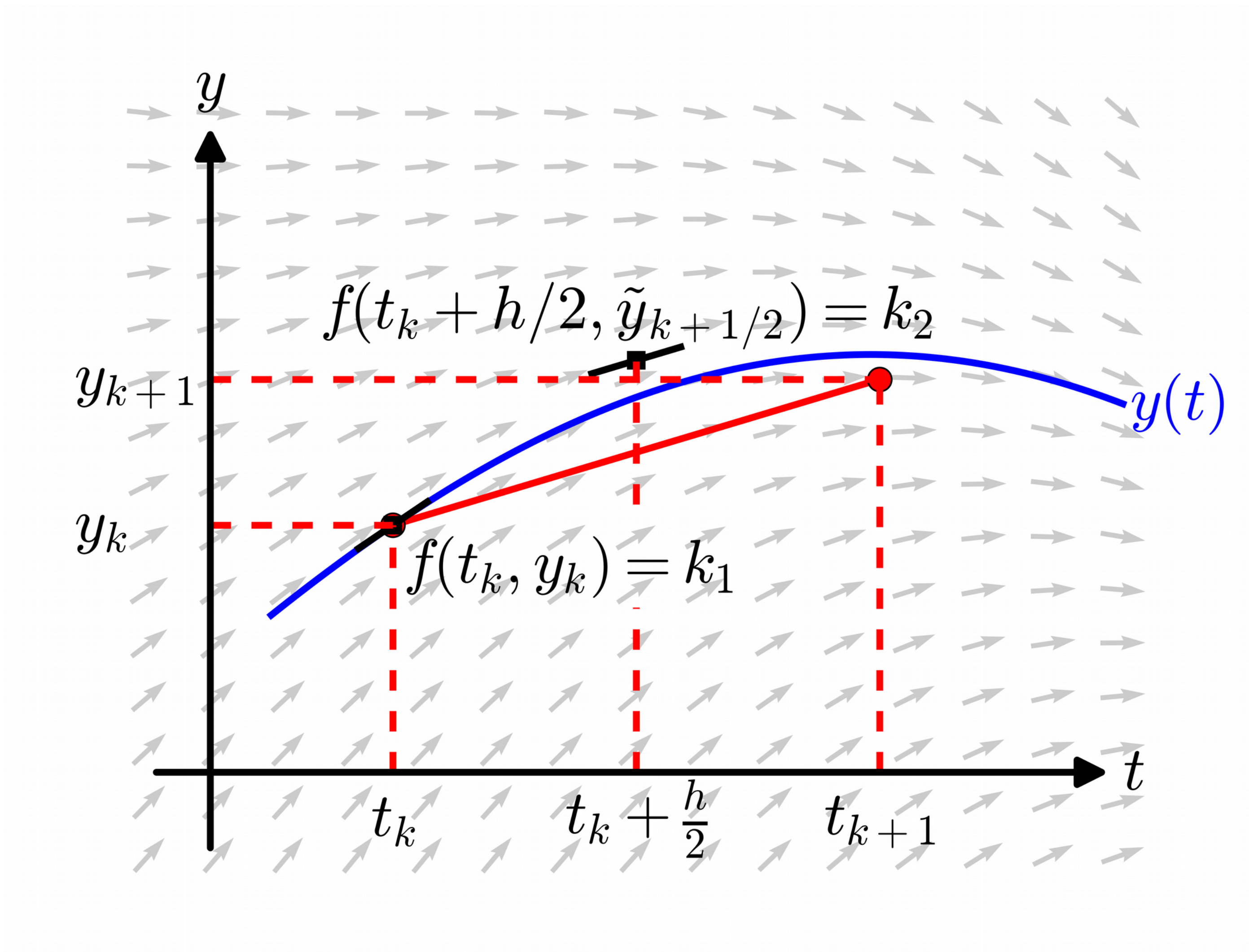
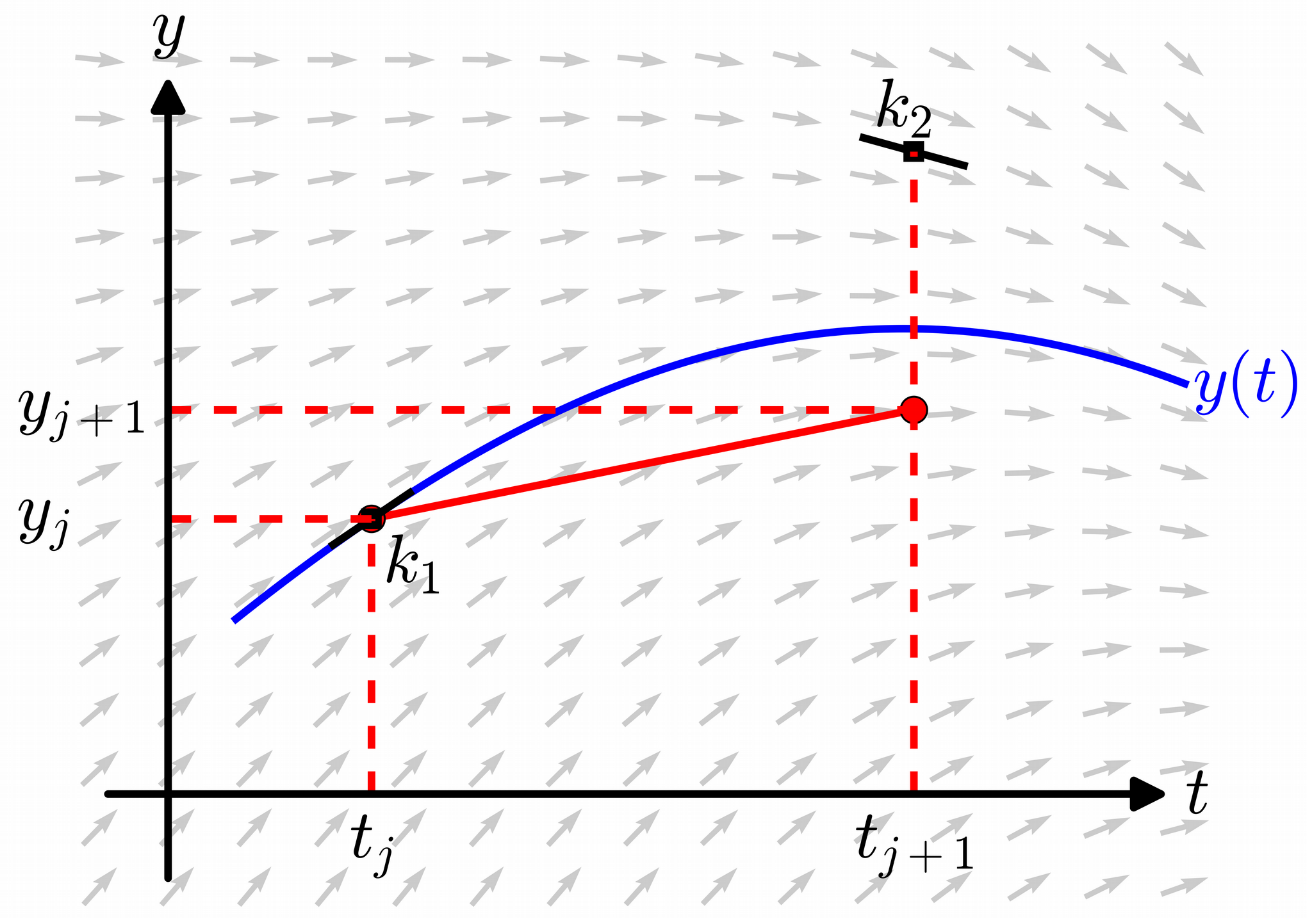
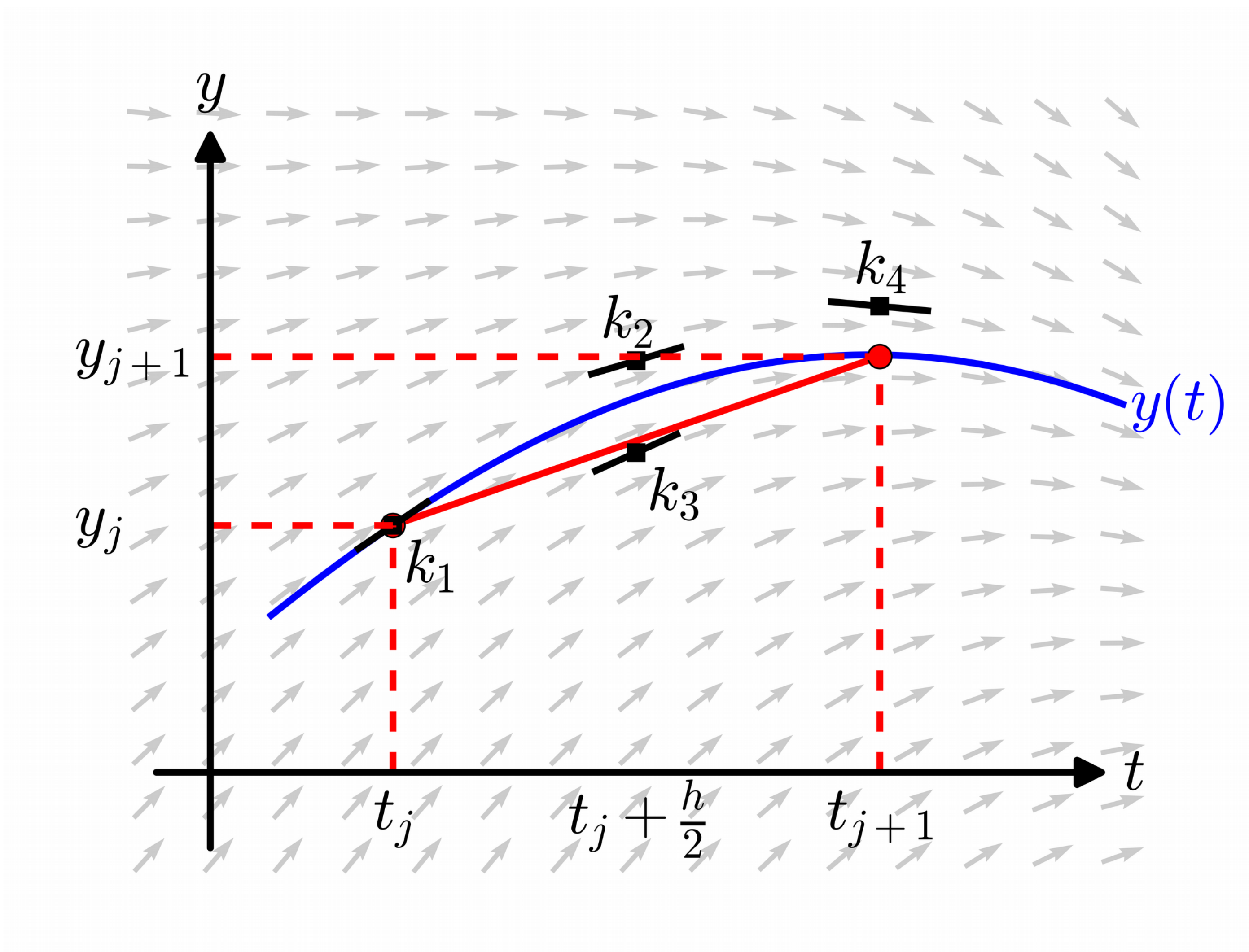
Verbesserter Euler



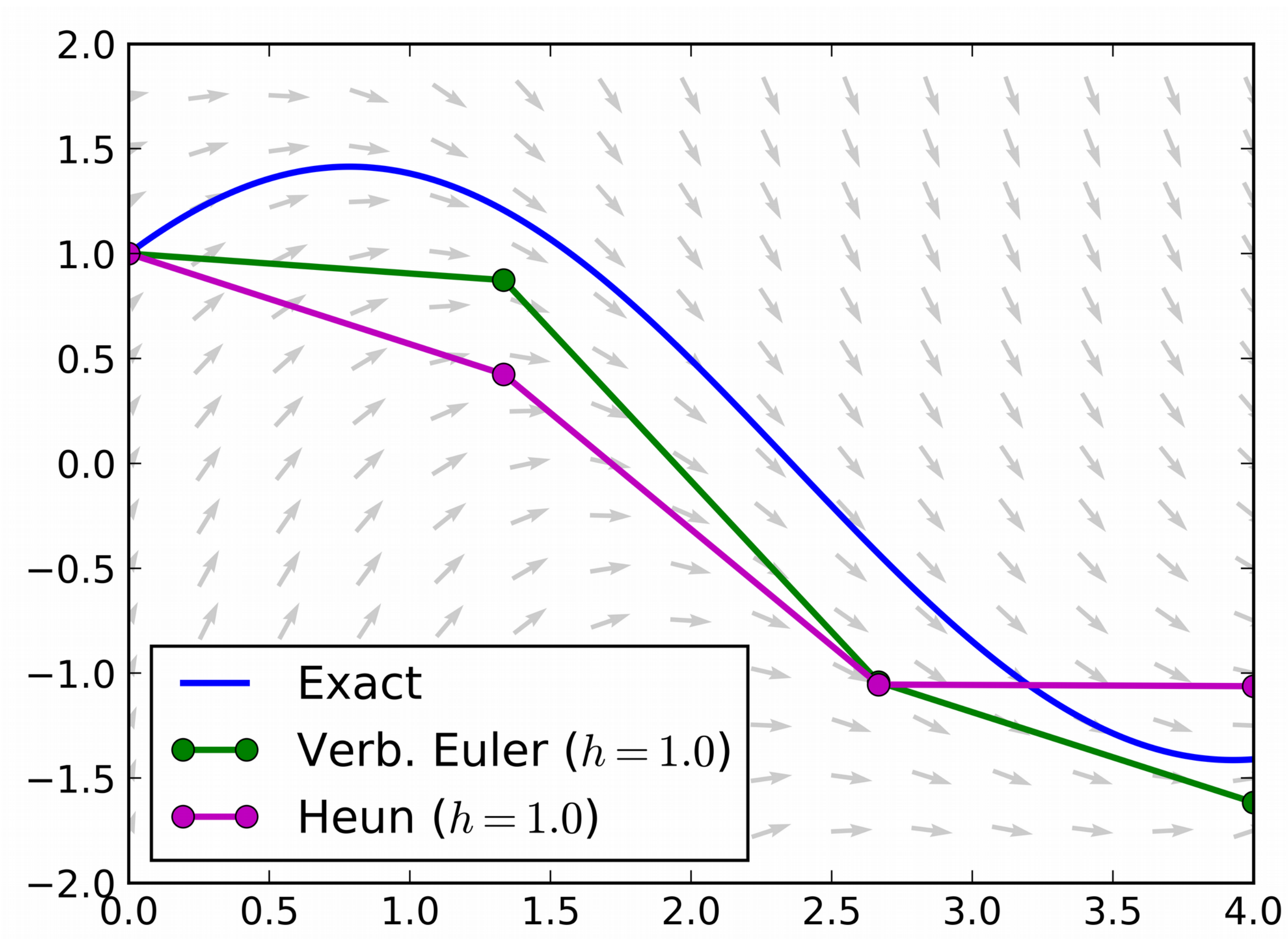
Heun‘s Methode



DIE Runge-Kutta Methode

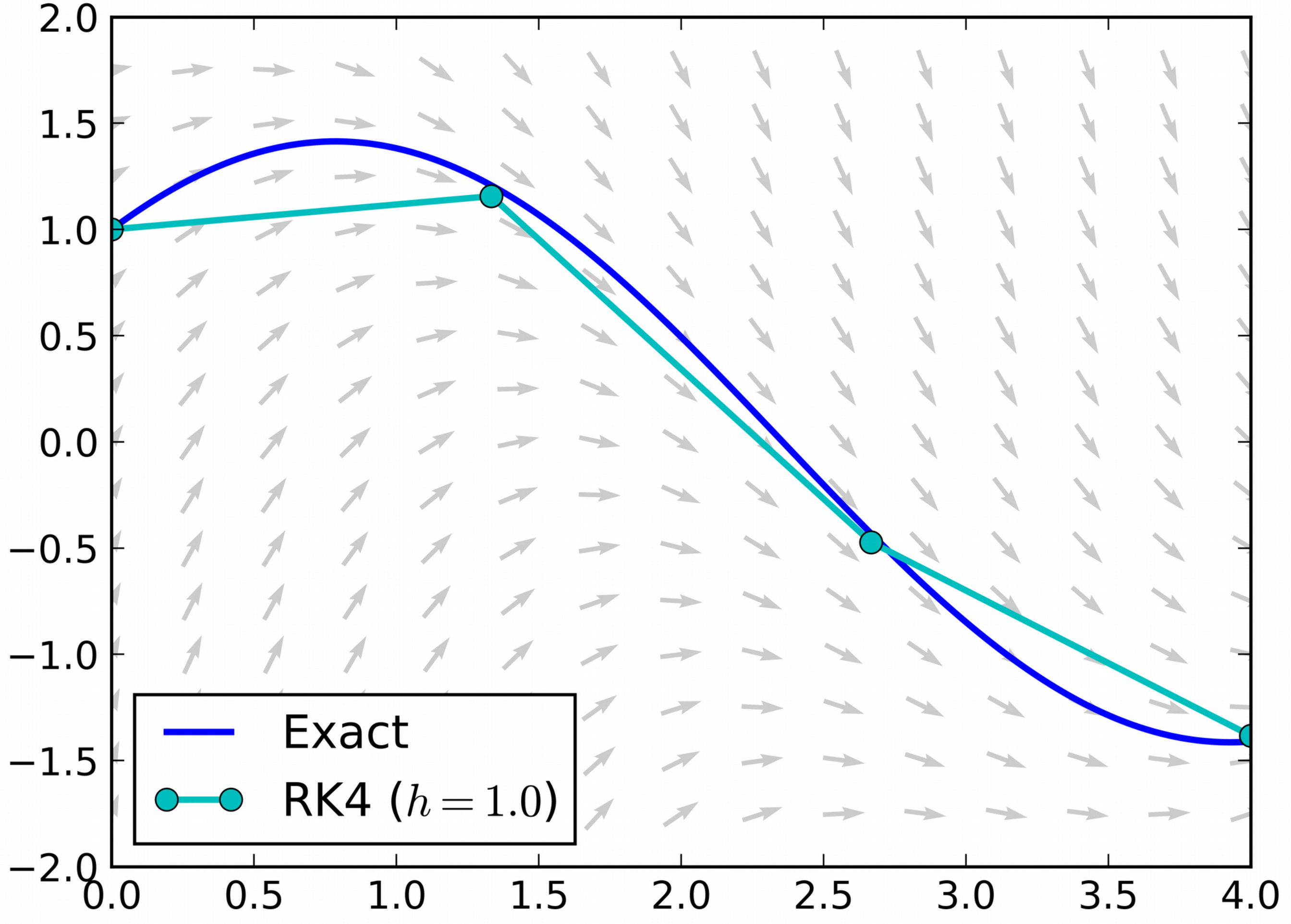


Verb. Euler & Heun



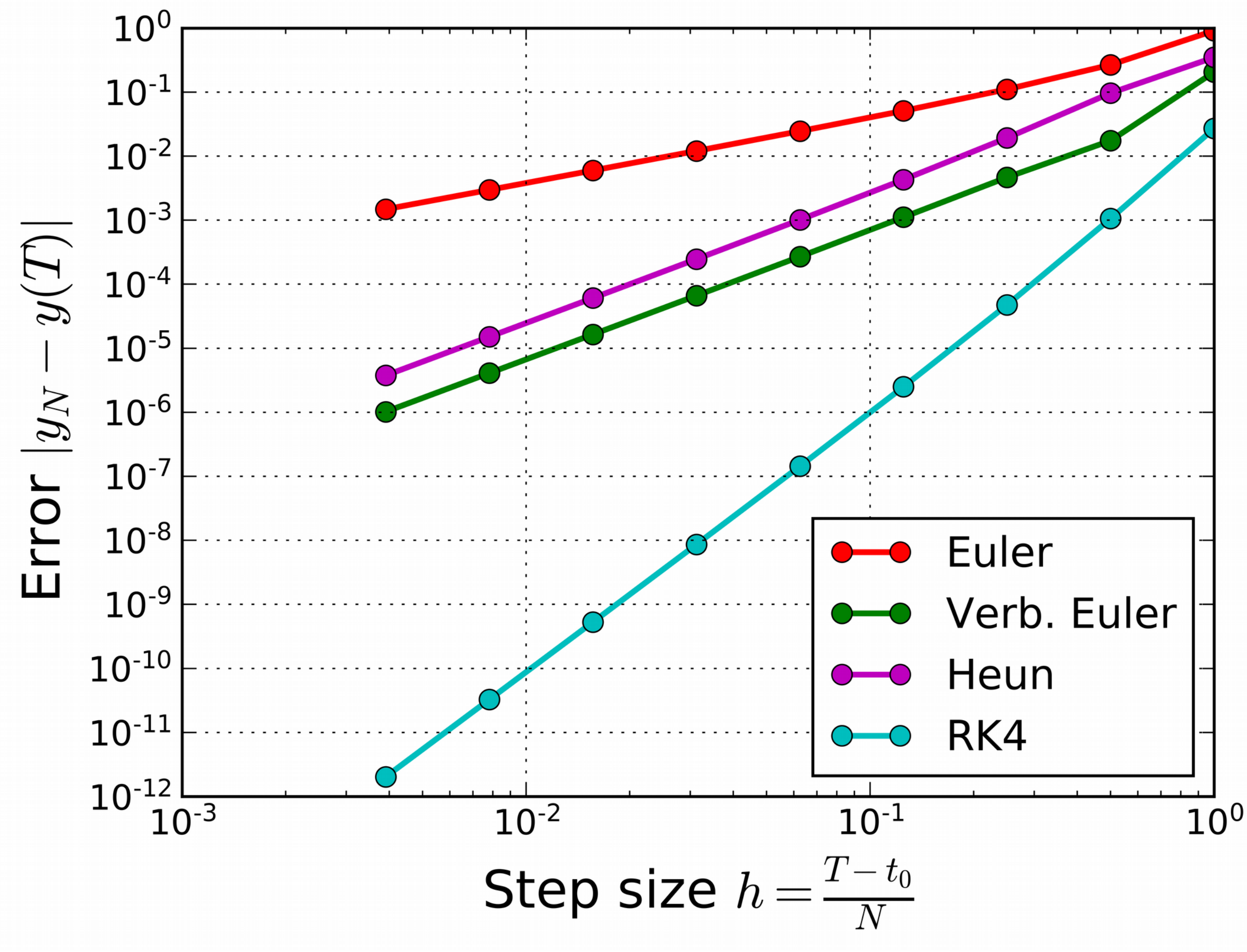
$$\begin{cases} \dot{y}(t) & = -y(t) + 2\cos(t) \\ y(t_0 = 0) & = 1 \end{cases}$$

DIE Runge-Kutta Methode



$$\begin{cases} \dot{y}(t) & = -y(t) + 2\cos(t) \\ y(t_0 = 0) & = 1 \end{cases}$$

Fehler



Error $|y_N - y(T)|$
Step size $h = \frac{T-t_0}{N}$
Euler
Verb. Euler
Heun
RK4
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}

Satz II.3: Falls die rechte Seite der DGL $\vec{f}(t,\vec{y})$ und die Verfahrens-Funktion $\vec{\phi}(t,\vec{y},t)$ Lipschitz-stetig in $\vec{y}$ sind, dann gilt für das ESV folgende (globale) Fehlerabschätzung

$$\epsilon = \max_{j=0,\dots,N} \|\vec{y}(t_j) - \vec{y}_j\|$$

$$\leq \Big(\underbrace{\|\vec{y}(t_0) - \vec{y}_0\|}_{\text{AW Fehler}} + \sum_{j=1}^{N} \underbrace{\|\vec{e}_j\|}_{\text{Fehler in jedem Schritt summieren sich schlimmstenfalls}} \Big) \cdot e^{\tilde{L}(t_N - t_0)}$$

wobei $\tilde{L}$ die Lipschitz-Konstante der Verfahrens-Funktion $\vec{\phi}$ ist.