Lecture 3: Topology optimization and automated generative design, perspectives and applications in the context of additive manufacturing

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Spring 2022 – Seminar for Applied Mathematics

ETH zürich

Outline

1. What is topology optimization ?
2. Industrial design processes: from CAD based design towards additive manufacturing
    - CAD based design
    - Additive manufacturing
    - Some future prospects
3. A non-exhaustive review of shape and topology optimization techniques
    - Parametric shape optimization
    - Geometric shape optimization
    - Topology optimization

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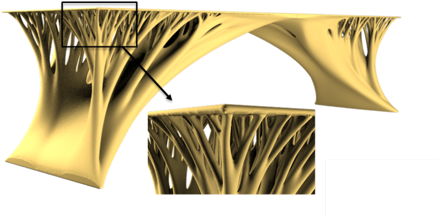
(a) Siemens (2017)

(b) APWorks (2016)



(c) M2DO (Kambampati et. al. 2018)

(d) AIRBUS (2010)

What is topology optimization ?

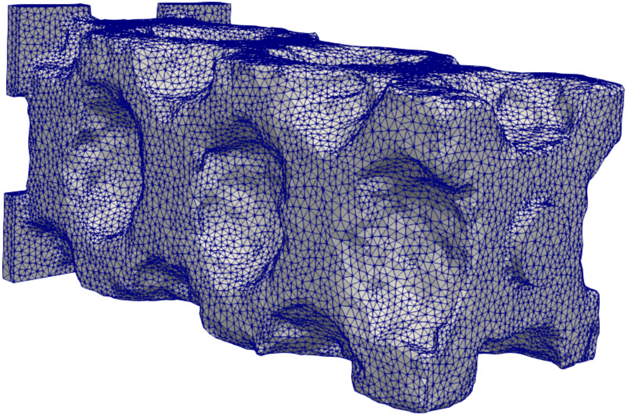


Figure: Optimization of the rigidity of a mechanical structure subject to flexural load

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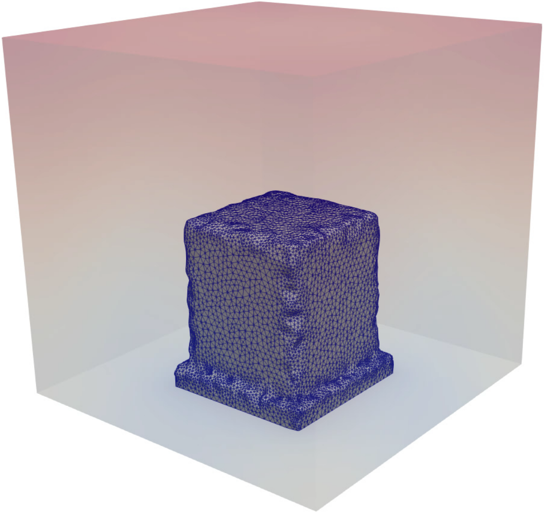

Figure: Minimization of the average temperature with a cooling material

What is topology optimization ?

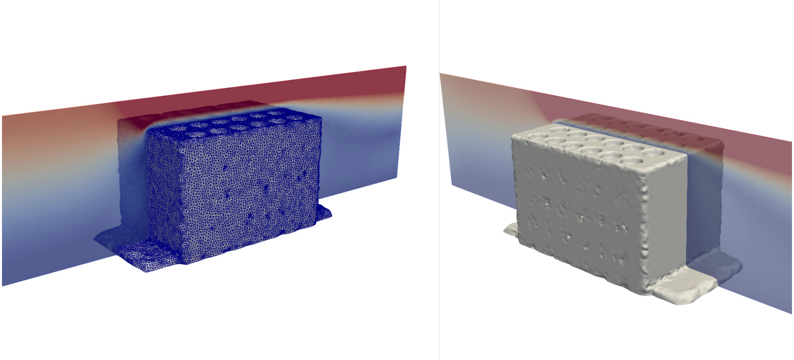

Figure: Topology optimization of a supporting mechanical structure subject to the pressure of an incoming flow

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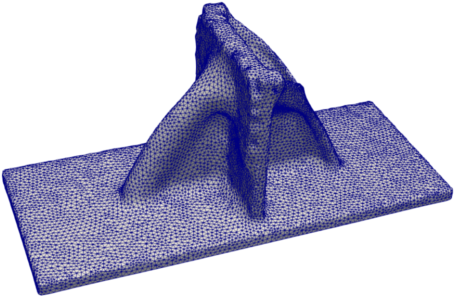

Figure: Optimized shape

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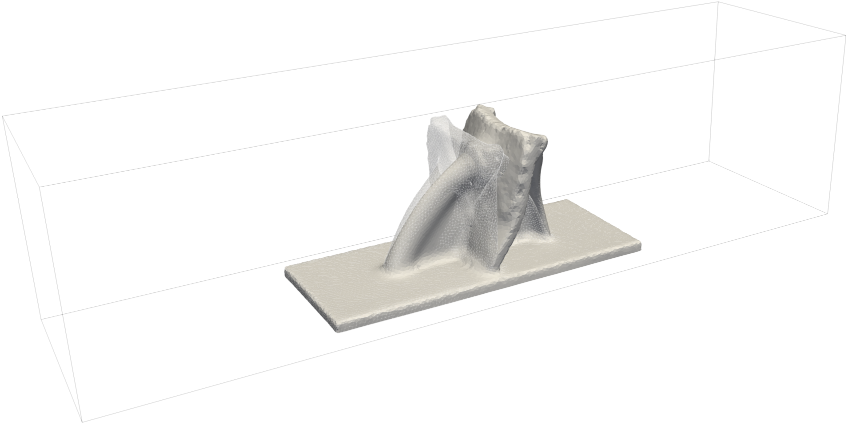

Figure: Elastic deformation

What is topology optimization ?

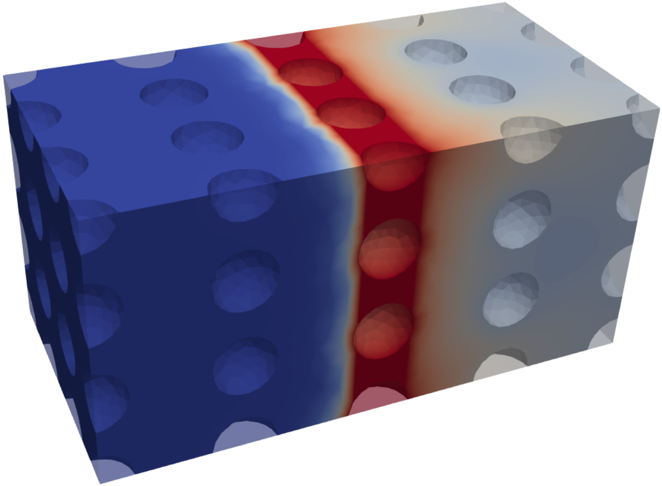

Figure: Maximization of the heat acquired by fluid pipes through a heated box.

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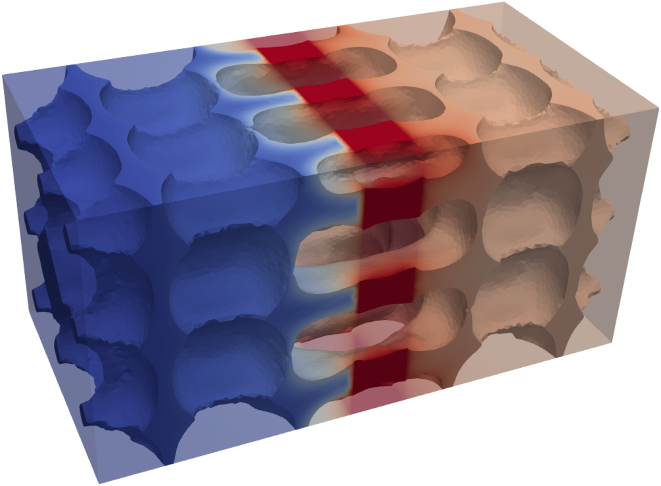

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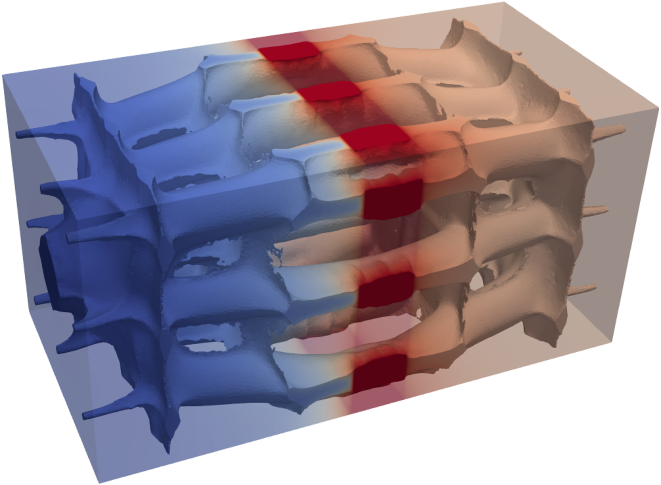

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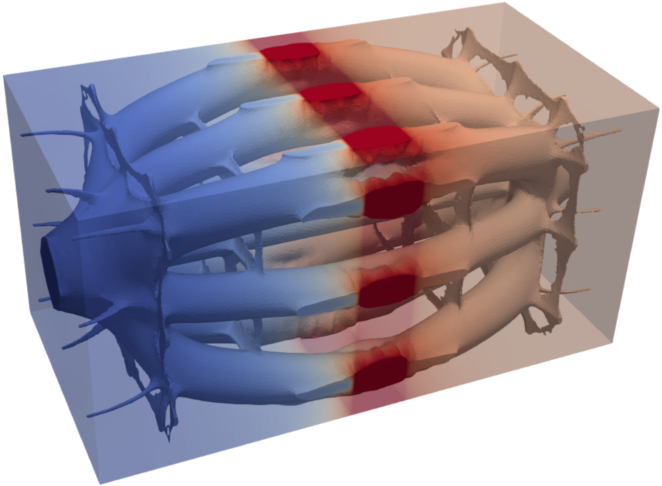

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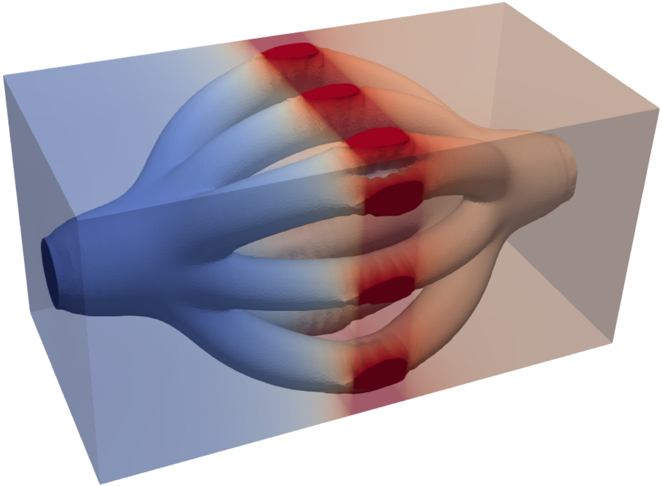

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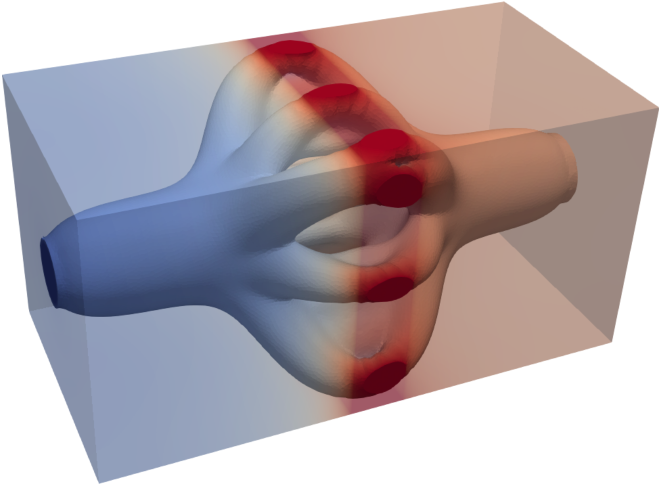

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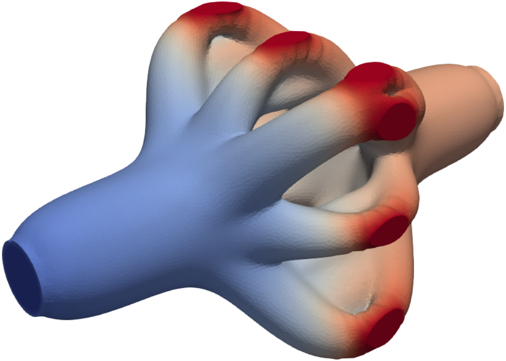

Figure: Optimized design.

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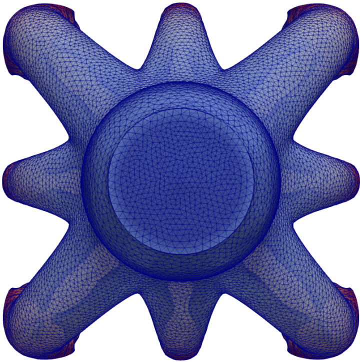

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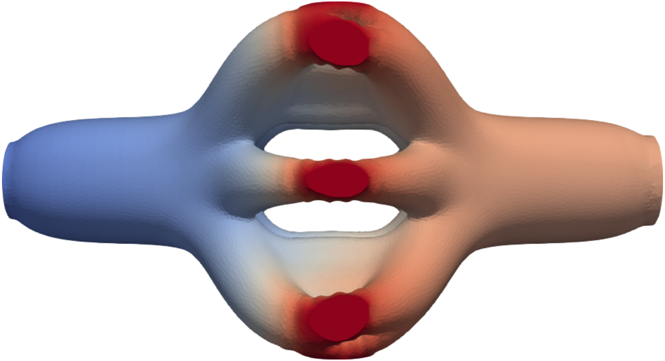

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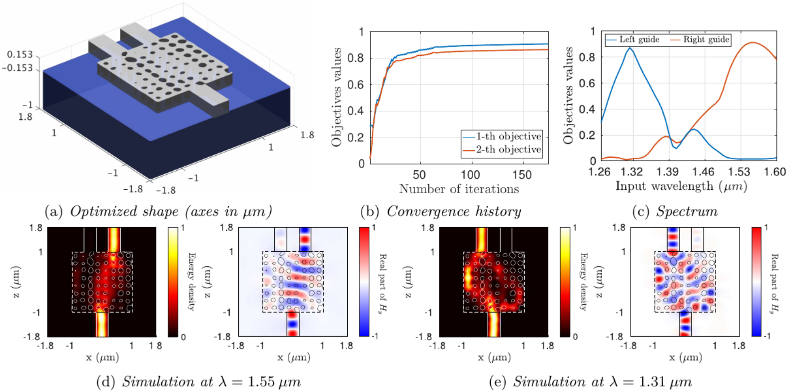

Figure: A topologically optimized photonic device: light duplexer. Figure from Lebbe, 2019

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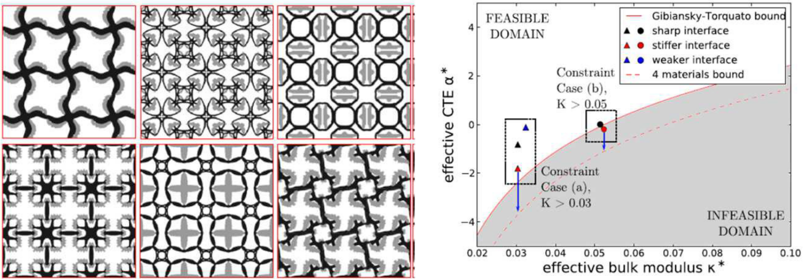

Figure: Architectured 2-D elastic material achieving optimal composite properties

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Industrial design processes

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- Industrial processes still rely heavily on classical design processes relying on CAD (*Computer Aided Design*) parametrizations

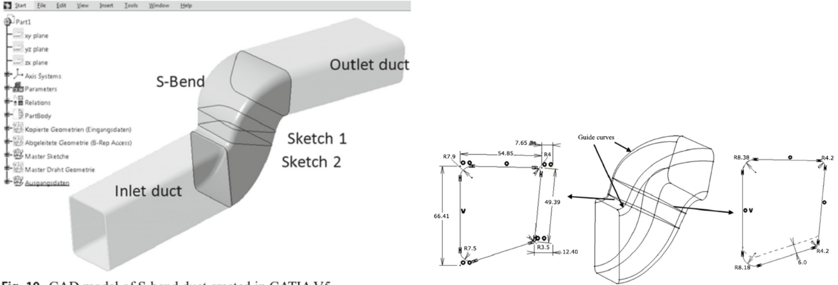
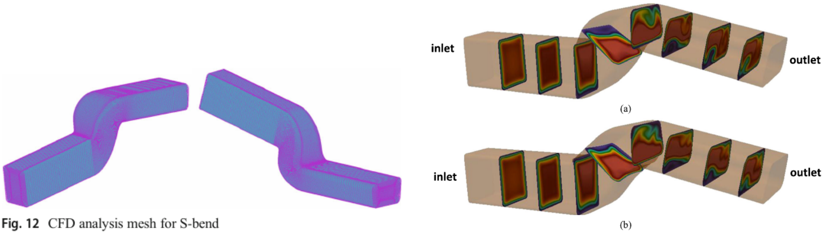


Fig. 10 CAD model of S-bend duct created in CATIA V5

Figure: CAD based optimization of a pipe for flow transfer. Figure from Agarwal et. al. 2019

CAD based design:

- Use a CAD parameterization of the design using e.g. Bezier curves, NURBS or spline



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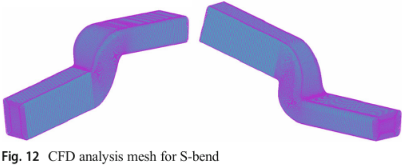
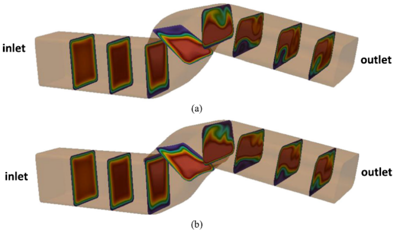
CAD based design optimization





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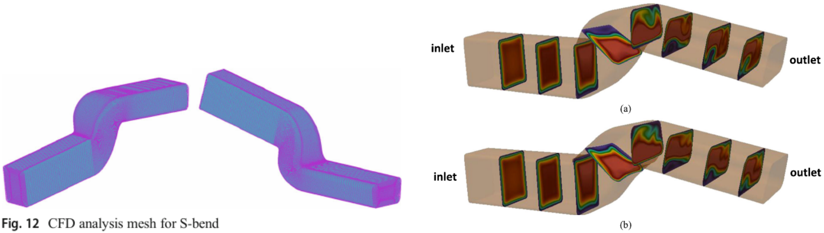

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- CAD based designs are easier to manufacture, but they lead to small modifications of the proposed design
- However the use of topology optimization is rising in high-tech industries due to the advent of additive manufacturing

Figure: Additive manufacturing: metal powder bed fusion (source: https://www.youtube.com/watch?v=Y-dTc8_3dU0).

It is now possible for industries to build complex topology optimized structures.

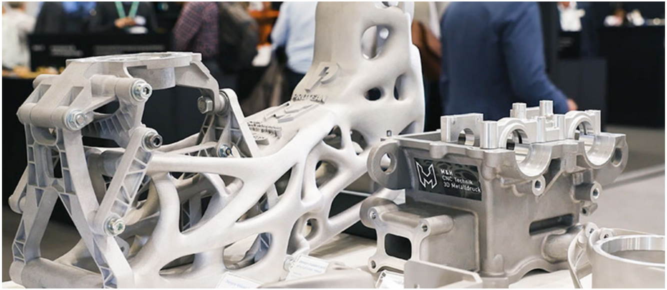

Figure: Altair

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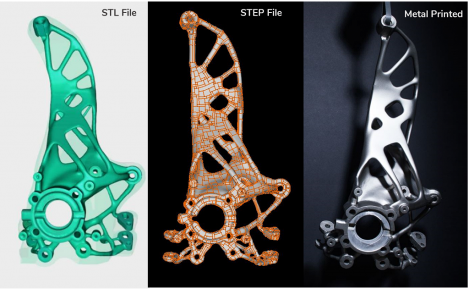

Figure: ParaMatters

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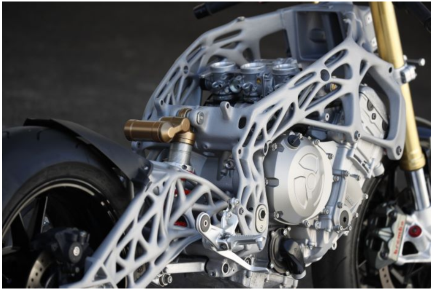

Figure: BMW

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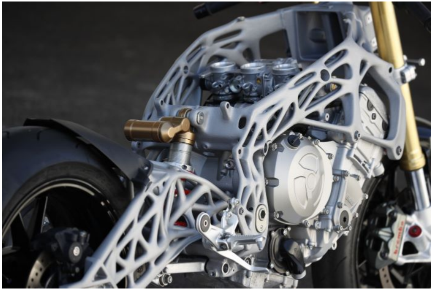

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Today, the technology is mature mainly for the design of lightweight mechanical structures and used in the automotive industry. Commercial software are available (Ansys, Comsol, Autodesk,. . .).

Industrial design processes

Additive manufacturing: 3D metal printing

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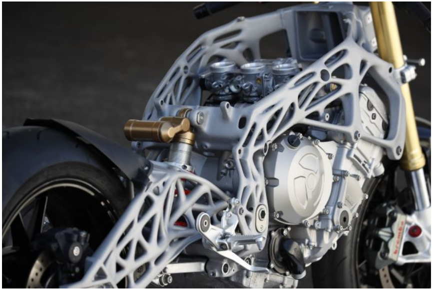

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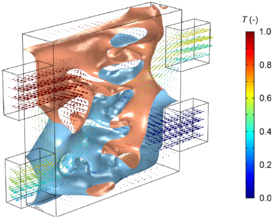
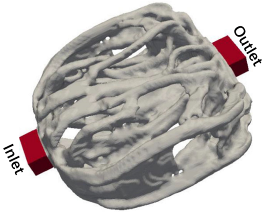
Today, the technology is mature mainly for the design of lightweight mechanical structures and used in the automotive industry. Commercial software are available (Ansys, Comsol, Autodesk,. . .).
However, one can expect from recent progresses that this technology will be ready in a short future for more complex physics involved e.g. in the aeronautic industry.

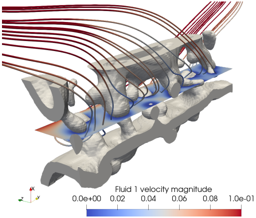


(a) Kobayashi et. al. (2020)



(b) Yu et. al. (2020)

(c) Savier (2019, United Technologies)



(d) Hoghoj et. al. (2020)

Figure: Fluid pipes optimized for convective heat transfer with density methods.

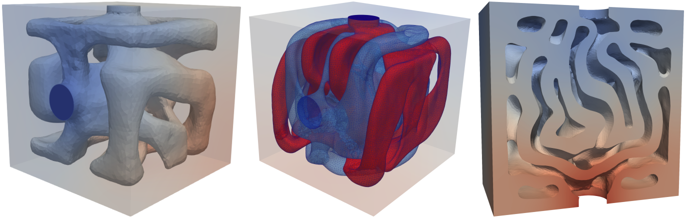


Figure: Topology optimized fluid-to-fluid heat exchanger devices with the method of Hadamard and a body-fitted mesh evolution algorithm.

Industrial design processes
Some future perspectives

- Optimization of the internal cooling channels of turbine blades

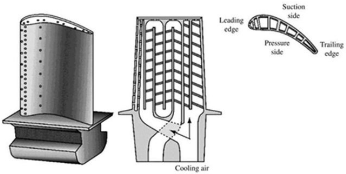


Figure: Internal cooling system of a turbine blade.

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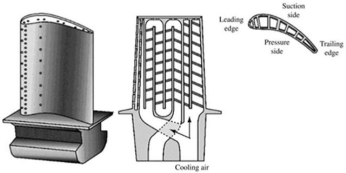

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- Multi-scale design of heat exchangers

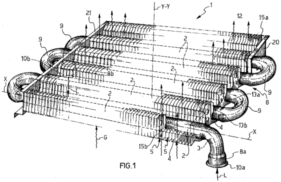

Figure: Industrial gas-liquid heat exchanger.

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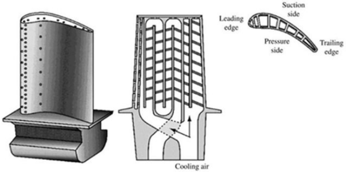

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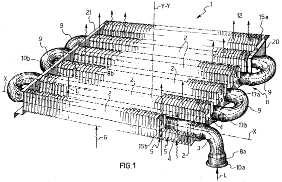

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- Manufacturing constraints in additive manufacturing: overhang, minimum thickness, residual stress constraints...

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$$s.t. \begin{cases} G_i(\Omega) = 0, & 1 \leq i \leq p \\ H_j(\Omega) \leq 0, & 1 \leq j \leq q \end{cases}$$

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In industrial applications, $J(\Omega)$, $G_i(\Omega)$ or $H_j(\Omega)$ involve the solution u_Ω defined with respect to a PDE model posed on Ω.

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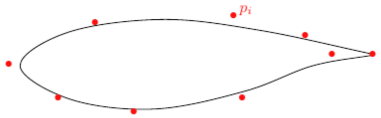
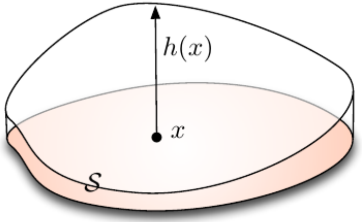
- A **mathematical representation of shapes** (through a finite set of parameters, a density function) and their variations
- A **numerical description of shapes** and a methodology to update them.

Methodologies fall broadly into three categories: parametric shape optimization , (Hadamard) geometric shape optimization , and homogenization-based topology optimization.

- Parametric shape optimization:



(a) Parametrized representation of an airfoil using NURBS control points



(b) Parameterized representation of the shape of a membrane through a height function $h(x)$

Figure: Figure from the lecture of Charles Dapogny

In this case, the objective and the constraints functions can be rewritten in terms of the parameters $p_1, \dots, p_N$ involved to represent the shape:

$$\min_{(p_1,\dots,p_N)} J(p_1,\dots,p_N)$$
$$s.t. \begin{cases} G_i(p_1,\dots,p_N) = 0, & 1 \leq i \leq p \\ H_j(p_1,\dots,p_N) \leq 0, & 1 \leq j \leq q \end{cases}$$

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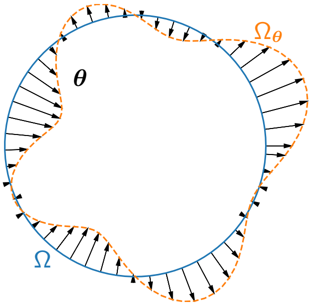
The variations of the shape are accounted for by the variations of the parameters,

$$\{p_i\}_{1\leq i \leq N} \mapsto \{p_i + \delta p_i\}_{1 \leq i \leq N},$$

and one can then use standard optimization methods in $\mathbb{R}^N$ to solve the problem.

- Non parametric shape optimization based on the boundary variation method of Hadamard:



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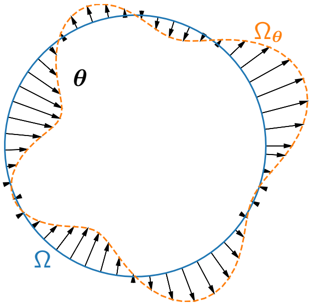

- Free deformations with a small amplitude are taken into account and parameterized by a vector field $\boldsymbol{\theta}$

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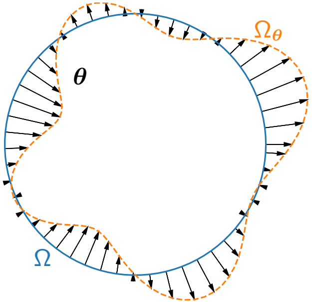
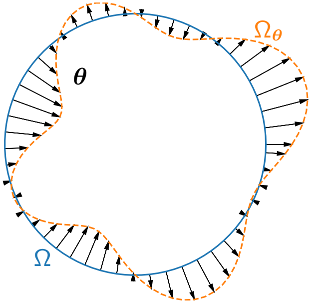


- Free deformations with a small amplitude are taken into account and parameterized by a vector field $\boldsymbol{\theta}$
- The number of arches or holes remains fixed; the method does not allow, *a priori*, changes of the topology.

- Non parametric shape optimization based on the boundary variation method of Hadamard:



- Free deformations with a small amplitude are taken into account and parameterized by a vector field θ
- The number of arches or holes remains fixed; the method does not allow, *a priori*, changes of the topology.
- In practice, one uses shape initializations featuring a given number of holes (looking like Emmental cheese)

- Several instantiations of this method are possible depending on the chosen numerical representation of shapes

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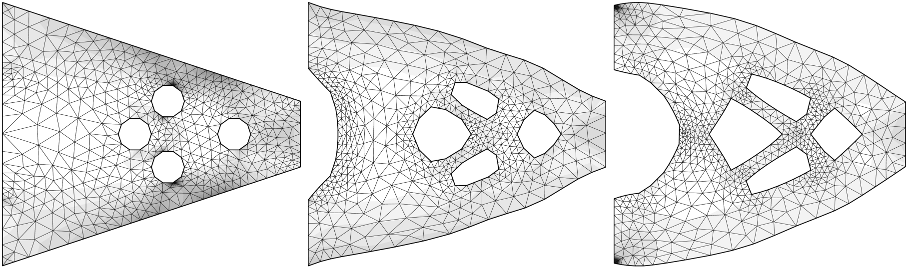


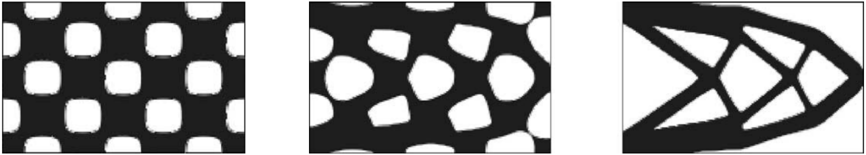
(a) Initial design (b) Intermediate design (c) Final design

Figure: Geometric shape optimization of the shape of a 2-d cantilever beam with the method of mesh deformation (figures from Allaire, 2007). No topological changes occur.

A non-exhaustive review of shape and topology optimization techniques

Geometric shape optimization

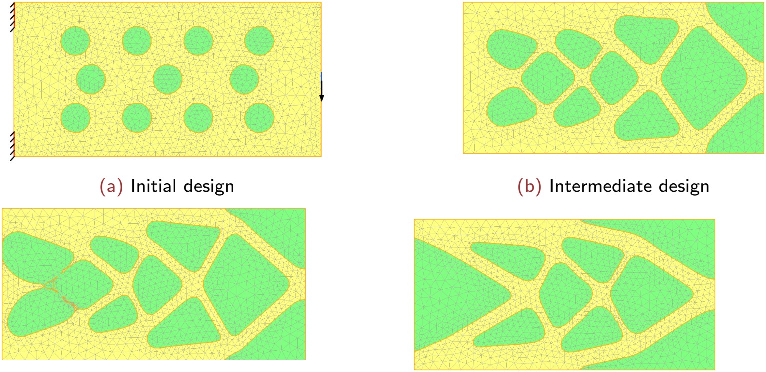
- Several instantiations of this method are possible depending on the chosen numerical representation of shapes



(a) Initial design (b) Intermediate design (c) Final design

Figure: Geometric shape optimization of a 2-d cantilever beam with the level set method (figures from Allaire, 2002). Topological changes occur: some holes have merged from the initial to the final design.

- Several instantiations of this method are possible depending on the chosen numerical representation of shapes



(a) Initial design

(b) Intermediate design

(c) Intermediate design featuring a topological change

(d) Final design

Figure: Geometric shape optimization of a 2-d cantilever beam with a level-set based mesh evolution method (figures from Dapogny, 2013). Topological changes are handled while keeping an explicit, conformal discretization of shapes.

Some further methods exist for optimizing intrinsically both the shape and the topology of the domain:

- Topological gradients

Figure: Topology optimization of a cantilever beam coupling geometric shape updates and hole nucleation with the topological gradient, which detects where to put new holes. Courtesy from Allaire

Some further methods exist for optimizing intrinsically both the shape and the topology of the domain:

- Density based topology optimization: the shape Ω is represented by a density function $\rho : D \mapsto (0, 1)$.

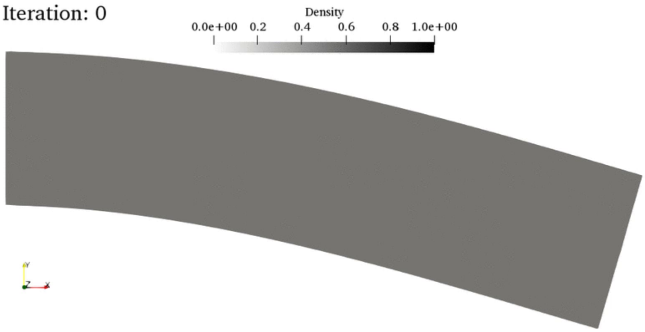


Figure: Topology optimization of a cantilever beam using the Solid Isotropic Material Penalization (SIMP) method. Figure from `https://comet-fenics.readthedocs.io`.

Some further methods exist for optimizing intrinsically both the shape and the topology of the domain:

- Homogenization based topology optimization.

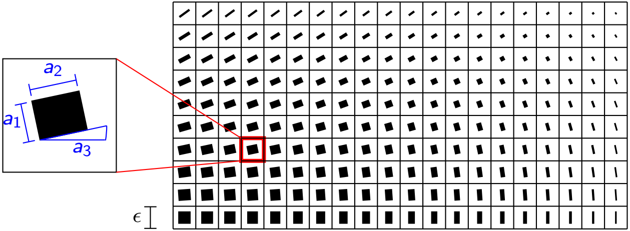
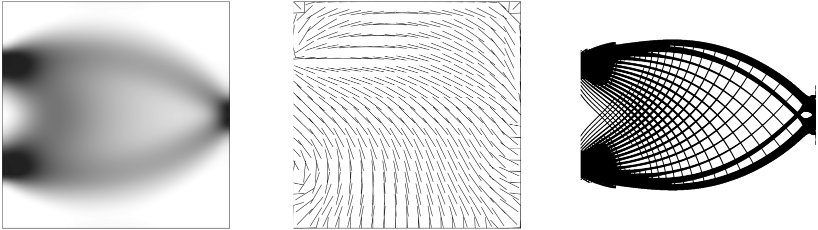


Figure: A periodic medium with a modulated parameterized microstructure.

Some further methods exist for optimizing intrinsically both the shape and the topology of the domain:

- Homogenization based topology optimization.



(a) Optimized density (b) Optimized orientation (c) Interpreted shape

Figure: Topology optimization of a 2-d cantilever beam by a homogenization method. Figure from Geoffroy-Donders, 2019.

Homework

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