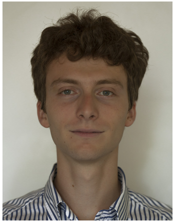
Francesca Da Lio
fdalio@ethz.ch

Gerard Orriols Gimenez
gerard.orriols@math.ethz.ch

Teaching Assistants

Chen Nuo

Niklas Canova



Antonio Casetta

Flavio Dalessi

Adrian Dawid

Administrative Information

- Course Webpage in Metaphor
- My Webpage: (here you find my Lecture Notes, Class Content and other Material)
- Course Catalogue

Information Lectures and Exercises

- Lectures: Wednesday 9-10, HG E 1.2, Friday 10-12, CAB G 11
- Exercise Classes: Monday 16-18 (more information in the Course Webpage in Metaphor). The **first exercise class will start on September 26th.**
- Forum: There is a **forum** for the students to discuss the current exercise sheet and topics from the lectures. You can access the forum here .

Evaluation

In Course Catalogue you will find the following information:

*Im Bachelor-Studiengang Mathematik (***Reglement 2021***) wird die Lerneinheit* **Analysis III zusammen mit Analysis IV geprüft**. *Im Bachelor-Studiengang Mathematik (***Reglement 2016***) wird vor dem ersten Versuch des Prüfungsblocks* 2 **Mass und Integral oder diese Lerneinheit** *gewählt, der zweite Versuch erfolgt mit der gleichen Lerneinheit wie der erste.* **Die Prüfungsanmeldung zu Analysis III statt Mass und Integral erfolgt über die Prüfungsplanstelle:** *exams@ethz.ch*

- Oral Exam: it lasts 20 minutes (18 minutes exam, 3 minutes discussion of the grade): it will consist in two questions where you will have to prove two results (sometime if I am not satisfied or I want to be sure for the maximal grade I ask a 3rd question).
- Written Exam: it should last 180 minutes: there will be 3 or 4 exercises concerning Analysis III. I will upload a mock exam (Probeprüfung) with a structure similar to the real exam (part Analysis III).

Mathematics is NOT a SPECTATOR SPORT

- ★ **Weakly Homeworks:** I really encourage active and regular participation to our weekly problem sessions: they will give you the opportunity to review the topics in smaller groups, to discuss problems and see some of them solved in great detail. I advise you to work in a timely manner. **Studying Mathematics is effective if it is a regular activity**. I advise you to attend as much as possible the lectures: they aim at guiding you in understanding the key concepts in each chapter
- ★ You can always ask me questions!!

Textbooks

- My Lecture Notes (in English) (which will be continuously updated. Remarks and comments are always welcome!). You will find the class notes in Polybox (I will send you by email the password)
- M. Struwe's Lecture Notes: Analysis III, Mass und Integral (in German)
- An additional recommended reference: L. Evans and R. Gariepy, Measure Theory and Fine Properties of Functions, Textbooks in Mathematics, CRC Press, 201. See also the webpage of the course for other references.
- For a review of some important notions of Analysis 1& 2 I recommended: Lecture notes of Analysis I and II by M. Struwe or Lecture notes of Analysis I and II by M. Einsiedler .

About this Course

The goal of this course is to provide notions of abstract measure and integral which are more general and robust than the notion of **Jordan measure** and **Riemann integral**.

Why do we need a finer concept of measure than the one we already have with the Jordan measure?

- ★ **From the point of view of geometry**, we may be interested in being able to **measure** as many quantities as possible in a natural way. For this we need a measure with which we can also measure countable unions of measurable quantities. The Jordan measure cannot do this, as some examples show.
- ★ **From the point of view of the analysis** we need a theory of integration which extends Riemann theory and concerns with a more general class of functions, not necessarily continuous or piecewise continuous (the so-called Borel or measurable functions).
- ★ Finally, abstract measure theory is also of fundamental importance for the field of stochastics, since calculating with probabilities is only possible in the language of measure theory.

A Preliminary Program

- Measure Spaces (Lebesgue Measure, Hausdorff Measure, Radon Measure)
- Measurable Functions: definition and properties
- Integration: definition, properties, theorems of convergence, Lebesgue L^p spaces
- Product Measures and Multiple Integrals. Fubini and Tonelli Theorems, Convolutions
- Differentiation of measures