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Computational Electromagnetics Course

Exercise Sheet 7: Function Spaces.

1. Theorem 3.4L of the lecture notes states that

$$\gamma_n : H(\mathrm{div};\Omega) \to H^{1/2}(\partial\Omega) \qquad \text{continous and surjective} \tag{1}$$

   Prove this theorem.

2. In theorem 3.5A the Helmholtz decompositions read as

$$\begin{aligned}(L^2(\Omega))^3 &= \mathrm{curl}\, H(\mathrm{curl};\Omega) \oplus \mathrm{grad}\, H_0^1(\Omega) \oplus \mathcal{H}^2(\Omega) & (2)\\ &= \mathrm{curl}\, H_0(\mathrm{curl};\Omega) \oplus \mathrm{grad}\, H^1(\Omega) \oplus \mathcal{H}_0^1(\Omega) & (3)\end{aligned}$$

   Construct co–homology spaces $\mathcal{H}^2(\Omega)$ and $\mathcal{H}_0^1(\Omega)$ such that (2) and (3) become L^2 orthogonal.

3. Lemma 3.6D states that with $\overline{C}, \underline{C} > 0$ only depending on ℓ and the shape regularity measure $\rho(\mathcal{T}_h)$

$$\underline{C} \sum_{b\in B(W_o^\ell(\mathcal{T}_h))} \alpha_b^2 \, \|b\|_{0,\Omega}^2 \leq \left\| \sum_b \alpha_b b \right\|_{0,\Omega}^2 \leq \overline{C} \sum_{b\in B(W_o^\ell(\mathcal{T}_h))} \alpha_b^2 \, \|b\|_{0,\Omega}^2 \tag{4}$$

   Prove this lemma.

4. The inverse estimate of Lemma 3.6D is Lemma 3.6E. This lemma states that

$$\|d\omega_h\|_{0,\Omega} \leq C \frac{1}{\min\limits_{T\in\mathcal{T}_h} h_T} \, \|\omega_h\|_{0,\Omega} \qquad \forall \omega_h \in W_0^\ell(\mathcal{T}_h) \tag{5}$$

   for $C = C(\rho(\mathcal{T}_h)) > 0$. Prove this lemma.

5. In section 3.6 of the lecture notes examples of squashed or flat elements are given. Give an example of a bad interpolation, on a flat tetrahedral element for Whitney 1–forms.