Berechnen wir zunächst die Ableitungen der Lösung

$$\dot y(t) \overset{\text{DGL}}{=} f(t,y(t))$$

$$\ddot y(t) = \frac{d}{dt} f(t,y(t))$$

Ketten-Regel!

$$= \frac{\partial f}{\partial t}(t,y(t)) + \frac{\partial f}{\partial y}(t,y(t))\cdot \underbrace{\dot y(t)}_{= f(t,y(t)) \text{ DGL!}}$$

$$= \frac{\partial f}{\partial t}(t,y(t)) + \frac{\partial f}{\partial y}(t,y(t))\cdot f(t,y(t))$$

$$\dddot y(t) = \frac{d}{dt}\left(\frac{\partial f}{\partial t}(t,y(t)) + \frac{\partial f}{\partial y}(t,y(t))\cdot f(t,y(t))\right)$$

$$= \frac{\partial^2 f}{\partial t^2}(t,y(t)) + \frac{\partial^2 f}{\partial y\,\partial t}(t,y(t))\cdot \underbrace{\dot y(t)}_{= f(t,y(t))}$$

$\frac{\partial^2 f}{\partial t\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial t}$

$$+ \left(\frac{\partial^2 f}{\partial t\,\partial y}(t,y(t)) + \frac{\partial^2 f}{\partial y^2}(t,y(t))\cdot \underbrace{\dot y(t)}_{= f(t,y(t))}\right)\cdot f(t,y(t))$$

$$+ \frac{\partial f}{\partial y}(t,y(t))\cdot\left(\frac{\partial f}{\partial t}(t,y(t)) + \frac{\partial f}{\partial y}(t,y(t))\cdot \underbrace{\dot y(t)}_{= f(t,y(t))}\right)$$

$$= \frac{\partial^2 f}{\partial t^2}(t,y(t)) + 2\cdot\frac{\partial^2 f}{\partial t\,\partial y}(t,y(t))\cdot f(t,y(t))$$

$$+ \frac{\partial^2 f}{\partial y^2}(t,y(t))\cdot f(t,y(t))^2 + \frac{\partial f}{\partial t}(t,y(t))\cdot\frac{\partial f}{\partial y}(t,y(t))$$

$$+ \left(\frac{\partial f}{\partial y}(t,y(t))\right)^2\cdot f(t,y(t))$$

Usw.