SS03 Prof. R. Hiptmair, Dr. P.D. Ledger

Computational Electromagnetics Course

Exercise Sheet 8: Magentostatics and Maxwell Eigenvalue Problems.

1. The variational approach to the computation of the offset field $\boldsymbol{h}_s$ in magnetostatics is shown in section 4.3.2 of the lecture notes to be the following, seek $\boldsymbol{h}_s \in H_0(\mathrm{curl};\Omega_j)$ and $\boldsymbol{\mu} \in (L^2(\Omega)^3)$ such that

$$\int_{\Omega_j} \boldsymbol{h}_s \cdot \boldsymbol{h}' \,\mathrm{d}x + \int_{\Omega_j} \mathrm{curl}\, \boldsymbol{h}' \cdot \boldsymbol{\mu} \,\mathrm{d}x = 0 \tag{1}$$

$$\int_{\Omega_j} \mathrm{curl}\, \boldsymbol{h}_s \cdot \boldsymbol{\mu}' \,\mathrm{d}x = \int_{\Omega_j} \boldsymbol{j} \cdot \boldsymbol{\mu}' \,\mathrm{d}x \tag{2}$$

   for all $\boldsymbol{h}' \in H_0(\mathrm{curl};\Omega_j)$ and $\boldsymbol{\mu}' \in (L^2(\Omega)^3)$ where $\boldsymbol{h}_s$ is an offset field, $\boldsymbol{\mu}$ is a Lagrange multiplier and $\boldsymbol{j}$ is a source term. Show that this variational statement has a unique solution for $\boldsymbol{h}_s$ but not for $\boldsymbol{\mu}$.

2. The construction of spanning trees is considered in section 4.3.4 of the lecture notes. Describe how one can program the technique of spanning trees in order to obtain a scalar surface potential for a curl free element of $W_0^1(\mathcal{T}_h)$ where $\mathcal{T}_h$ is a triangulation of a topologically trivial surface.

3. Lemma 4.3.4F of the lecture notes gives the interpolation estimate for fields with a discrete curl. For $1/2 < s \leq 1$, $\boldsymbol{v} \in H^s \cap H_0(\mathrm{curl};\Omega)$ and $\mathrm{curl}\, \boldsymbol{v} \in W_{0,0}^2(\mathcal{T}_h)$

$$\|\boldsymbol{v} - I^1\boldsymbol{v}\|_{0,\Omega} \leq h^s C |\boldsymbol{v}|_{s,\Omega} \qquad \text{with } C = C(\Omega, s, \rho(\mathcal{T}_h)),\ h := h(\mathcal{T}_h) \tag{3}$$

   Fill in the details of the proof.

4. Algorithm 4.3.5.A of the lecture notes describes how to construct co–tree gauging which can then be used to obtain a solution for the the discretised problem (4.3.3a). Discuss how this algorithm can be programmed in an efficient manner.

5. When co-tree gauging is used in equation (4.3.5c) poor conditioning of the matrix can occur for tensor product type meshes. Give an example of when this poor conditioning can arise.

6. The variational statement for the $\boldsymbol{e}$ based continuous Maxwell eigenvalue problem reads: seek $\boldsymbol{e} \in H_0(\mathrm{curl};\Omega)$ $\omega \neq 0$ such that

$$\int_\Omega \frac{1}{\mu} \mathrm{curl}\, \boldsymbol{e} \cdot \mathrm{curl}\, \boldsymbol{e}' \,\mathrm{d}x = \omega^2 \int_\Omega \epsilon \boldsymbol{e} \cdot \boldsymbol{e}' \,\mathrm{d}x \qquad \boldsymbol{e}' \in H_0(\mathrm{curl};\Omega) \tag{4}$$

under the assumptions of a bounded domain $\Omega \subset A_3$ filled with homogenous non–conducting material and with a perfectly conducting boundary $\partial\Omega$. Derive this variational statement assuming harmonic time dependence.