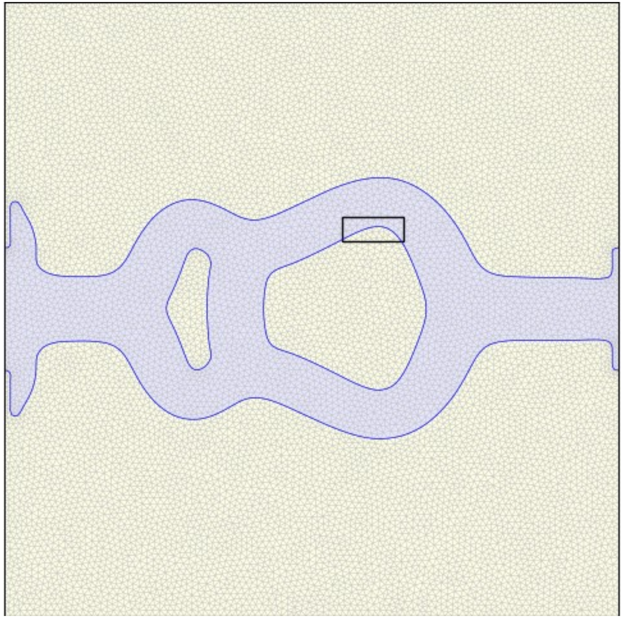

OPTIMISATION TOPOLOGIQUE DE SYSTÈMES THERMIQUES, HYDRAULIQUES, MÉCANIQUES

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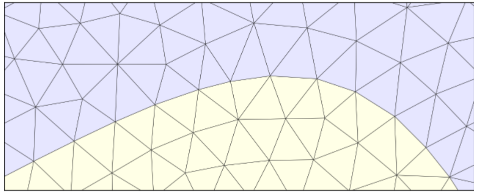
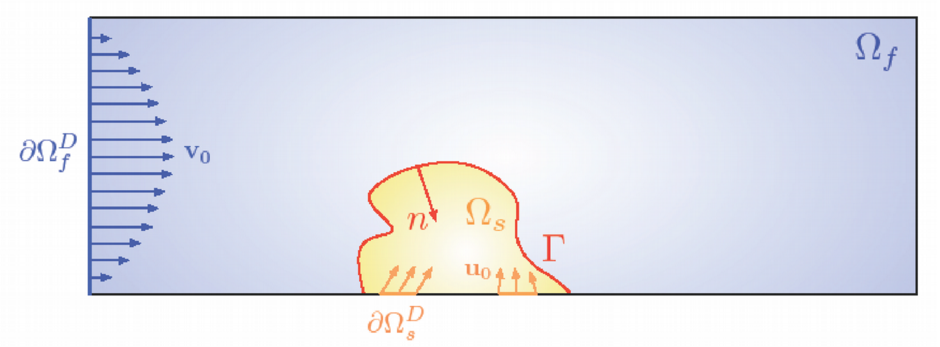
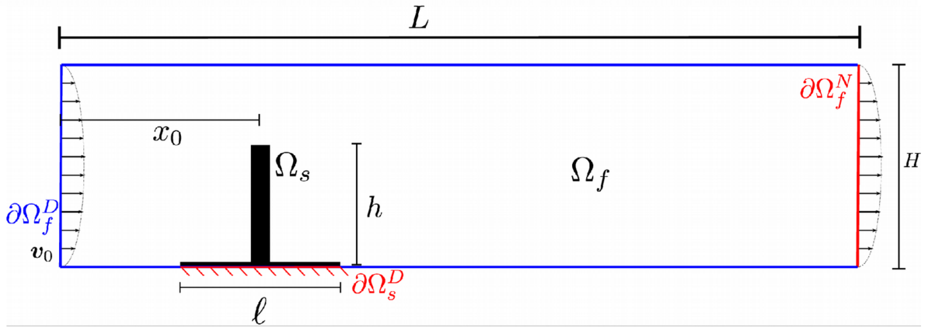
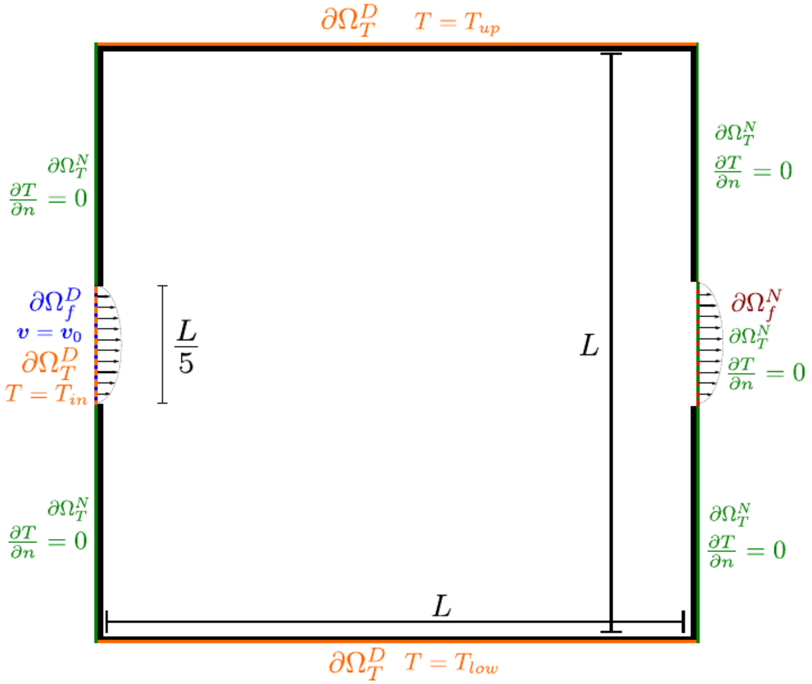
9e Journées FreeFem++



Cadre de travail







Cadre de travail : 3 physiques couplées en cascade

$$\begin{cases} -\mathrm{div}(\sigma_f(\boldsymbol{v},p)) + \rho \boldsymbol{v}\cdot\nabla\boldsymbol{v} = \boldsymbol{f}_f & \text{in } \Omega_f \\ \mathrm{div}(\boldsymbol{v}) = 0 & \text{in } \Omega_f \\ \boldsymbol{v} = \boldsymbol{v}_0 & \text{on } \partial\Omega_f^D \\ \sigma_f(\boldsymbol{v},p)\cdot\boldsymbol{n} = 0 & \text{on } \partial\Omega_f^N \\ \boldsymbol{v} = 0 & \text{on } \Gamma, \end{cases}$$

Domaine fluide :
Navier Stokes

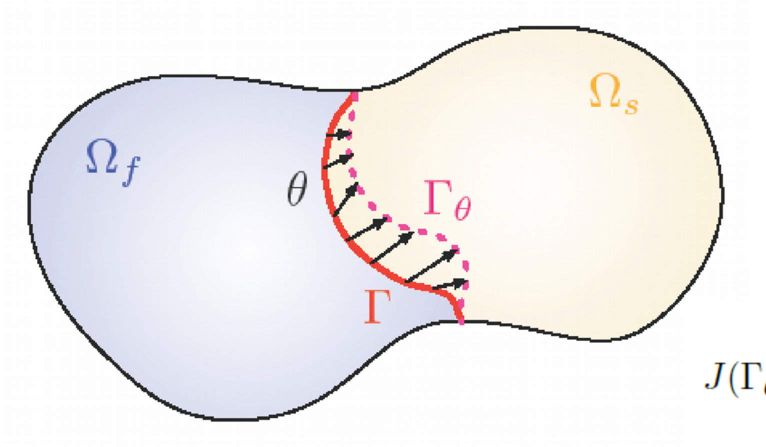
$$\begin{cases} -\mathrm{div}(k_f\nabla T_f) + \rho\boldsymbol{v}\cdot\nabla T_f = Q_f & \text{in } \Omega_f \\ -\mathrm{div}(k_s\nabla T_s) = Q_s & \text{in } \Omega_s \\ T = T_0 & \text{on } \partial\Omega_T^D \\ -k_f\dfrac{\partial T_f}{\partial n} = q_f & \text{on } \partial\Omega_f^N \\ -k_s\dfrac{\partial T_s}{\partial n} = q_s & \text{on } \partial\Omega_s^N \\ T_f = T_s & \text{on } \Gamma \\ -k_f\dfrac{\partial T_f}{\partial n} = -k_s\dfrac{\partial T_s}{\partial n} & \text{on } \Gamma, \end{cases}$$

Domaine fluide + solide :
Conduction-Convection

$$\begin{cases} -\mathrm{div}(\sigma_s(\boldsymbol{u},T_s)) = \boldsymbol{f}_s & \text{in } \Omega_s \\ \boldsymbol{u} = \boldsymbol{u}_0 & \text{on } \partial\Omega_s^D \\ \sigma_s(\boldsymbol{u},T_s)\cdot\boldsymbol{n} = \boldsymbol{g} & \text{on } \partial\Omega_s^N \\ \sigma_s(\boldsymbol{u},T_s)\cdot\boldsymbol{n} = \sigma_f(\boldsymbol{v},p)\cdot\boldsymbol{n} & \text{on } \Gamma. \end{cases}$$

Domaine solide :
Thermoélasticité
+contraintes fluide structure

Méthode de dérivation de formes d'Hadamard



$$\min_{\Gamma} J(\Gamma, \boldsymbol{v}(\Gamma), p(\Gamma), T(\Gamma), \boldsymbol{u}(\Gamma))$$

$$\Gamma_{\boldsymbol{\theta}} = (I + \boldsymbol{\theta})\Gamma, \text{ where } \boldsymbol{\theta} \in W^{1,\infty}(\Omega, \mathbb{R}^d), \ ||\boldsymbol{\theta}||_{W^{1,\infty}(\mathbb{R}^d,\mathbb{R}^d)} < 1.$$

$$J(\Gamma_{\boldsymbol{\theta}}) = J(\Gamma) + \frac{\mathrm{d}J}{\mathrm{d}\boldsymbol{\theta}}(\boldsymbol{\theta}) + o(\boldsymbol{\theta}), \text{ where } \frac{|o(\boldsymbol{\theta})|}{||\boldsymbol{\theta}||_{W_0^{1,\infty}(\Omega,\mathbb{R}^d)}} \xrightarrow{\boldsymbol{\theta}\to 0} 0,$$

$$\text{Find } \boldsymbol{\theta} \in H_0^1(\Omega, \mathbb{R}^d), \text{ such that } \forall \boldsymbol{\theta}' \in H_0^1(\Omega, \mathbb{R}^d), \ a(\boldsymbol{\theta}, \boldsymbol{\theta}') = -\frac{\mathrm{d}J}{\mathrm{d}\boldsymbol{\theta}}(\boldsymbol{\theta}').$$

Méthode de dérivation de formes d’Hadamard

$$
\begin{aligned}
&\left[\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}} J(\Gamma_{\boldsymbol{\theta}}, \boldsymbol{v}(\Gamma_{\boldsymbol{\theta}}), p(\Gamma_{\boldsymbol{\theta}}), T(\Gamma_{\boldsymbol{\theta}}), \boldsymbol{u}(\Gamma_{\boldsymbol{\theta}}))\right](\boldsymbol{\theta}) \\
&\quad = \overline{\frac{\partial \mathfrak{J}}{\partial \boldsymbol{\theta}}}(\boldsymbol{\theta}) + \int_\Gamma (\boldsymbol{f}_f \cdot \boldsymbol{w} - \sigma_f(\boldsymbol{v}, p) : \nabla \boldsymbol{w} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{w}, q) \nabla \boldsymbol{v} \cdot \boldsymbol{n} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{v}, p) \nabla \boldsymbol{w} \cdot \boldsymbol{n})(\boldsymbol{\theta} \cdot \boldsymbol{n}) \mathrm{d}s \\
&\quad + \int_\Gamma \left(k_s \nabla T_s \cdot \nabla S_s - k_f \nabla T_f \cdot \nabla S_f + Q_f S - Q_s S_s - 2k_s \frac{\partial T_s}{\partial n}\frac{\partial S_s}{\partial n} + 2k_f \frac{\partial T_f}{\partial n}\frac{\partial S_f}{\partial n}\right)(\boldsymbol{\theta} \cdot \boldsymbol{n}) \mathrm{d}s \\
&\quad\quad + \int_\Gamma (\sigma_s(\boldsymbol{u}, T_s) : \nabla \boldsymbol{r} - \boldsymbol{f}_s \cdot \boldsymbol{r} - \boldsymbol{n} \cdot Ae(\boldsymbol{r}) \nabla \boldsymbol{u} \cdot \boldsymbol{n} - \boldsymbol{n} \cdot \sigma_s(\boldsymbol{u}, T_s) \nabla \boldsymbol{r} \cdot \boldsymbol{n})(\boldsymbol{\theta} \cdot \boldsymbol{n}) \mathrm{d}s,
\end{aligned}
$$

$\boldsymbol{w}, S, \boldsymbol{r}, q$ sont des états « adjoints »

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Méthode de dérivation de formes d'Hadamard

Formulation faible pour la variable d'état u:

$$\text{Find } \boldsymbol{u} \in \boldsymbol{u}_0 + V_{\boldsymbol{u}}(\Gamma) \text{ such that } \forall \boldsymbol{r} \in V_{\boldsymbol{u}}(\Gamma),$$

$$\int_{\Omega_s} \sigma_s(\boldsymbol{u}, T_s) : \nabla \boldsymbol{r} \mathrm{d}x = \int_{\Omega_s} \boldsymbol{f}_s \cdot \boldsymbol{r} \mathrm{d}x + \int_{\partial \Omega_s^N} \boldsymbol{g} \cdot \boldsymbol{r} \mathrm{d}s - \int_{\Gamma} \boldsymbol{r} \cdot \sigma_f(\boldsymbol{v}, p) \cdot \boldsymbol{n} \mathrm{d}s,$$

Formulation faible pour la variable adjointe r:

$$\text{Find } \boldsymbol{r} \in V_{\boldsymbol{u}}(\Gamma) \text{ such that } \forall \boldsymbol{r}' \in V_{\boldsymbol{u}}(\Gamma), \int_{\Omega_s} Ae(\boldsymbol{r}) : \nabla \boldsymbol{r}' \mathrm{d}x = \frac{\partial \mathfrak{J}}{\partial \hat{\boldsymbol{u}}}(\boldsymbol{r}').$$

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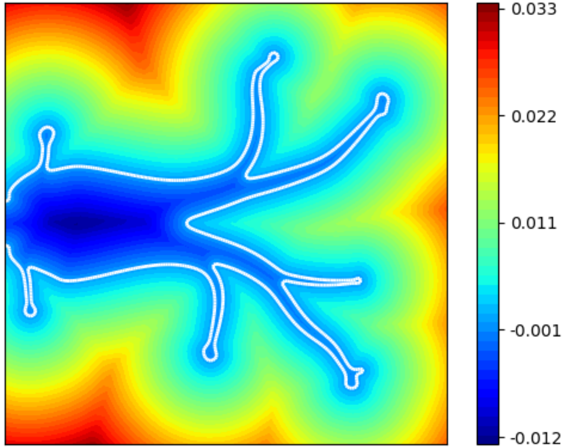
Algorithme et implémentation

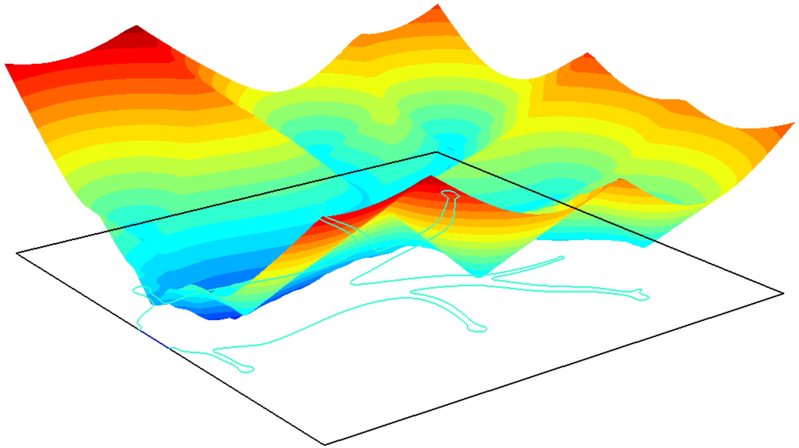
Étant donné un maillage T adapté à la décomposition $\Omega = \Omega_s \cup \Omega_f$

1. Résoudre les équations d'états et les équations adjointes sur le maillage (**FreeFem++**) et calculer une direction de descente θ
2. Générer une level set ϕ associée à la décomposition $\Omega = \Omega_s \cup \Omega_f$

Algorithme et implémentation

Level set ϕ associée à la décomposition
$$\begin{cases} \phi(x) < 0 \text{ if } x \in \Omega_f, \\ \phi(x) = 0 \text{ if } x \in \Gamma, \\ \phi(x) > 0 \text{ if } x \in \Omega_s. \end{cases}$$





Algorithme et implémentation

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2. Générer une level set ϕ associée à la décomposition $\Omega = \Omega_s \cup \Omega_f$ (**mshdist**)
3. Advecter la level set sur le maillage T et adapter le maillage (**advect+mmg2d**)

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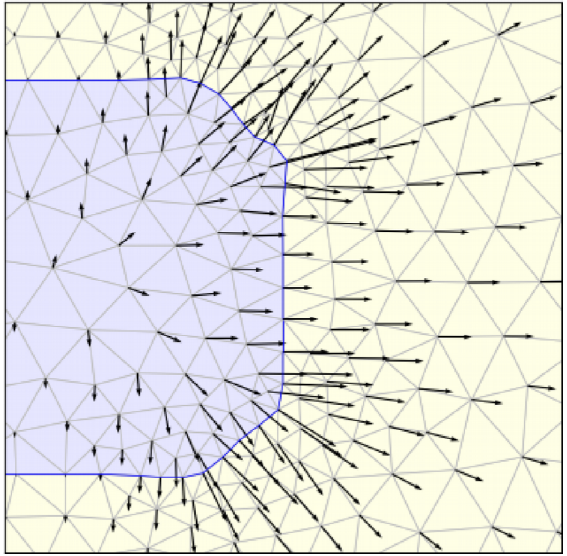
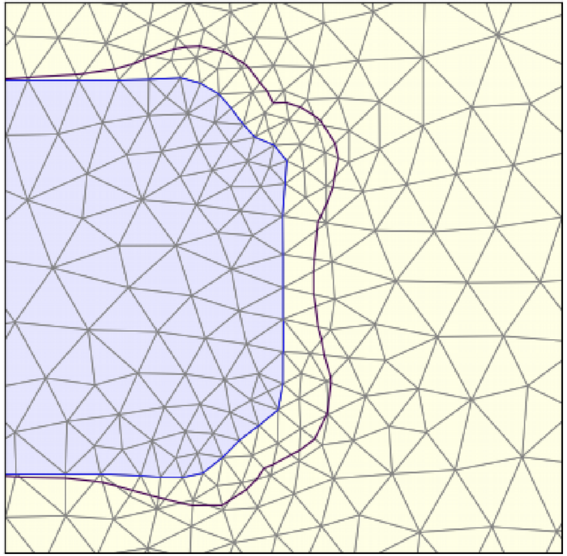
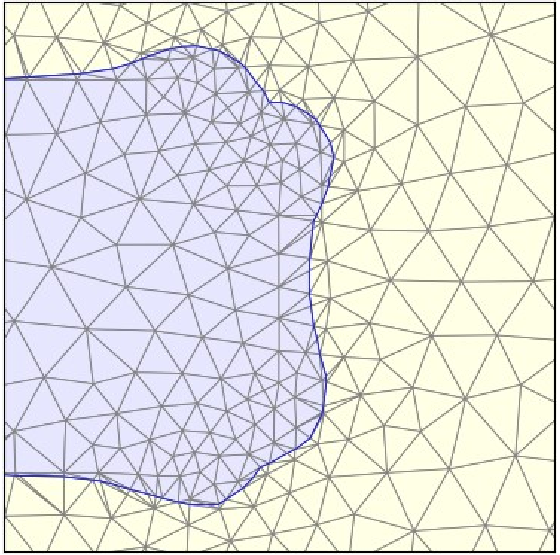
Variations de formes avec méthode hybride level-set + remaillage



Calcul du gradient de forme sur maillage conforme



Advection de la level set sur le maillage conforme (**advect**)



Discrétisation de la ligne de niveau 0 (**mmg2d**)

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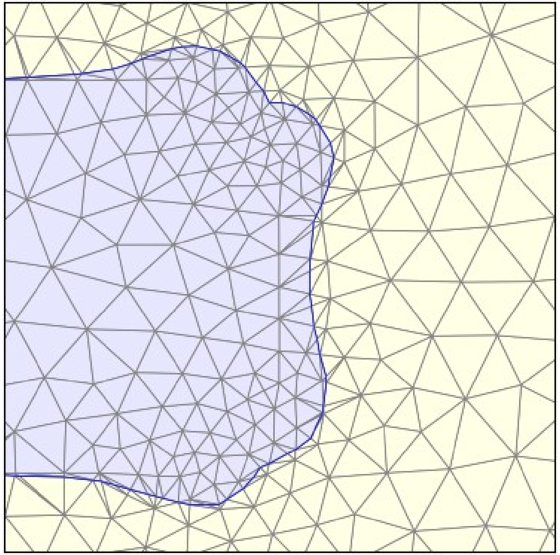
Variations de formes avec méthode level-set + remaillage



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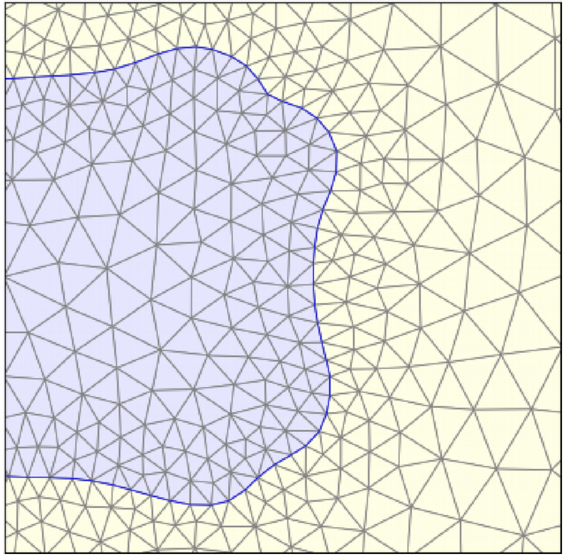

Adaptation du maillage (**mmg2d**)

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