# True Approximations for $k$-Center with Covering Constraints

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## Motivation

- Surge of interest in fairness-inspired $k$-center versions
- Fairness conditions naturally lead to covering constraints
- Current techniques only give pseudo-approximations

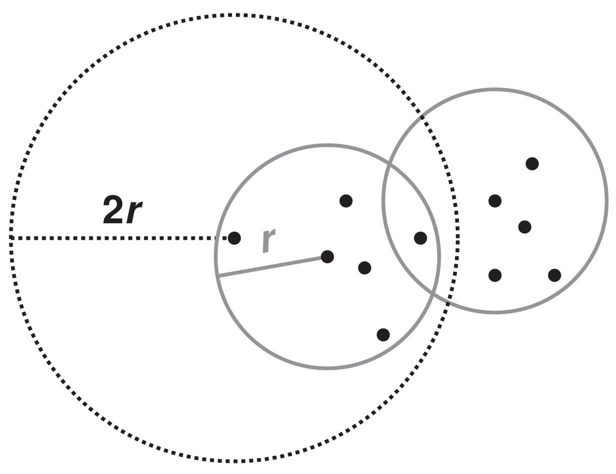
**How to deal with covering constraints in $k$-center problems?**

## Recap: The $k$-Center problem (classical version)

Task: cover all points of a metric space with $k$ balls of smallest possible radius

2-approximation can be achieved by:

- Pick arbitrary point
- Remove ball of radius $2r$
- Repeat



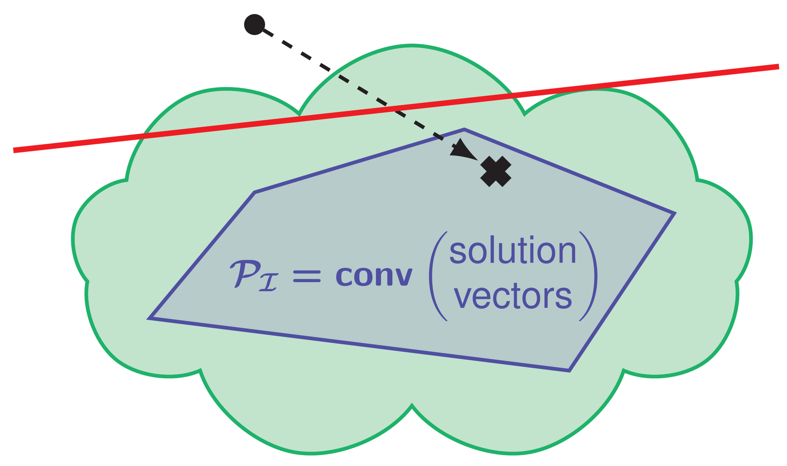

## Recap: round-or-cut (classical framework we build upon)

Rounding technique based on Ellipsoid Algorithm:

Find [cloud] such that we can

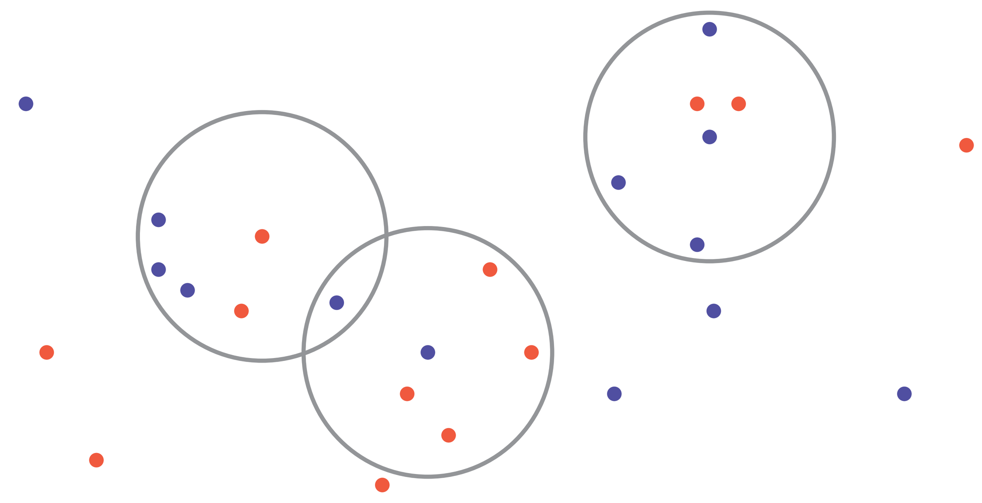
- either round solution (if in [cloud])
- or separate from target polytope

Then use this in Ellipsoid iterations




## $\gamma$-Colorful $k$-Center Problem ($\gamma$CkC), introduced by [1]: an illustrative example for our approach

- Input: metric space $X$, color classes $X_1, \dots, X_\gamma \subseteq X$ with covering requirements $m_1, \dots, m_\gamma$
- Output: centers $C \subseteq X$ with $|C| = k$ and $|\bigcup_{c \in C} B(c, r) \cap X_\ell| \geq m_\ell$
- Goal: minimize $r$



$$\mathcal{P} = \left\{ (x, y) \in [0,1]^X \times [0,1]^X \;\middle|\; \begin{array}{ll} \sum_{u \in X} y(u) \leq k & \\ \sum_{v \in B(u,r)} y(v) \geq x(u) & \forall u \in X \\ \sum_{u \in X_\ell} x(u) \geq m_\ell & \forall \ell \in [\gamma] \end{array} \right\}$$

($x$: extent to which points are covered by centers; $y$: extent to which points are opened as centers)

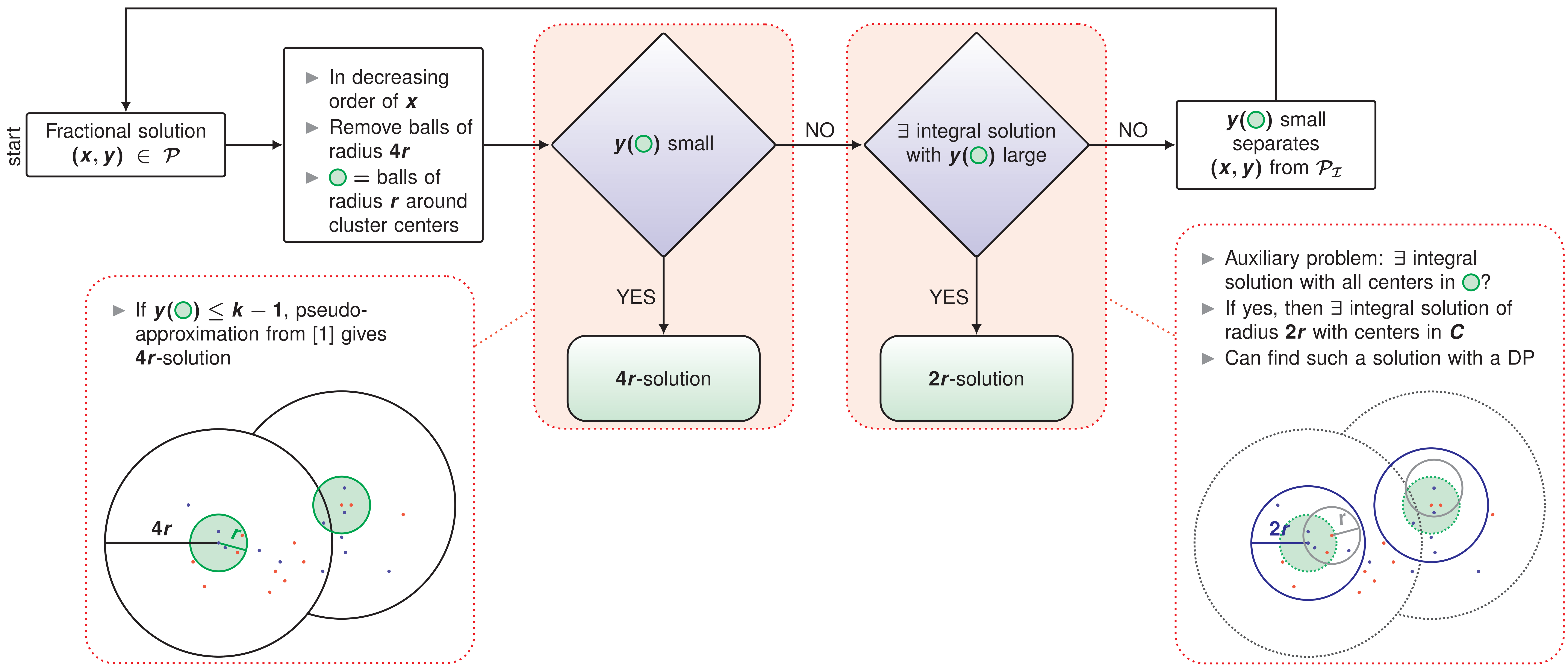
## Prior Work

- $(17 + \epsilon)$-approximation for plane [1]
- 2-pseudo-approximation opening $k + \gamma - 1$ centers [1]
- 2-pseudo-approximation for Fair $\gamma$CkC [2]
- Round-or-cut first used by [3] in this context

## Our Results for (Fair) $\gamma$CkC

- 4-approximation for Fair $\gamma$**CkC** for any metric for $\gamma = O(1)$ (Fair $\gamma$**CkC** is a probabilistic generalization of $\gamma$**CkC** by [2])
- $\gamma$**CkC** is inapproximable for unbounded $\gamma$ if $P \neq NP$, and for $\gamma = \omega(\log |X|)$ under *ETH*

## Algorithm: Illustration for $\gamma$CkC for $\gamma = 2$




## Further Result

- 5-approximation for supplier version

## Open Questions

- Best guarantee?
- Knapsack/Matroid $\gamma$**CkC**?

## References

[1] S. Bandyapadhyay, T. Inamdar, S. Pai, and K. R. Varadarajan, "A constant approximation for Colorful $k$-Center", *ESA*, 2019.
[2] D. G. Harris, T. Pensyl, A. Srinivasan, and K. Trinh, "A lottery model for center-type problems with outliers", *ACM TALG*, 2019.
[3] D. Chakrabarty and M. Negahbani, "Generalized center problems with outliers", *ACM TALG*, 2019.