Program of Analysis III-Masstheorie HS24 ETH, D-MATH

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For the preparation of the **written and oral exam** you should take as guideline the class content that you find in

https://people.math.ethz.ch/~fdalio/Analysis3MeasureTheoryHS24

The **written examination** will consist of three exercises, one of which will include a set of **multiple choice questions** as in the bonus exercises of the series in HS24. Below, you find the list of definitions and results (the enumeration is taken from the Lecture Notes of the course) that you should know for the oral and written exam. We advise you to look at also the class notes of the lectures (that you find in polybox). In the written examination you might be asked to provide a <u>definition</u> and/or the <u>statement</u> of a result mentioned below. You will not be explicitly asked the proof of a result, but you should know how to apply it. In any case, it is always advisable to review the proofs (those pointed in violet) listed below at least once in order to more effectively solve the exercises.

The **oral examination** lasts 20 minutes (the time for the discussion of the grade is included) and you will be asked in general the statement and the proof of two results among those mentioned below (those pointed out in red and in violet). In the case we are not completely satisfied by the answers or if we are not completely sure for the maximal grade we will ask (if the time permits it) a third question. It is very important to be very precise in the statement of a result and to explain clearly and rigorously

the key steps of its proof.

CHAPTER 1: Remark 1.1.1, Definition 1.1.2, Remark 1.1.3, Remark 1.1.6, Definition 1.1.7, Definition 1.2.1, Remark 1.2.2, Definition 1.2.3, Exercise 1.2.4, Definition 1.2.5, Definition 1.2.7, Definition 1.2.9, Remark 1.2.10, Theorem 1.2.11 (proof), Definition 1.2.12, Theorem 1.2.14 (proof), Definition 1.2.16, Theorem 1.2.18 (proof) , Theorem 1.2.20 (proof), Definition 1.3.1, Remark 1.3.3, Lemma 1.3.4, Definition 1.3.5, Definition 1.3.6, Theorem 1.3.7 (proof), Theorem 1.3.8 (proof), Corollary 1.3.9, Theorem 1.4.1 (proof), Definition 1.4.3, Corollary 1.4.4, Section 1.5: Vitali Set (construction and proof that it is not Lebesgue measurable), Exercise 1.5.1, Exercise 1.5.2, Section 1.6: Cantor Set (construction and proof for oral and written that it has Lebesgue measure zero and it is uncountable), Definition 1.7.1, Theorem 1.7.2, Theorem 1.7.4, Theorem 1.7.5, Definition 1.8.1, Definition 1.8.2, Theorem 1.8.3 (proof), Remark 1.8.4 , Lemma 1.8.5 (proof), Example 1.8.6, Definition 1.8.8, Remark 1.8.9, Definition 1.9.1, Example 1.9.2.

CHAPTER 2: Definition 2.2.1, Remark 2.2.2, Remark 2.2.3, Theorem 2.2.5 (proof), Theorem 2.2.6 (proof), Remark 2.2.8, Theorem 2.3.1 (Egoroff) (proof), Exercise 2.3.2, Theorem 2.3.3 (Lusin), Remark 2.3.4, Definition 2.4.1, Theorem 2.4.2 (proof), Remark 2.4.3, Theorem 2.4.4 (proof).

CHAPTER 3: Definition 3.1.1 , Definition 3.1.2, Definition 3.1.3, Definition 3.1.4, Proposition 3.1.6 (proof), Proposition 3.1.7 (proof), Definition 3.1.8, Proposition 3.1.9 (proof), Proposition 3.1.10 (proof), Corollary 3.1.11, Theorem 3.1.12, Corollary 3.1.13 (proof), Theorem 3.1.14, Corollary 3.1.15, Lemma 3.1.16, Corollary 3.1.17, Proposition 3.1.18, Example 3.2.1, Proposition 3.2.2 (proof), Theorem 3.3.1 (Fatou's Lemma)(proof oral), Example 3.3.2, Theorem 3.3.3 (Monotone Convergence Theorem) (proof), Theorem 3.3.5 (Dominated Convergence Theorem (proof), Theorem 3.5.3 (proof), Definition 3.6.1, Theorem 3.6.2 (Vitali's Theorem)(proof), Remark 3.6.3. Definition 3.7.1, Remark 3.7.2, Remark 3.7.3, Theorem 3.7.5 (L^p spaces are

Banach spaces) (proof), Lemma 3.7.6 (Young Inequality) (proof), Corollary 3.7.7 (Hölder Inequality) proof), Corollary 3.7.8, Corollary 3.7.10 (Minkowski Inequality)(proof), Lemma 3.7.13 (proof), Theorem 3.7.15, Remark 3.7.16.

CHAPTER 4: Definition 4.1.1, Theorem 4.1.5 (Fubini), Remark 4.1.11, Applications in Section 4.2, Theorem 4.3.2, Lemma 4.4.1 (proof), Lemma 4.4.2, Definition 4.4.3 , Remark 4.4.4, Theorem 4.4.5 (proof), Corollary 4.4.6 (proof), Definition 4.4.7, Proposition 4.4.8 (proof , see also the alternative proof in the class notes).

For the preparation of the written examination we also advise to revise **all the bonus exercises** from exercises sheets of HS24 and the following additional exercises:

Serie 1: Ex 3,5,6.

Serie 2: Ex 1,3.

Serie 3: Ex 2,4.

Serie 4: Ex 4.

Serie 5: Ex 1,2.

Serie 6: Ex 3,4.

Serie 7: Ex 2,3.

Serie 8: Ex 2,4,5.

Serie 9: Ex 3,4,5,6

Serie 10: Ex 4,5,8.

Serie 11: Ex 1,4,5,6.

Serie 12: Ex 2,3,6.

Serie 13: Ex 2,3,5,8.