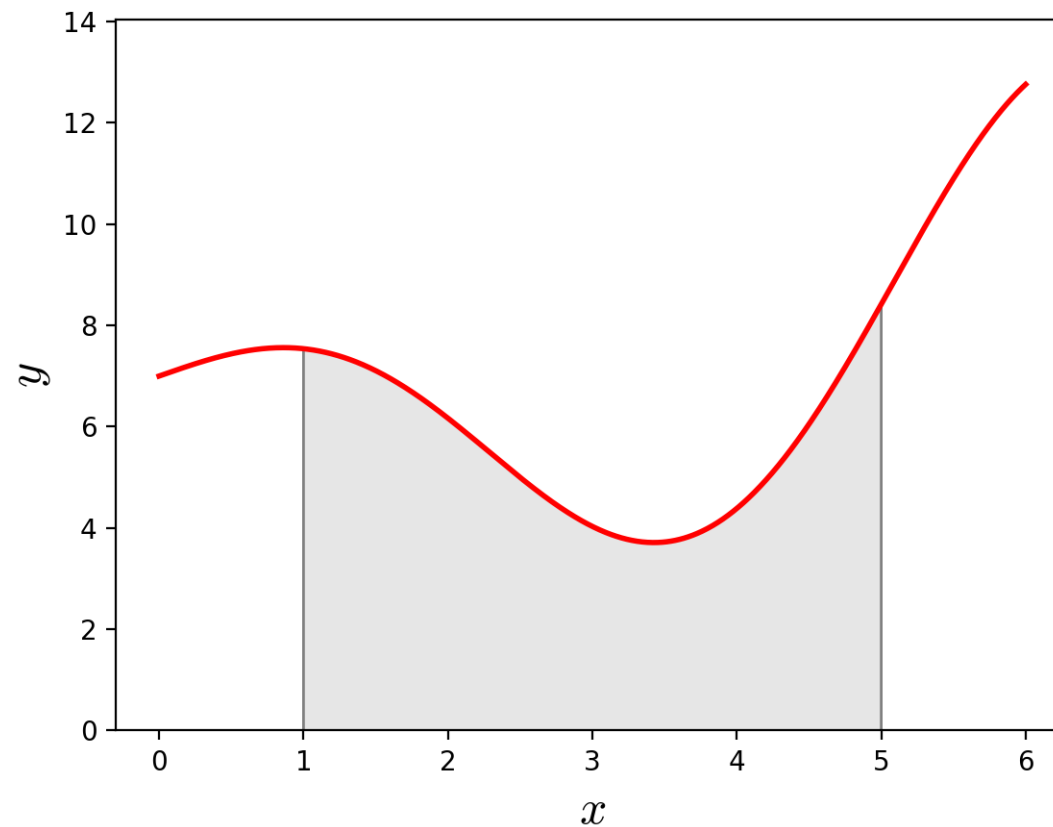


# Quadratur-Regeln

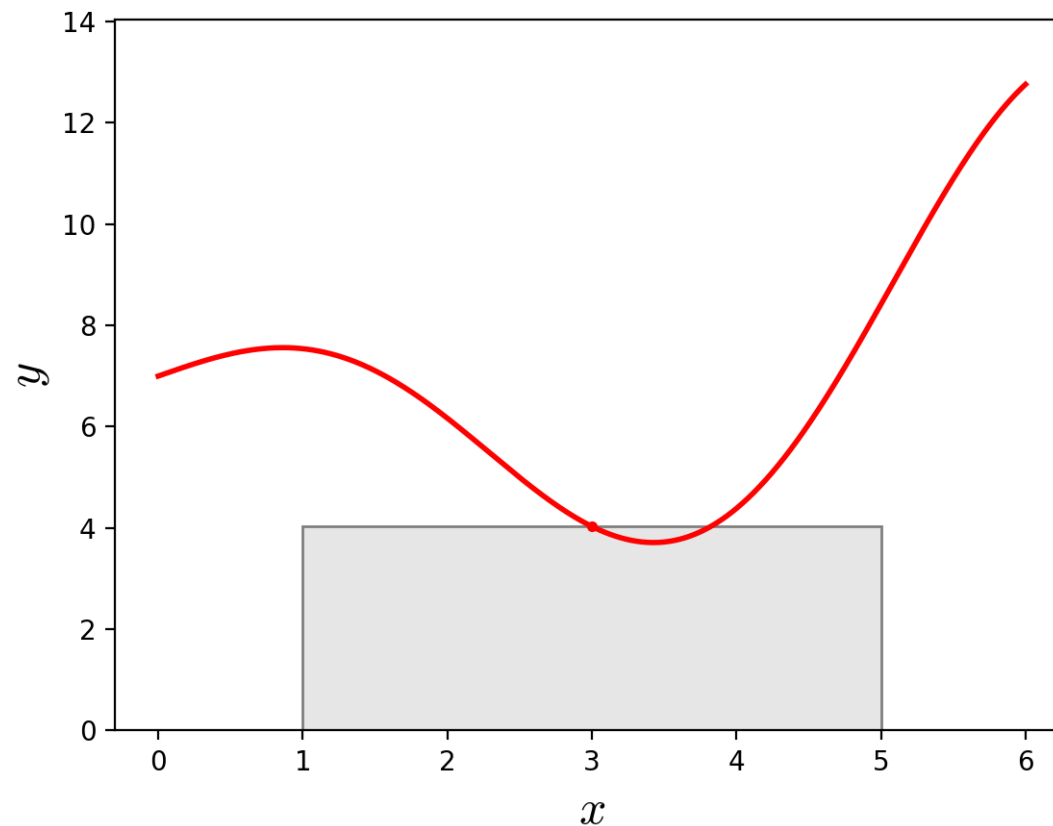
$$f(x) = x \cos(x) + 7$$



$$I[f] = \int_1^5 f(x) dx = 22.1073\dots$$

# Mittelpunktsregel

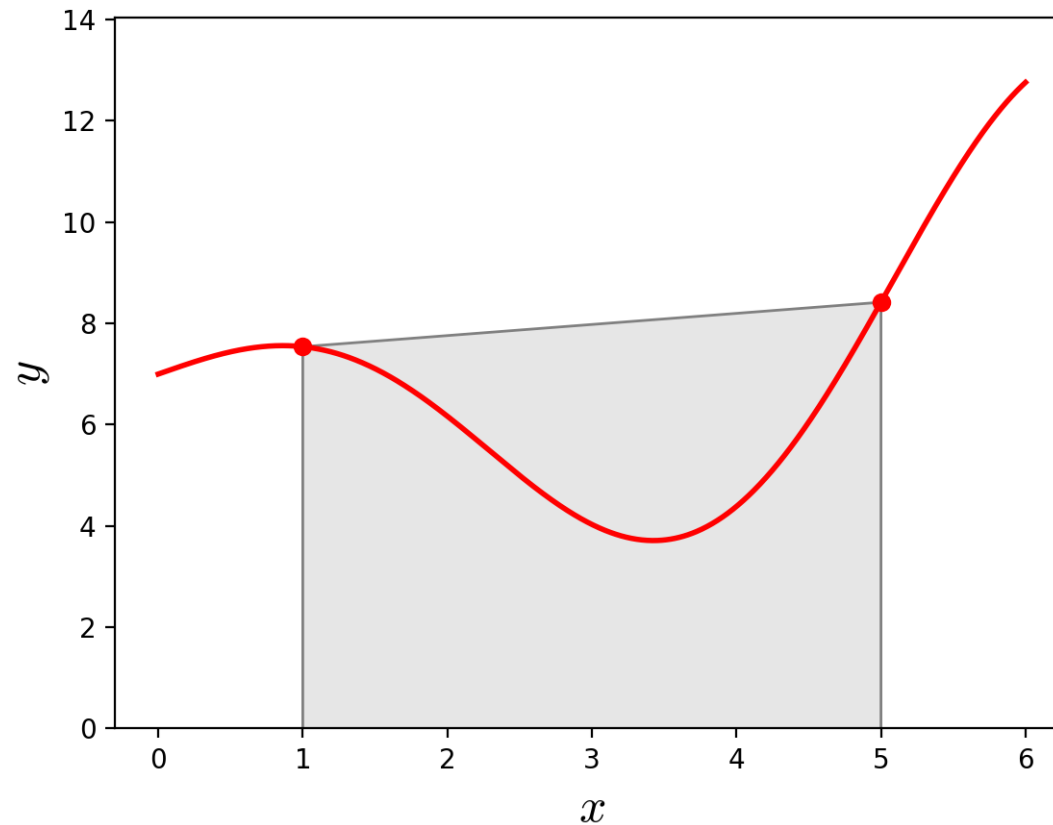
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_0[f] = 16.1201\dots \quad E[f] = 5.9872\dots$$

# Trapezregel

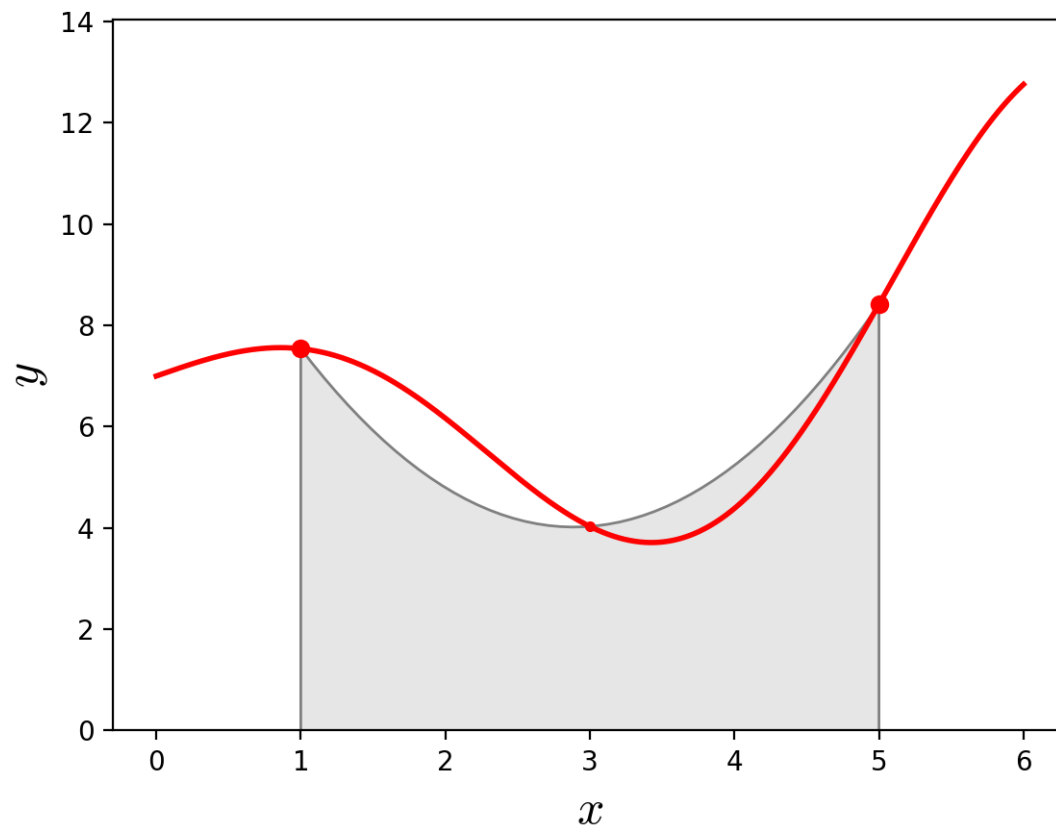
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_0[f] = 31.9172\dots \quad E[f] = 9.8100\dots$$

# Simpson-Regel

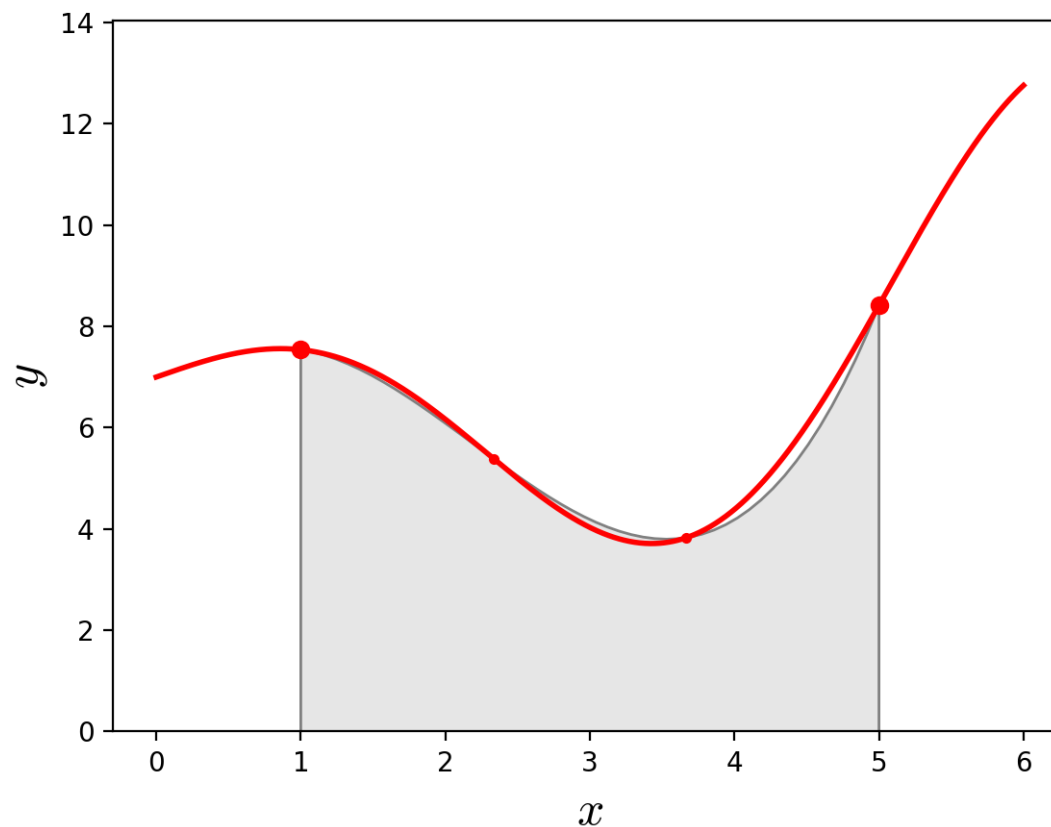
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_0[f] = 21.3858\dots \quad E[f] = 0.7215\dots$$

# 3/8-Regel

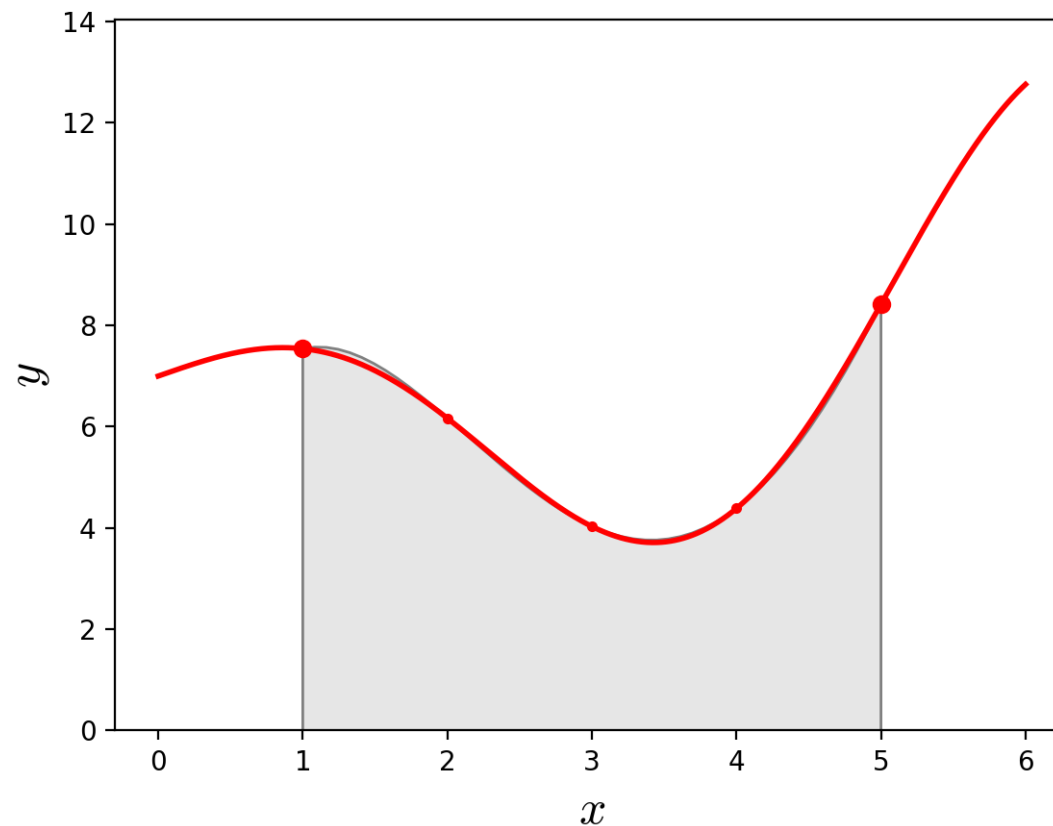
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_0[f] = 21.8026\dots \quad E[f] = 0.3047\dots$$

# Milne-Regel

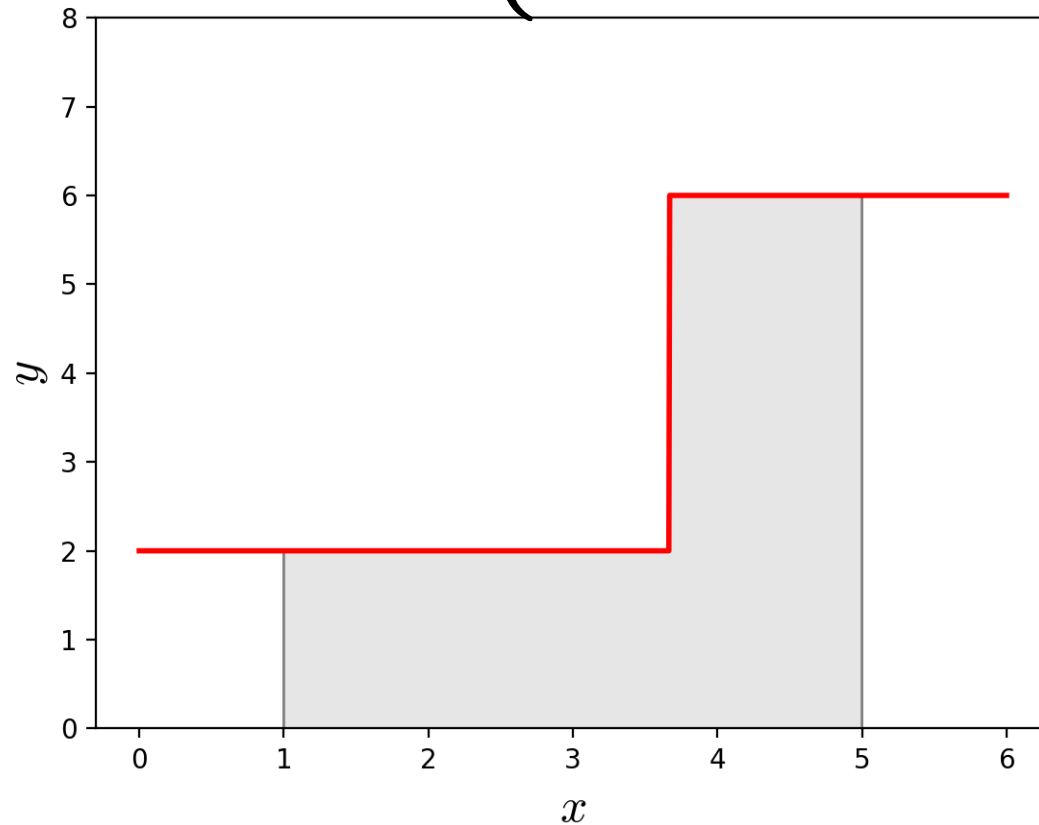
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_0[f] = 22.1231\dots \quad E[f] = 0.0159\dots$$

# Quadrature

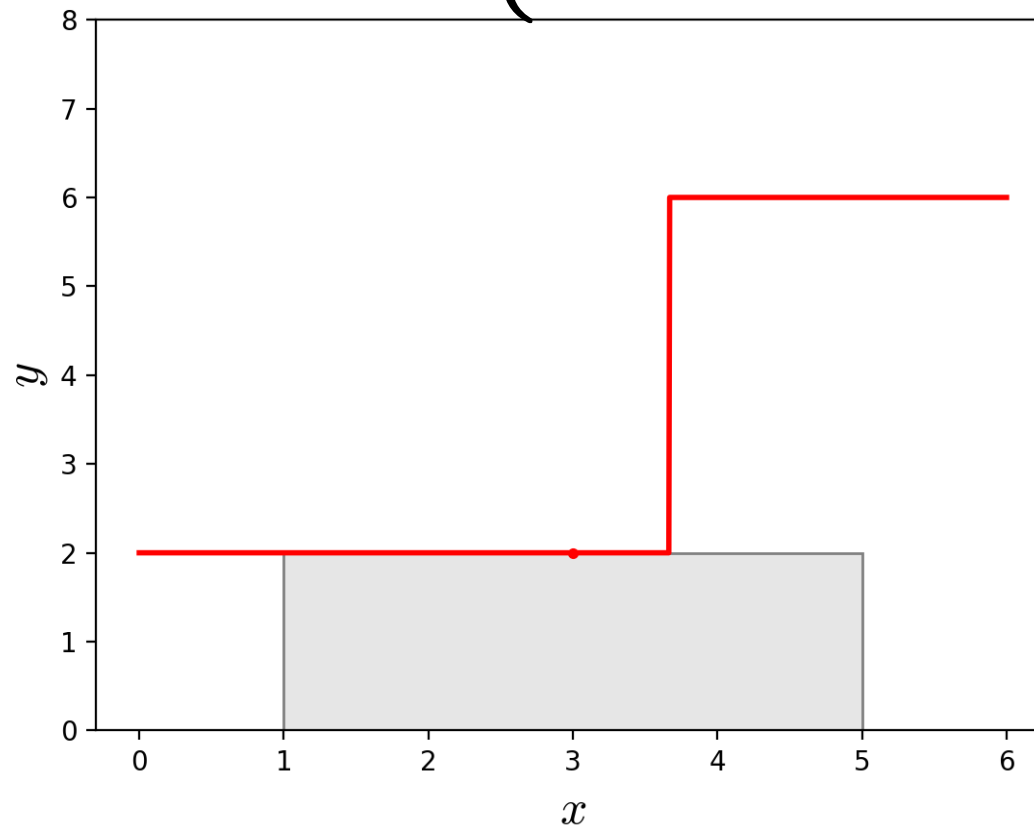
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = \int_1^5 f(x) dx = 15.4336\dots$$

# Mittelpunktsregel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



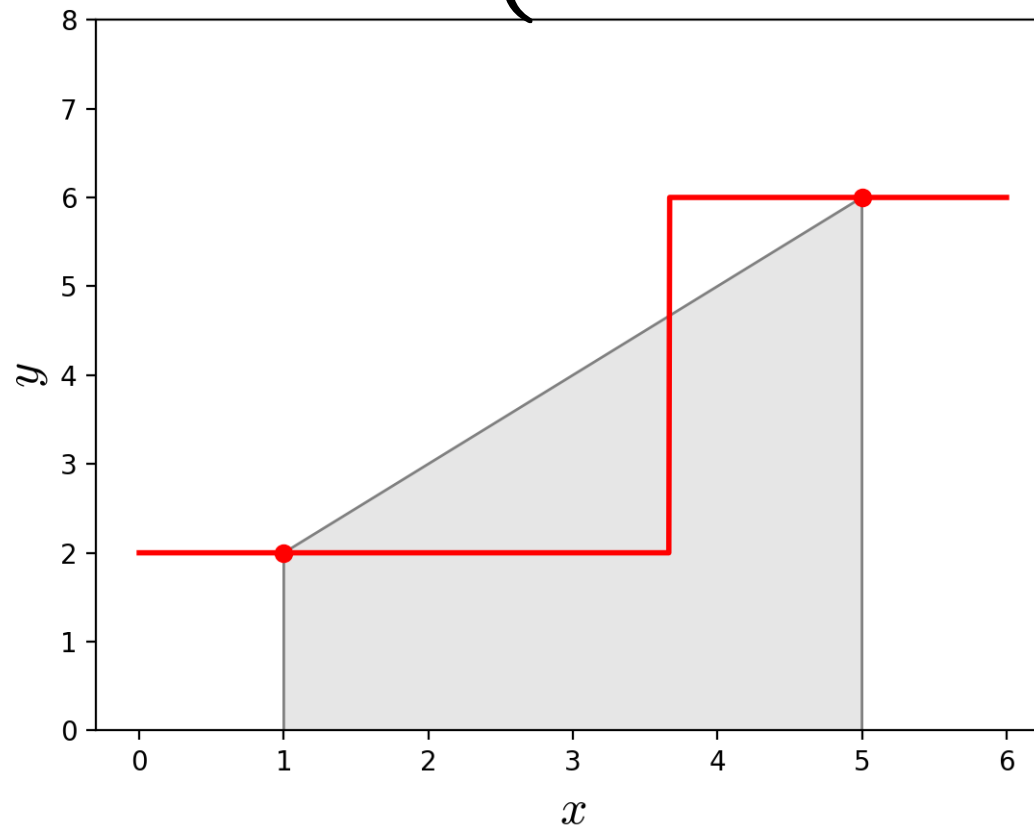
$$I[f] = 15.4336... \quad Q_0[f] = 8$$

$$E[f] = 7.4336...$$



# Trapezregel

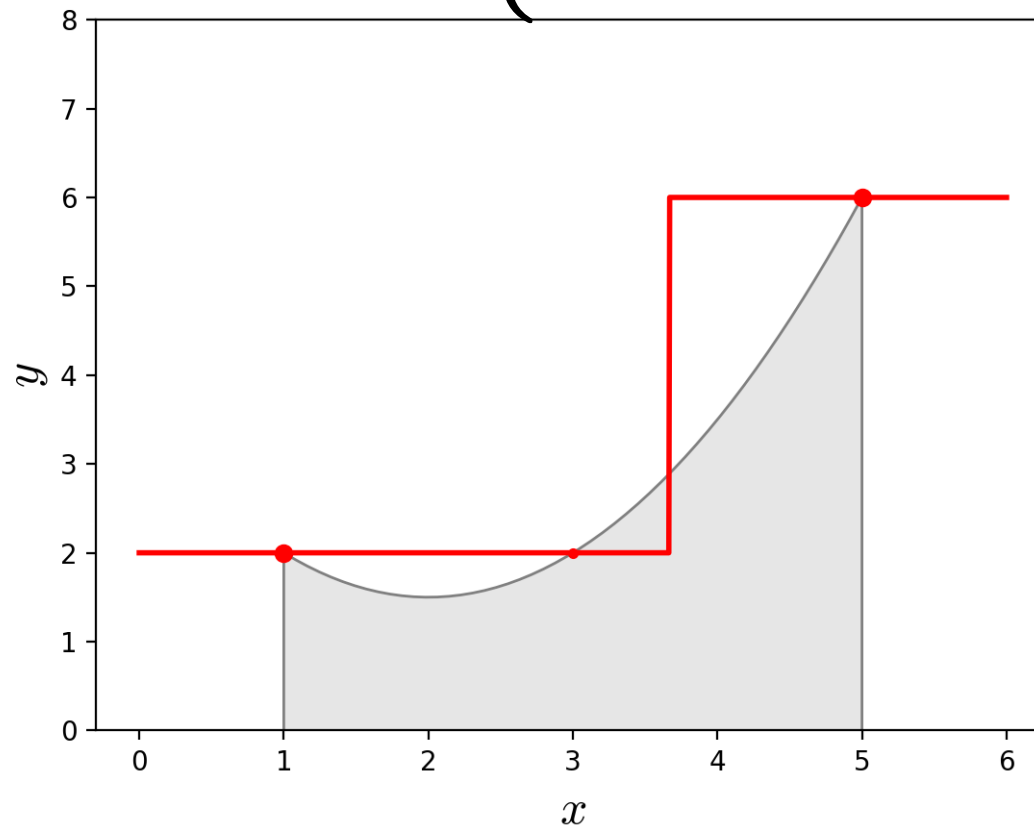
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = 15.4336... \quad Q_0[f] = 16 \quad E[f] = 0.5664...$$

# Simpson-Regel

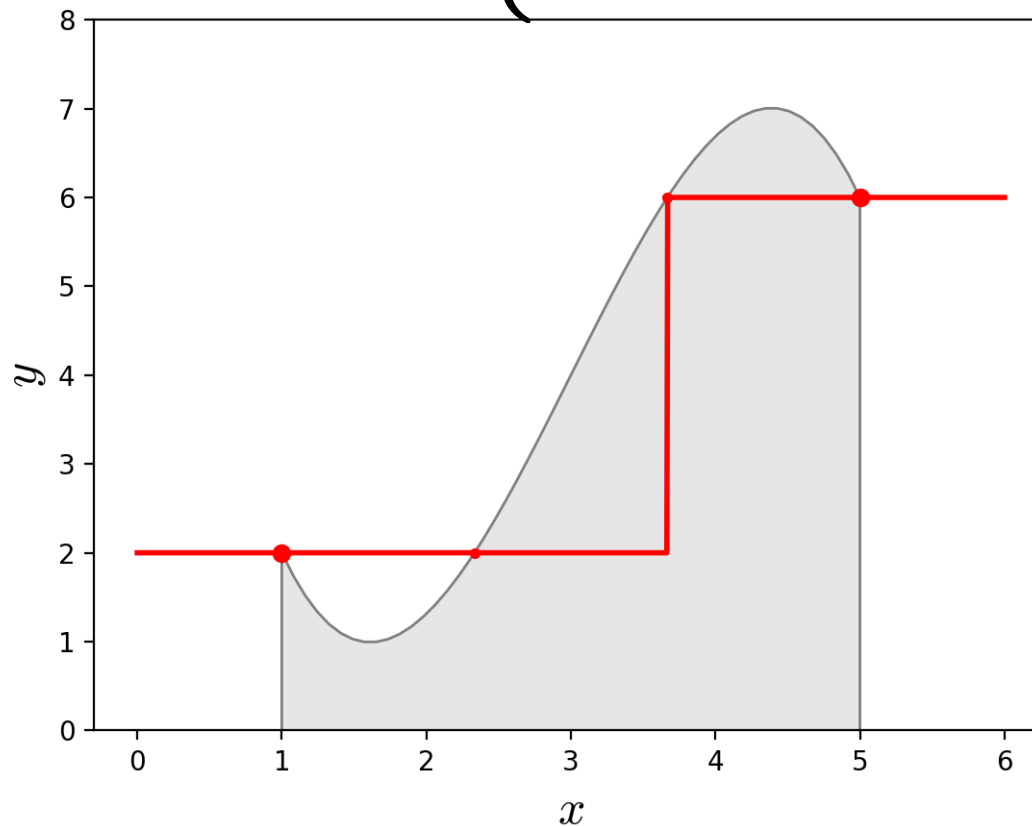
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = 15.4336\dots \quad Q_0[f] = 10.6666\dots \quad E[f] = 4.7670\dots$$

# 3/8-Regel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

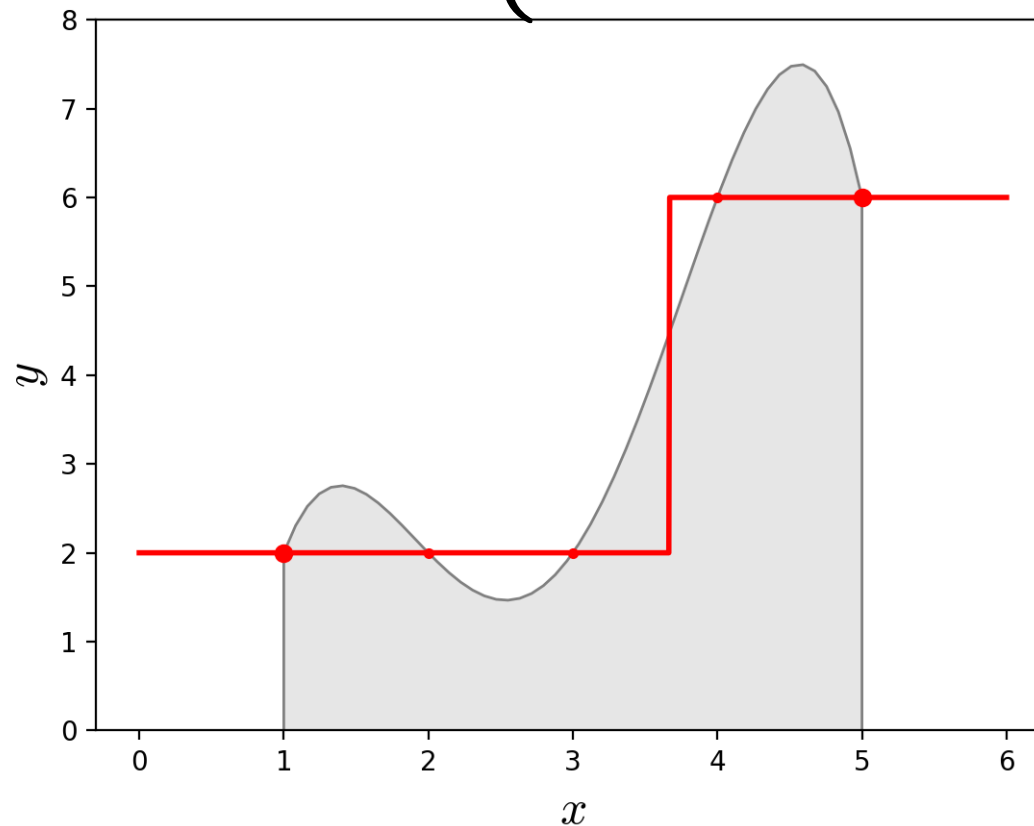


$$I[f] = 15.4336\dots \quad Q_0[f] = 16$$

$$E[f] = 0.5664\dots$$

# Milne-Regel

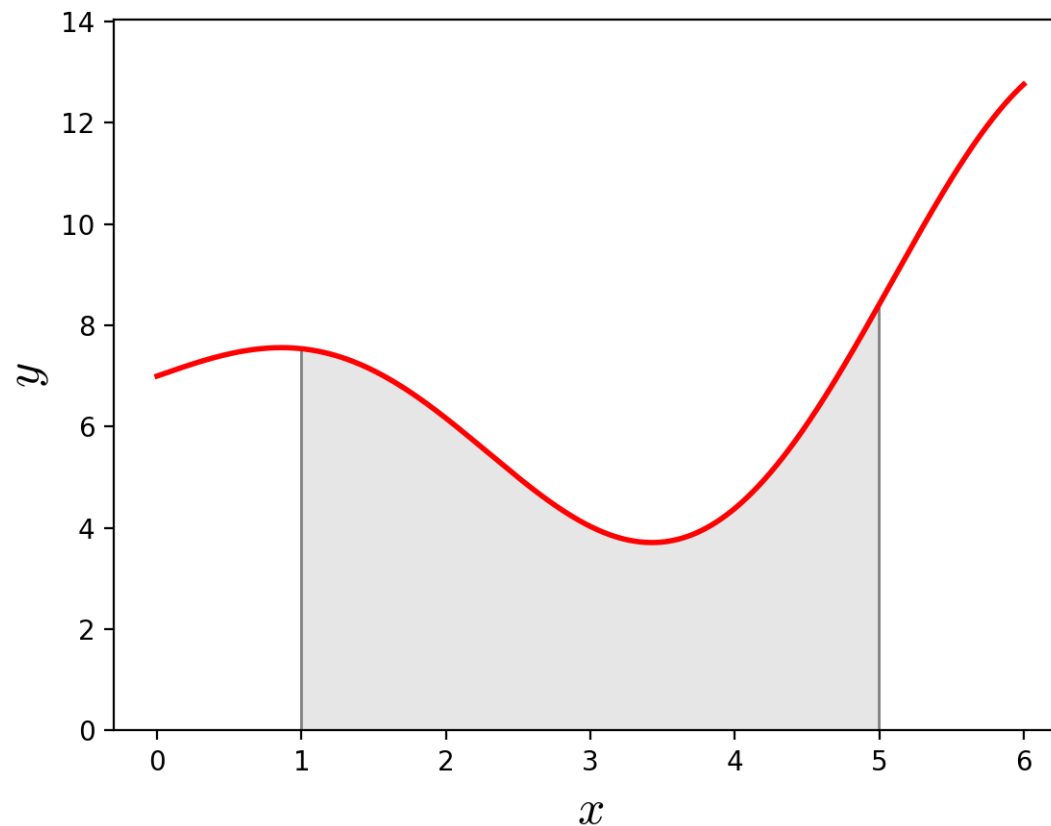
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = 15.4336\dots \quad Q_0[f] = 14.9333\dots \quad E[f] = 0.5003\dots$$

# Summierte Quadratur-Regeln

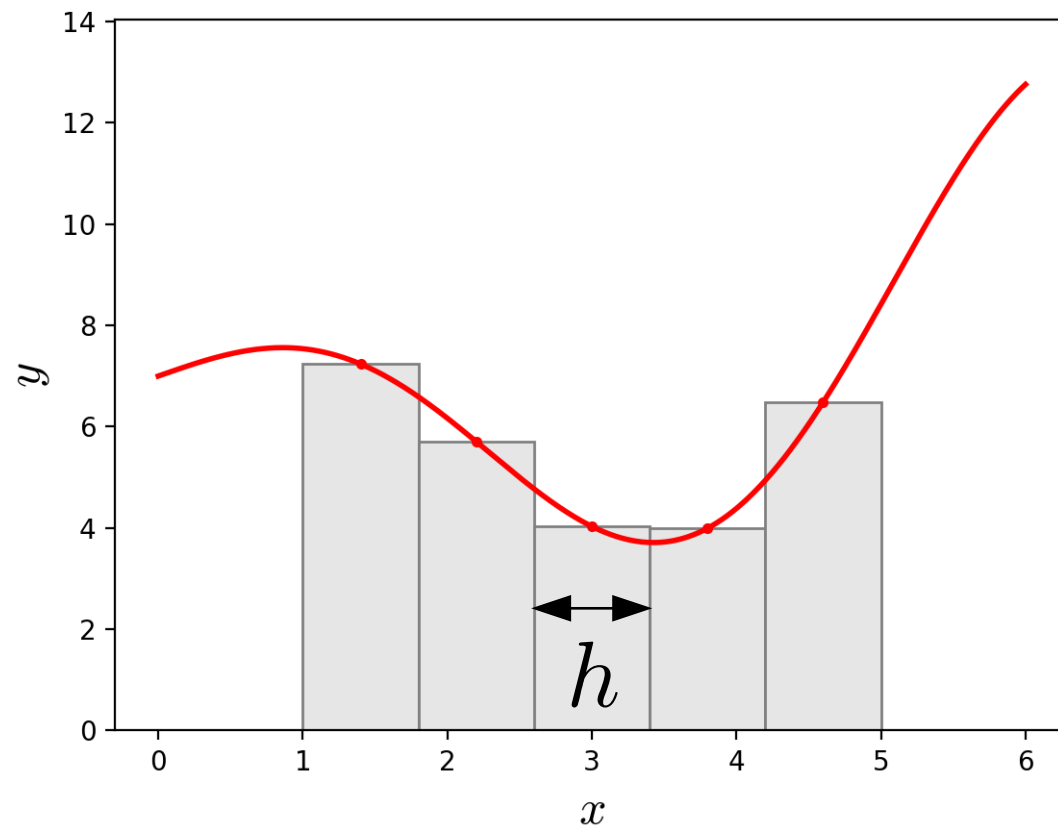
$$f(x) = x \cos(x) + 7$$



$$I[f] = \int_1^5 f(x) dx = 22.1073\dots$$

# Summierte Mittelpunktsregel

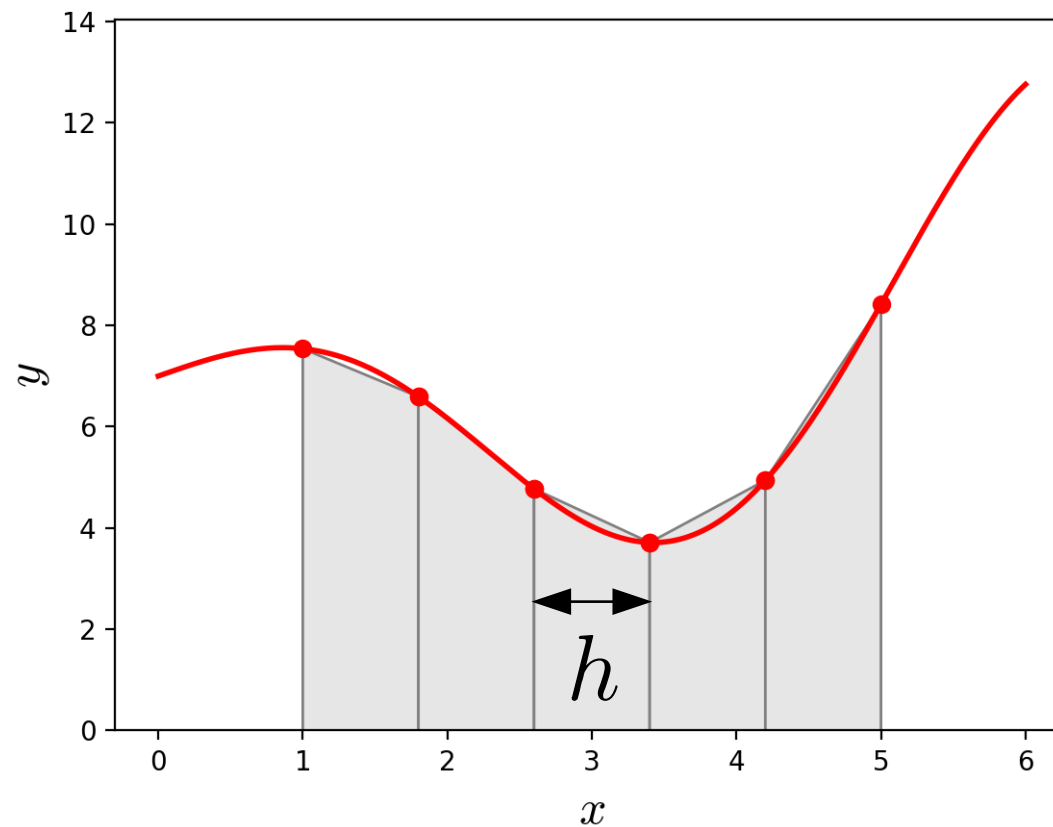
$$f(x) = x \cos(x) + 7$$



$$N = 5$$

# Summierte Trapezregel

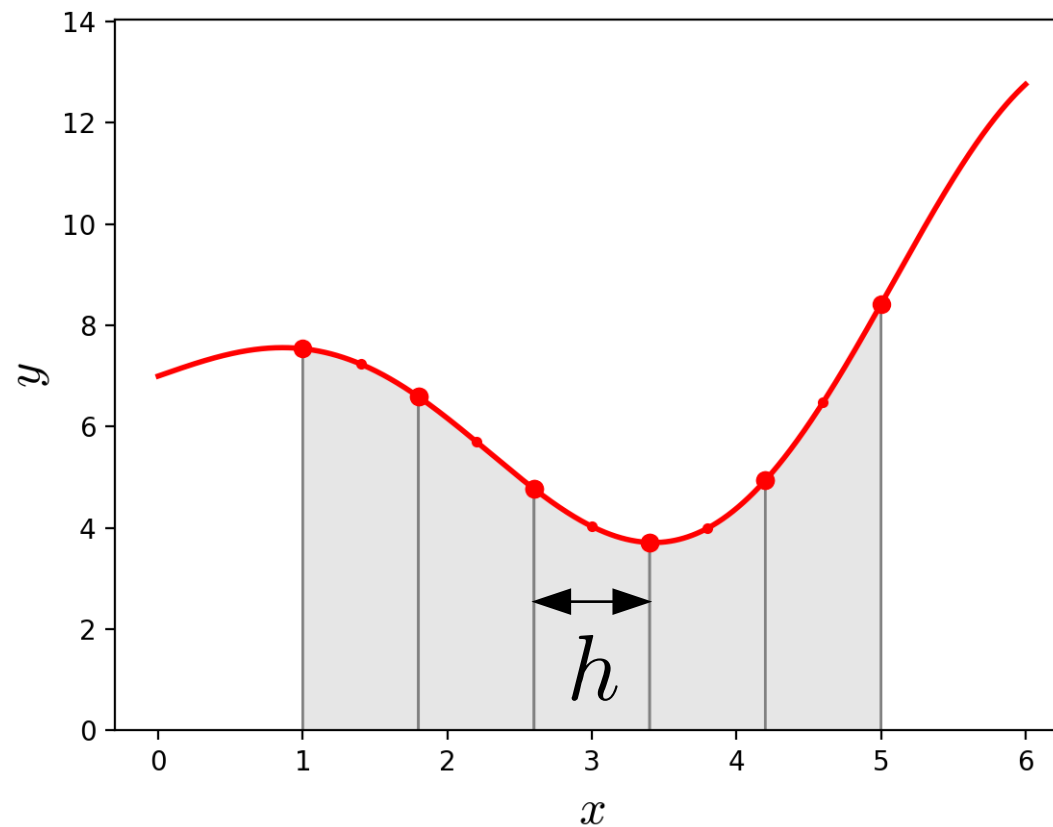
$$f(x) = x \cos(x) + 7$$



$$N = 5$$

# Summierte Simpson-Regel

$$f(x) = x \cos(x) + 7$$

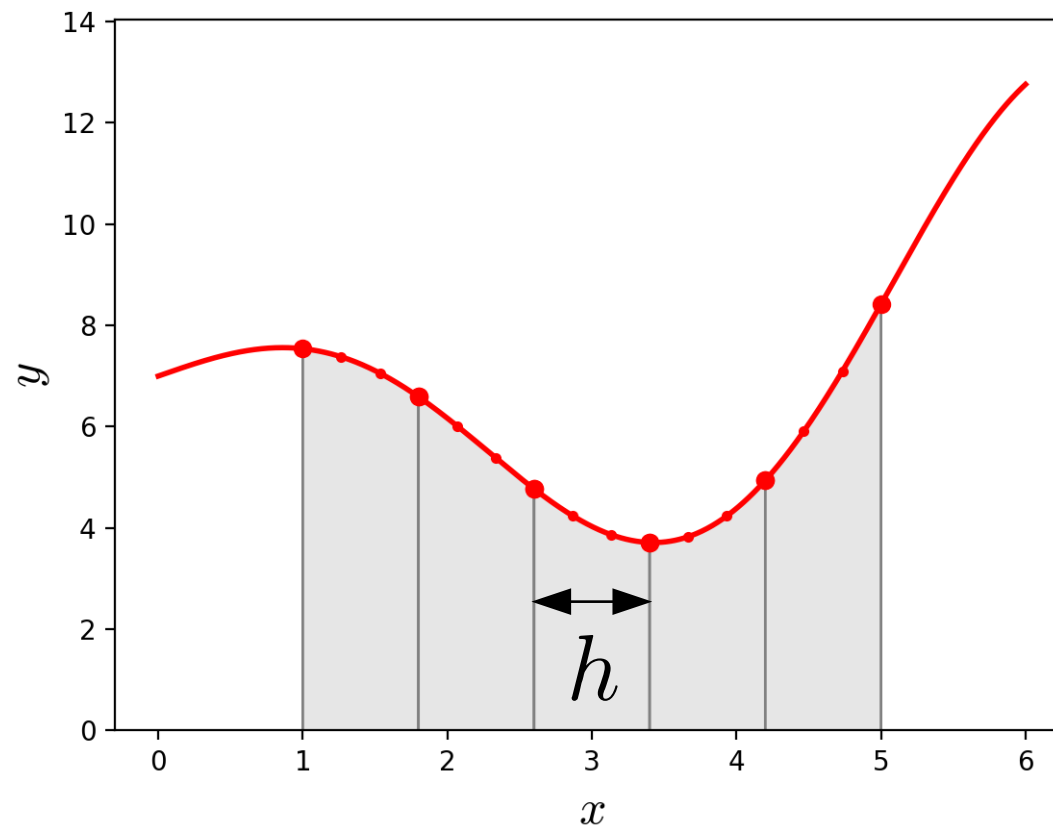


$$N = 5$$



# Summierte 3/8-Regel

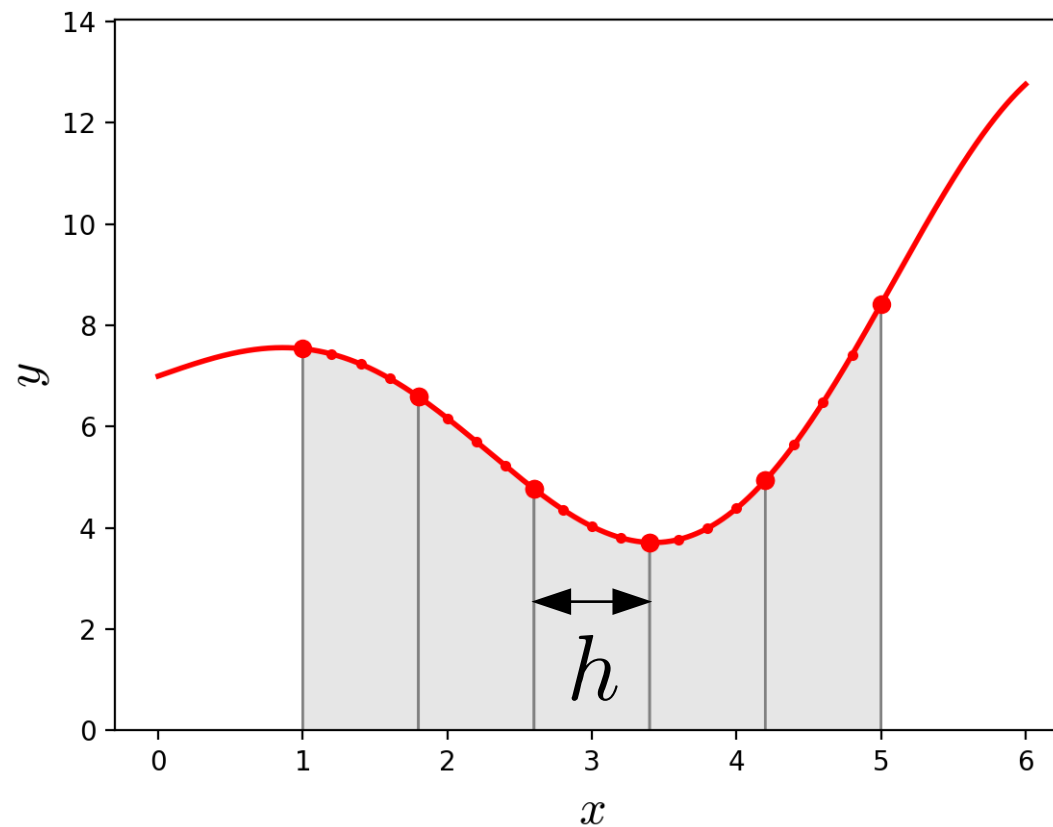
$$f(x) = x \cos(x) + 7$$



$$N = 5$$

# Summierte Milne-Regel

$$f(x) = x \cos(x) + 7$$



$$N = 5$$

# Summierte Quadratur-Regeln

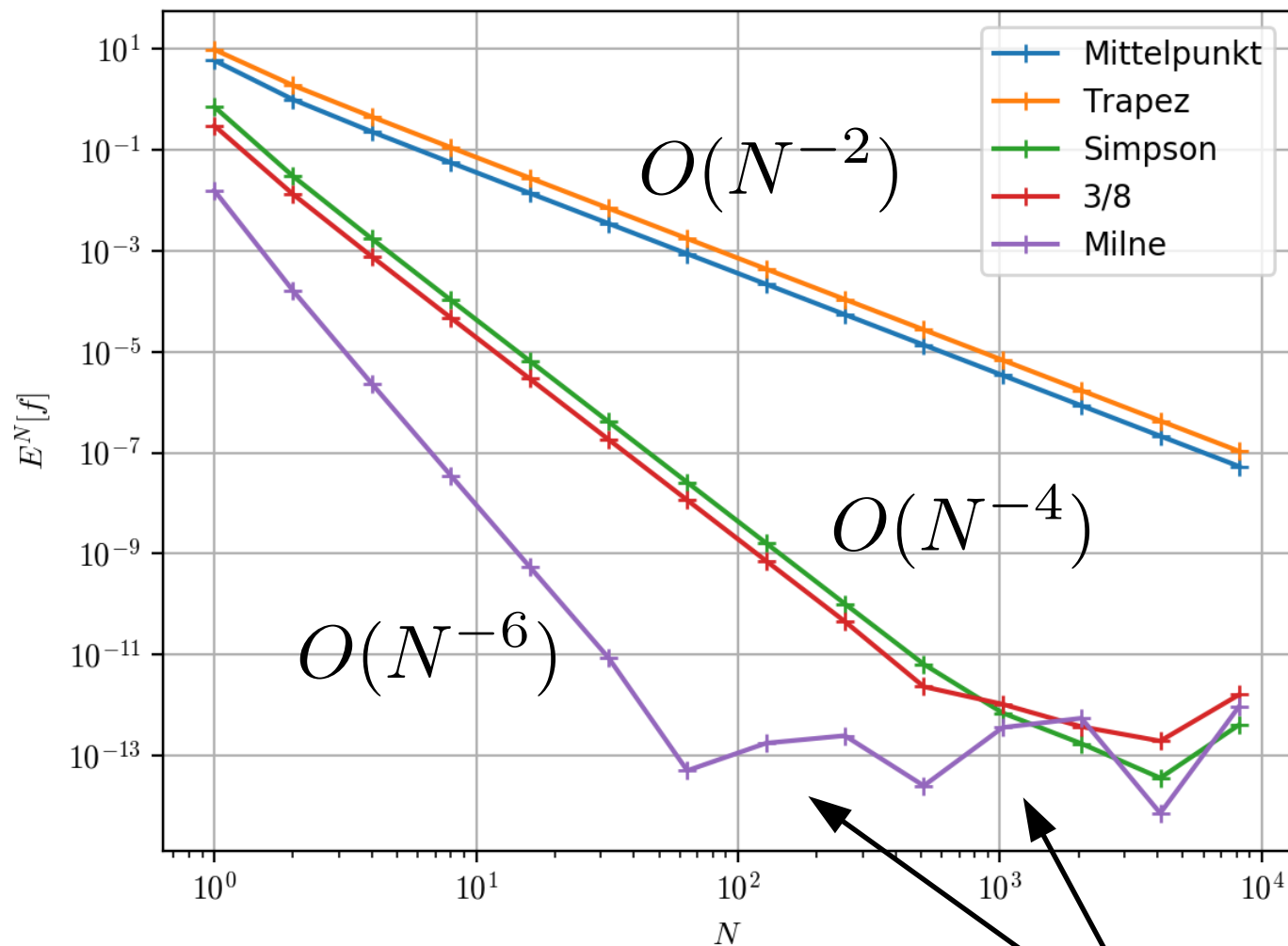
$$f(x) = x \cos(x) + 7$$

	Mittelpunkts	Trapez	Simpson	3/8	Milne	
$N$	$h$	$E_0^N[f]$	$E_1^N[f]$	$E_2^N[f]$	$E_3^N[f]$	$E_4^N[f]$
1	1.00E+00	5.99E+00	9.81E+00	7.21E-01	3.05E-01	1.59E-02
2	5.00E-01	1.00E+00	1.91E+00	3.02E-02	1.33E-02	1.62E-04
4	2.50E-01	2.30E-01	4.55E-01	1.74E-03	7.69E-04	2.32E-06
8	1.25E-01	5.64E-02	1.12E-01	1.06E-04	4.72E-05	3.54E-08
16	6.25E-02	1.40E-02	2.80E-02	6.61E-06	2.94E-06	5.50E-10
32	3.12E-02	3.50E-03	7.01E-03	4.13E-07	1.83E-07	8.62E-12
64	1.56E-02	8.76E-04	1.75E-03	2.58E-08	1.15E-08	4.97E-14
128	7.81E-03	2.19E-04	4.38E-04	1.61E-09	7.17E-10	1.74E-13

$$h = \frac{b - a}{N} \quad \dots$$

# Summierte Quadratur-Regeln

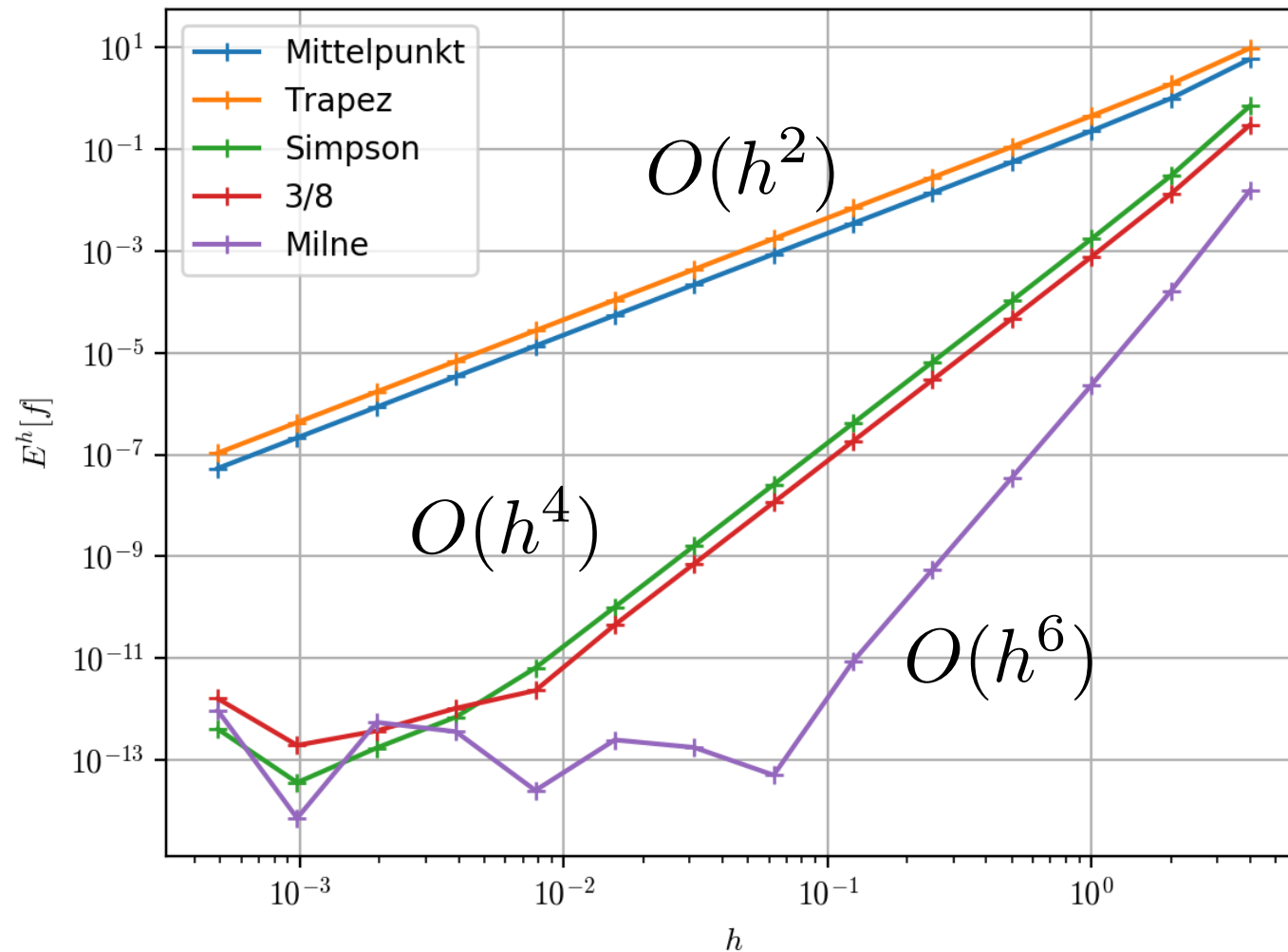
$$f(x) = x \cos(x) + 7$$



**Maschinengenauigkeit!**

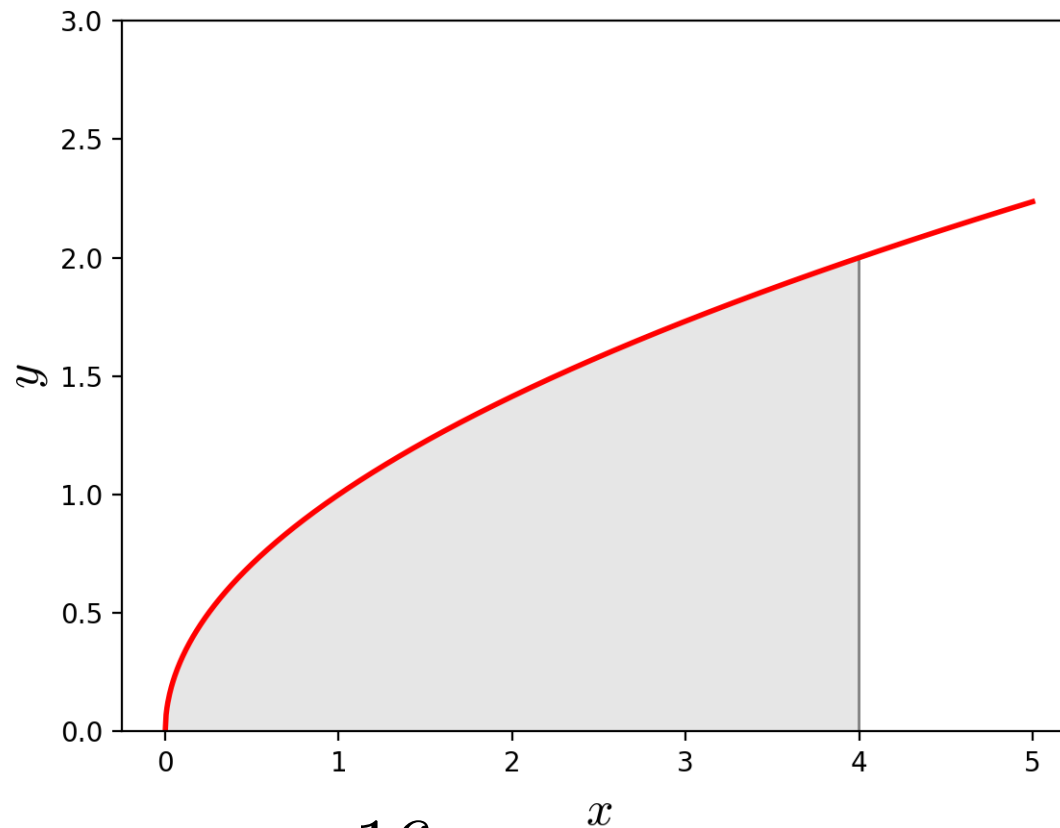
# Summierte Quadratur-Regeln

$$f(x) = x \cos(x) + 7$$



# Summierte Quadratur-Regeln

$$f(x) = \sqrt{x}$$

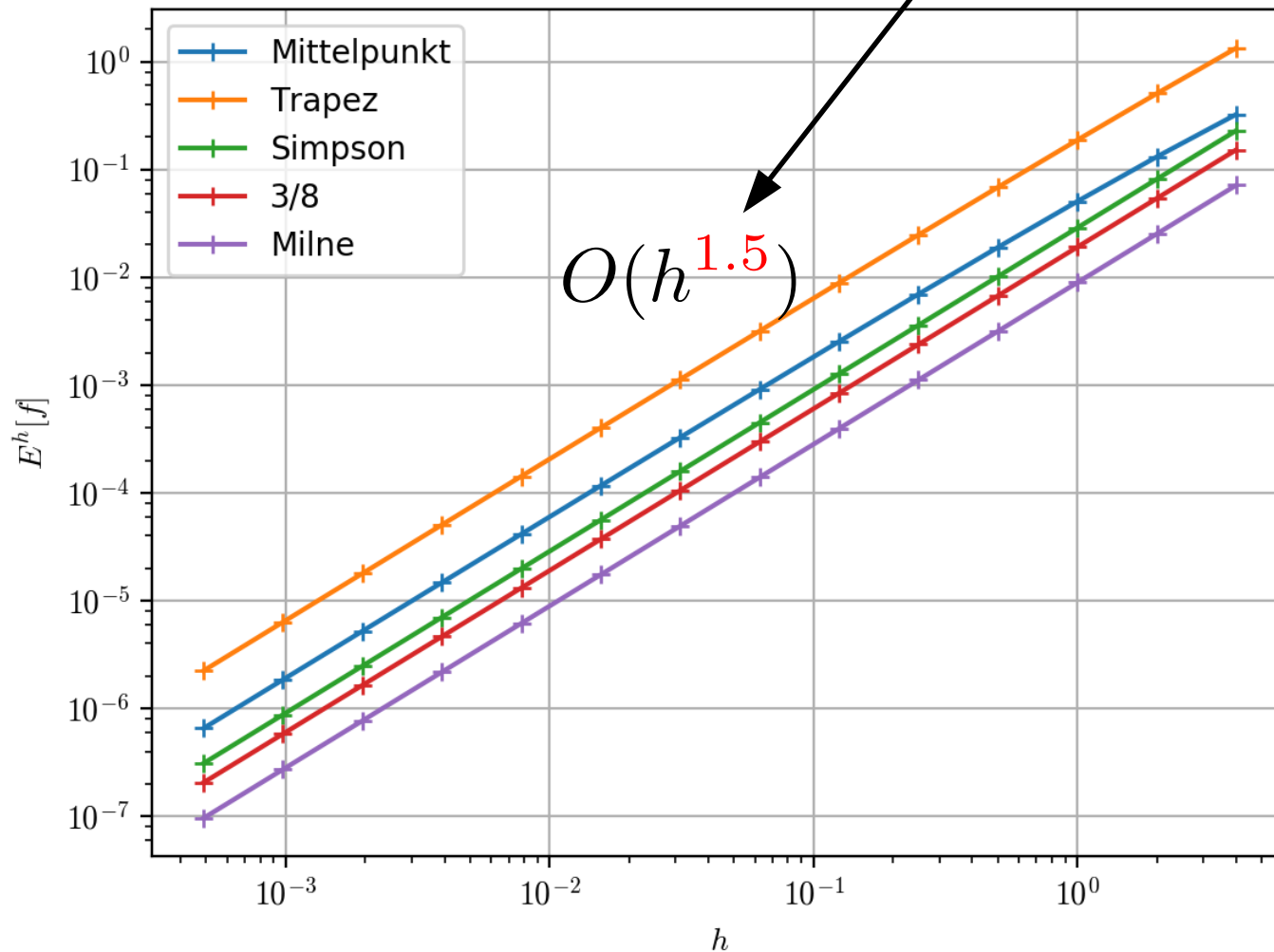


$$I[f] = \int_0^4 f(x) dx = \frac{16}{3}$$

# Summierte Quadratur-Regeln

$$f(x) = \sqrt{x}$$

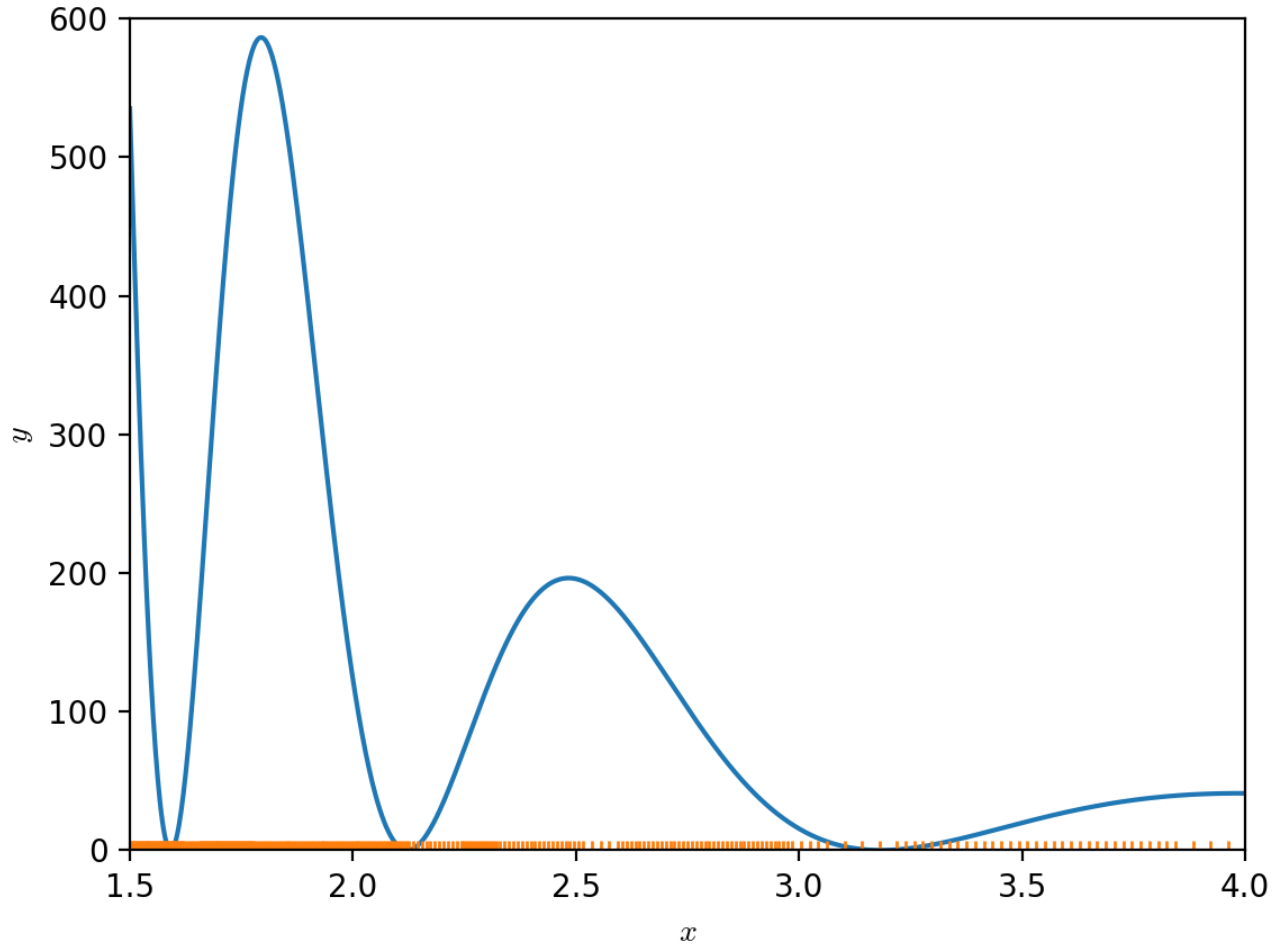
$$? f'(x) = \frac{1}{2\sqrt{x}} \xrightarrow{x \rightarrow 0} \infty$$



...Glattheit!

# Adaptive Quadratur

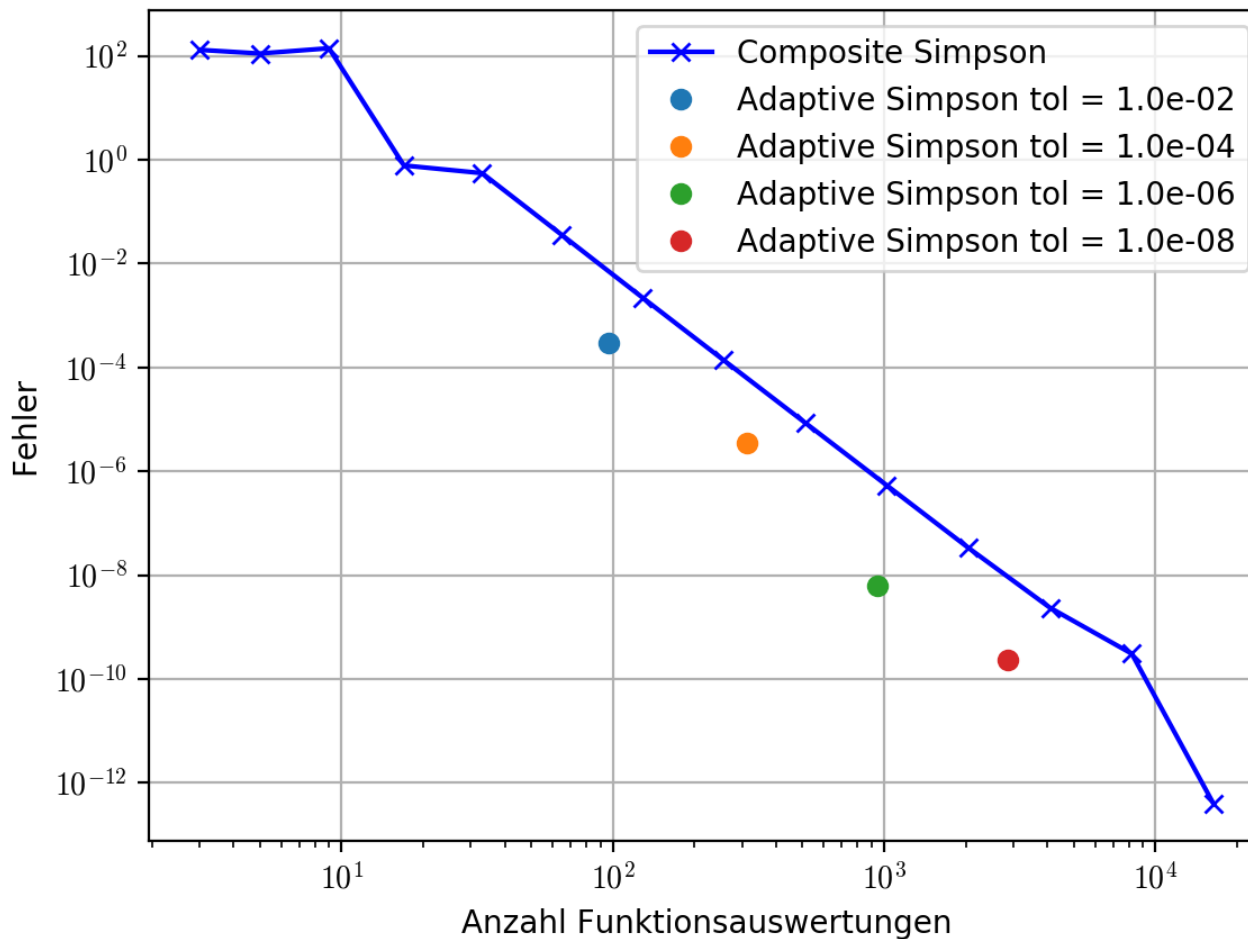
$$I[f] = \int_{3/2}^4 \left( \frac{200}{2x^3 - x^2} \right) (5 \sin(20/x))^2 dx$$





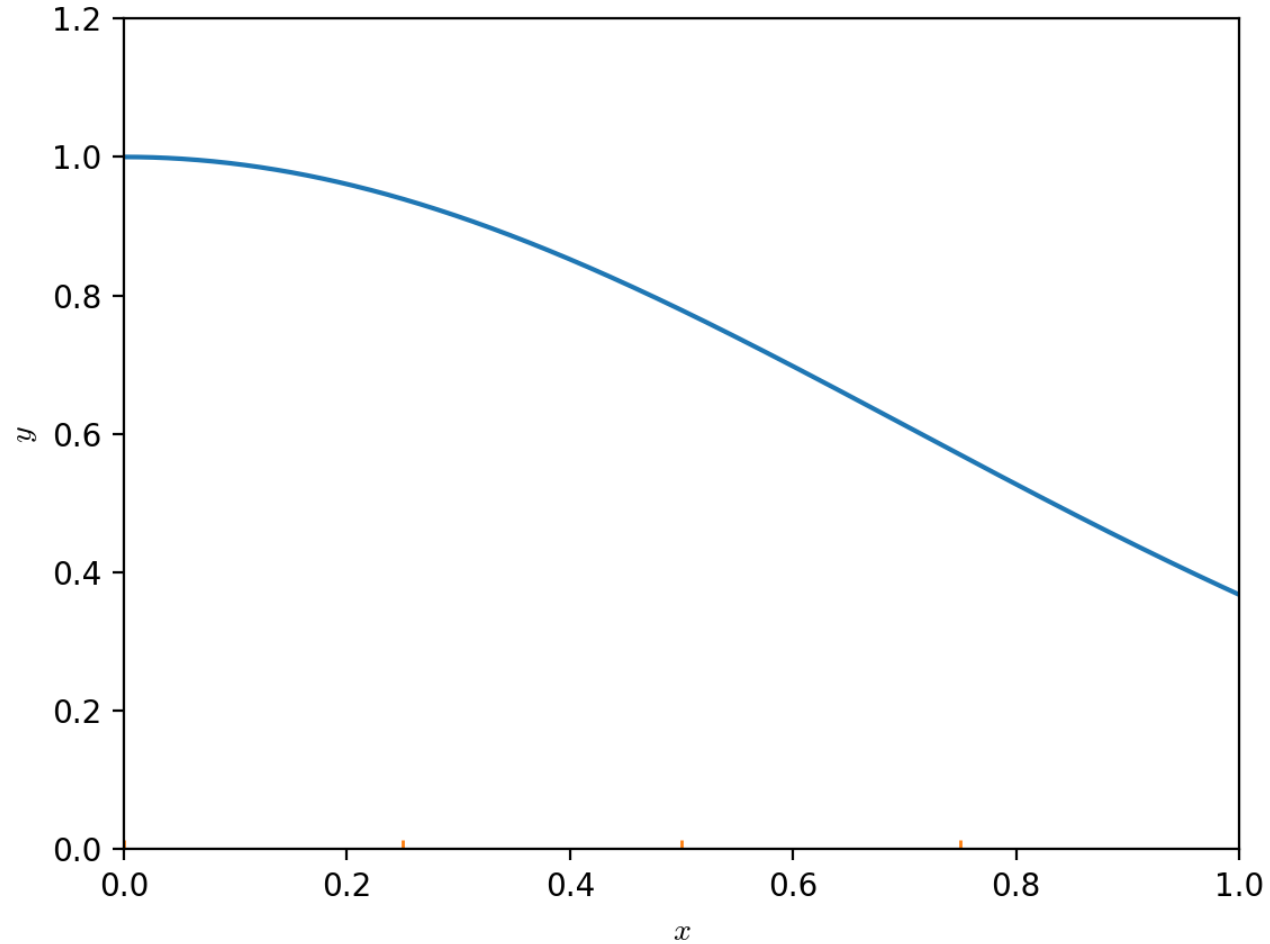
# Adaptive Quadratur

$$I[f] = \int_{3/2}^4 \left( \frac{200}{2x^3 - x^2} \right) (5 \sin(20/x))^2 dx$$



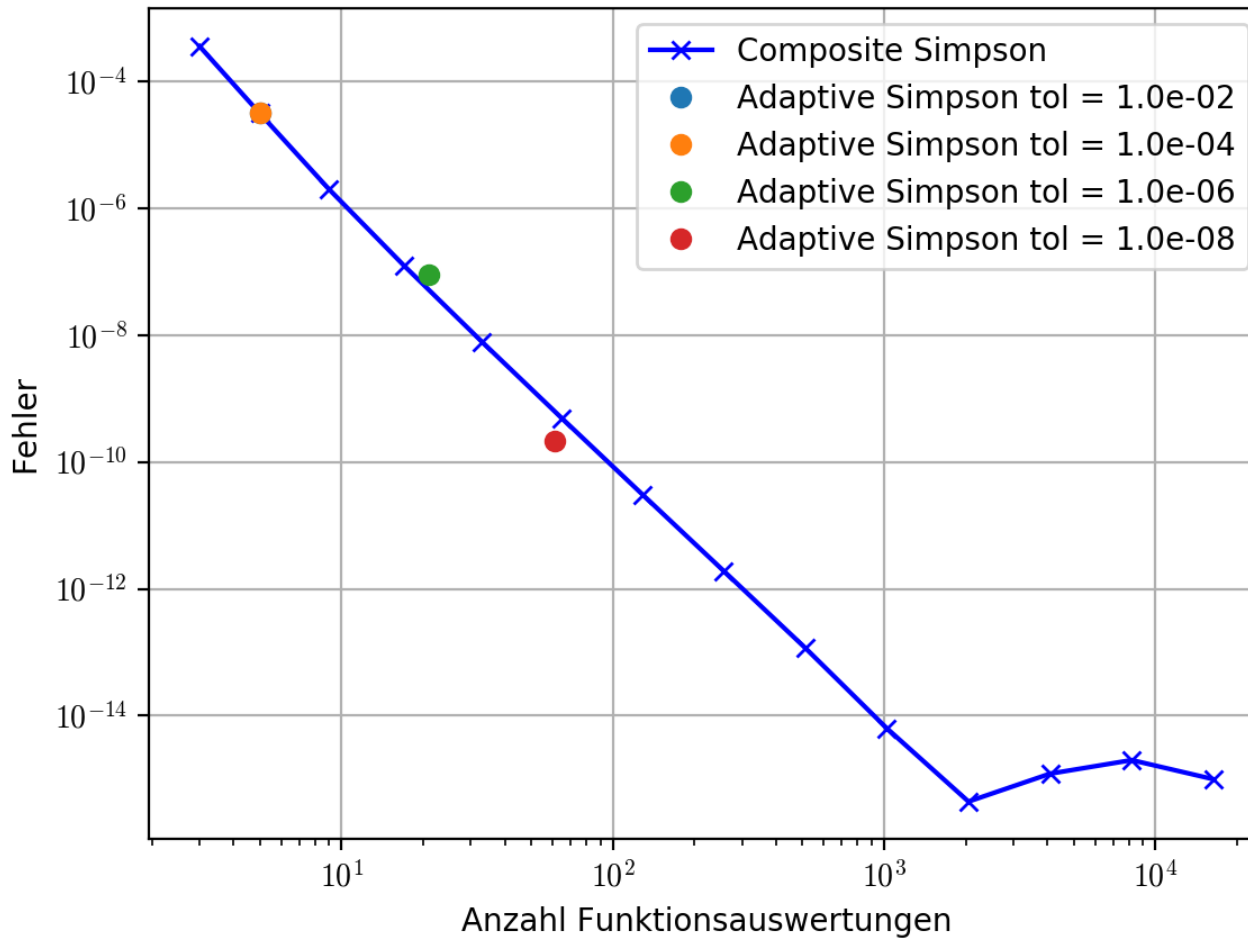
# Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x^2} dx$$



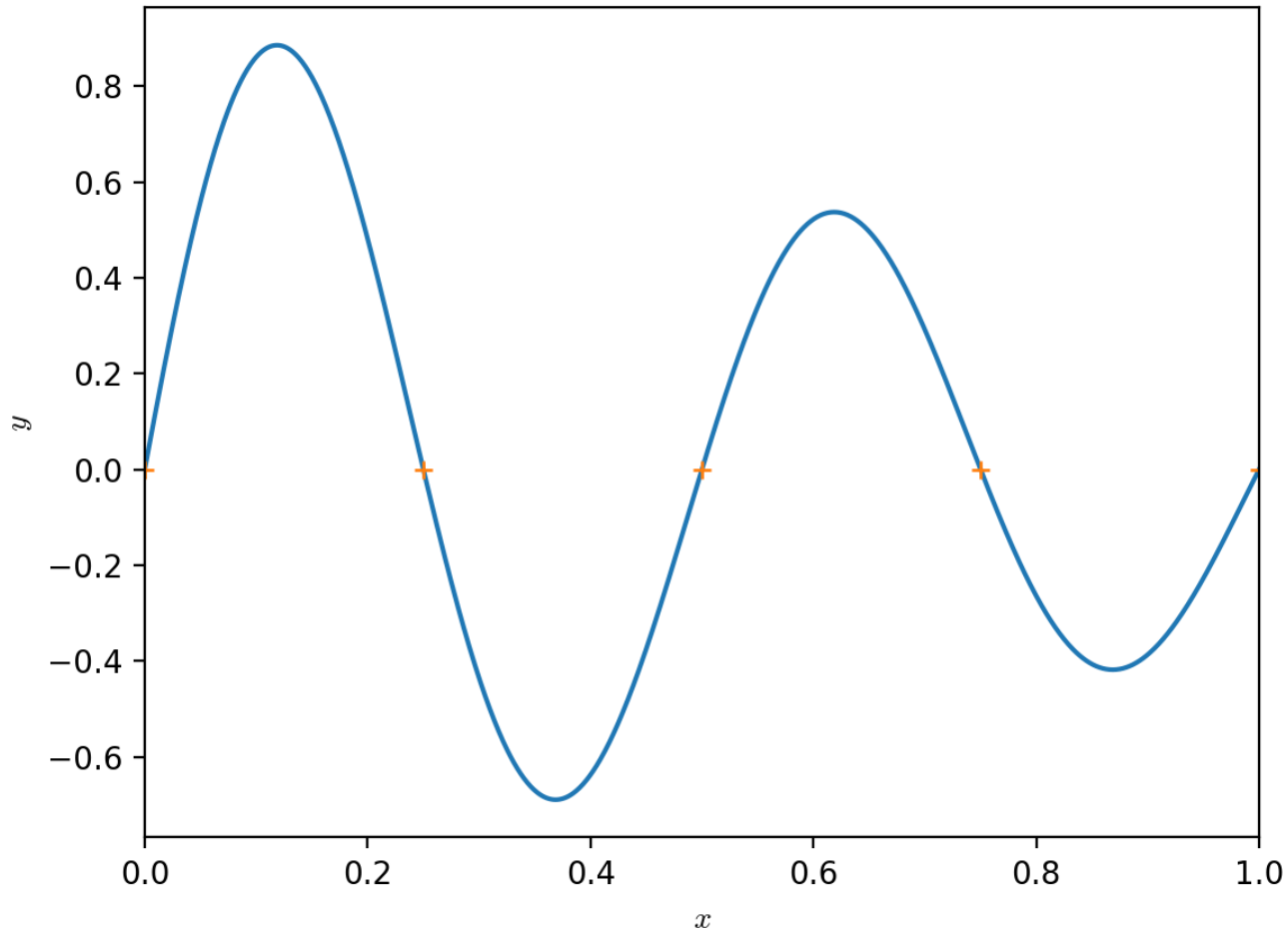
# Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x^2} dx$$



# Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x} \sin(4\pi x) dx$$



# Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x} \sin(4\pi x) dx$$

