SS03 Prof. R. Hiptmair, Dr. P.D. Ledger

Computational Electromagnetics Course

Exercise Sheet 3: Non–dimensional Parameters and Discrete Differentiation

1. A plane wave solution may be expressed as

$$\boldsymbol{e}(\boldsymbol{x}, t) = \boldsymbol{p} \cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^3$, $\boldsymbol{p} \in \mathbb{R}^3$ is the polarisation vector, $\boldsymbol{k} \in \mathbb{R}^3$ is the wave vector and $\omega \in \mathbb{R}$ is the frequency. Show that plane waves are solutions of Maxwell's equations in linear, isotropic, charge free mediums. Note the dispersion relationship is given by $|\boldsymbol{k}|^2 = \omega^2 \epsilon \mu$, and consider the case when $\boldsymbol{k} \parallel e_x$, $\boldsymbol{p} \parallel e_y$.

2. The non–dimensional parameter η is defined as

$$\eta^2 := \mu \epsilon \frac{L^2}{T^2} \tag{2}$$

where ϵ and μ denote the permittivity and permeability respectively of the medium. In addition L and T denote the characteristic length and time. By considering the characteristic length to be the wavelength and characteristic time to be the period, give an explanation of the physical meaning of η.

3. The non–dimensional relative dielectric relaxation time is defined as

$$\tau := \frac{\epsilon}{\sigma} \frac{1}{T} \tag{3}$$

where ϵ and σ denote the permittivity and conductivity respectively of the medium and T is the characteristic time. By using the continuity equation and the assumption of isotropic material parameters, give an explaination of the physical meaning of τ.

4. The non–dimensional relative penetration depth (skin depth) is defined as

$$\delta^2 := \frac{T}{\sigma \mu} \frac{1}{L^2} \tag{4}$$

where μ and σ denote the permeability and conductivity respectively of the medium. In addition L and T denote the characteristic length and time. By considering $\boldsymbol{e}(\boldsymbol{x}, t) = \boldsymbol{p} \exp(\mathrm{i}(\omega t - \boldsymbol{k} \cdot \boldsymbol{x})$, show that the physical meaning of δ is the half life of a wave propagating in a conductor.

5. For the mesh of 5 tetrahedra shown in Figure 1, construct the internal orientation of each tetrahedron and then the matrices D^0, D^1 and D^3.

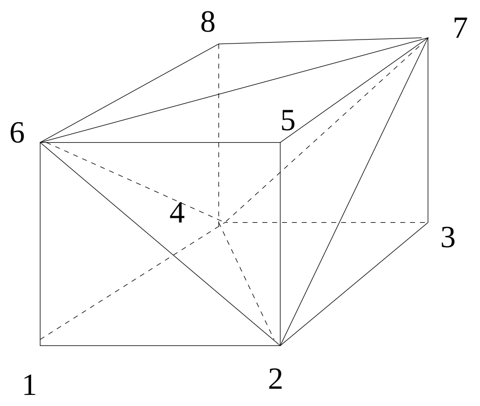

Figure 1: A cube split in to 5 tetrehadra.

6. Corollary 2.2C in the lecture notes states that for the discrete external derivative $d_h : \mathcal{C}^\ell(\mathcal{T}_h) \to \mathcal{C}^{\ell+1}(\mathcal{T}_h)$ and the incidence matrix $D^\ell \in \{-1, 0, 1\}^{N_{\ell+1} \times N_\ell}$

$$d_h \circ d_h = 0 \Leftrightarrow D^{\ell+1} D^\ell = 0 \tag{5}$$

where $\mathcal{T}_h$ donates a triangulation of a domain Ω. Prove that this corollary is true for a tetrahedral mesh.