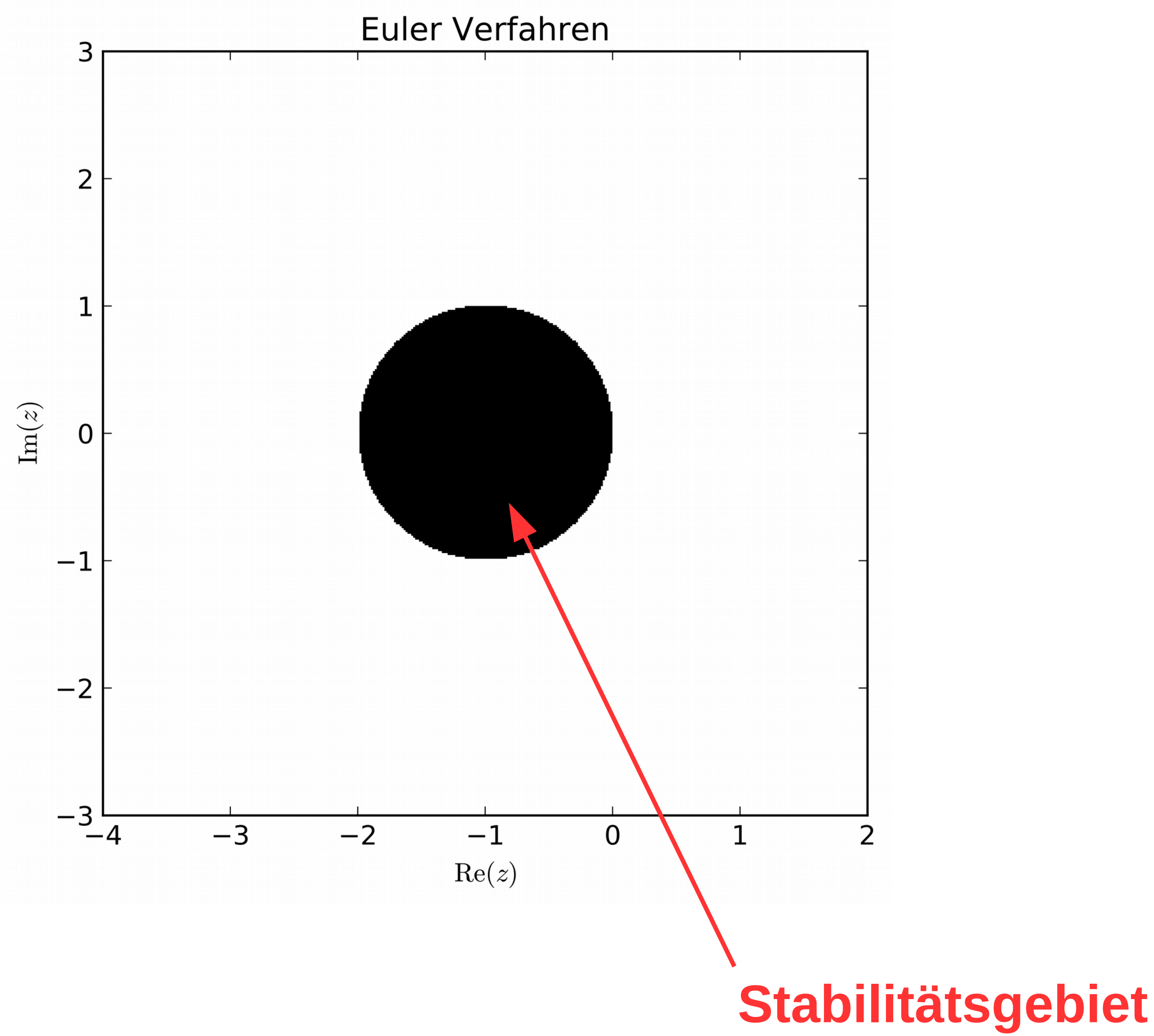
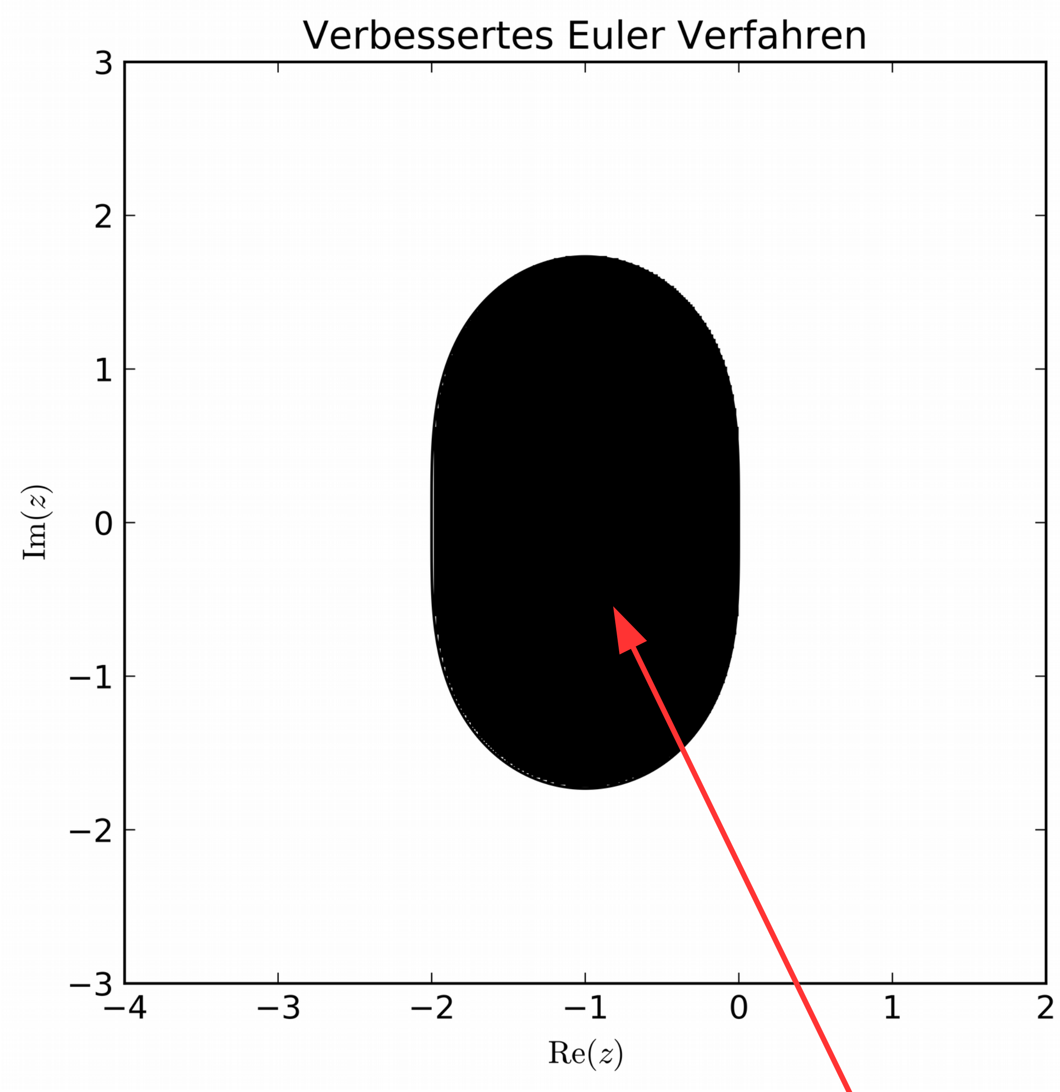
Stabilitätsgebiete



Stabilitätsgebiete



Stabilitätsgebiet

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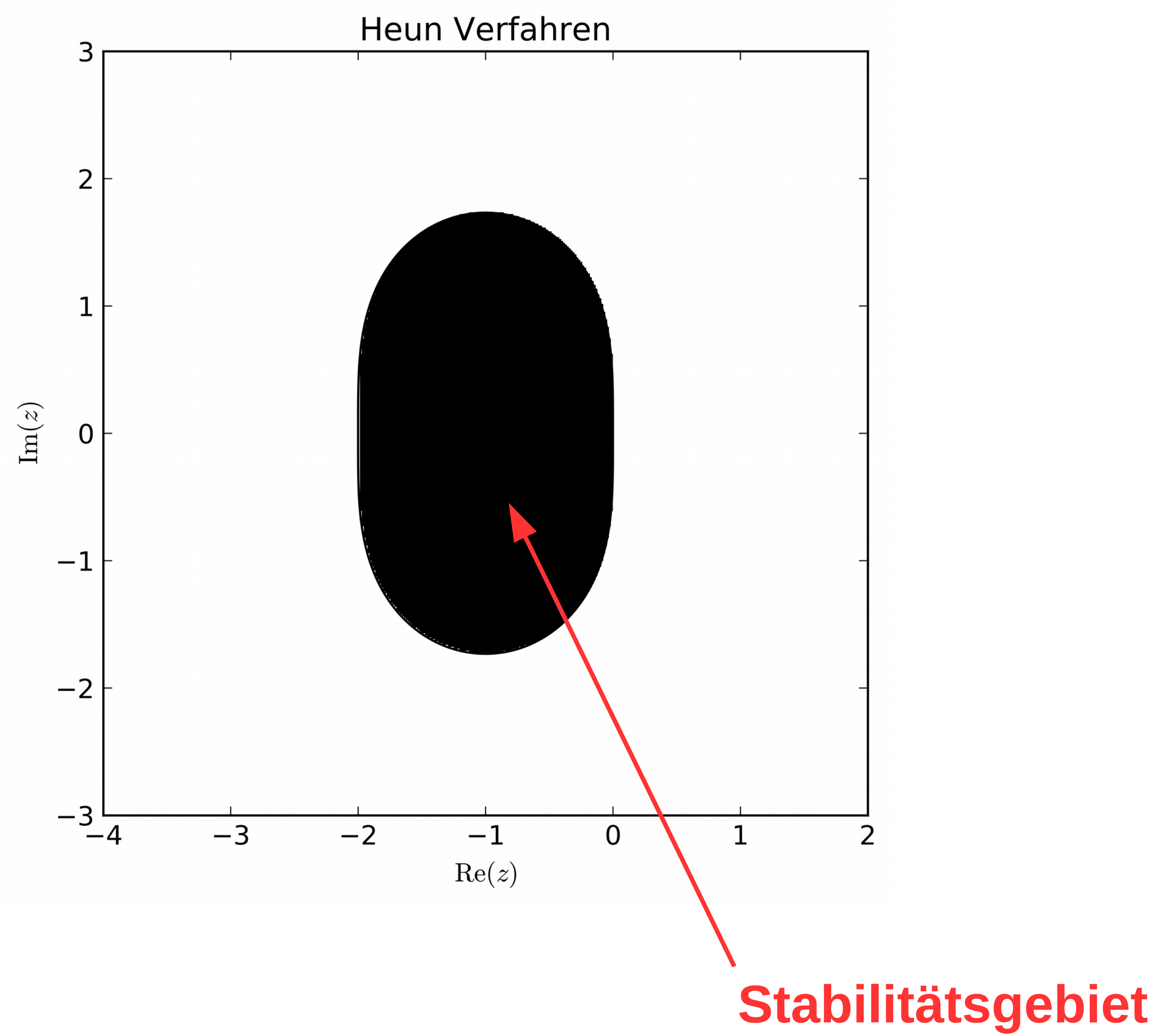

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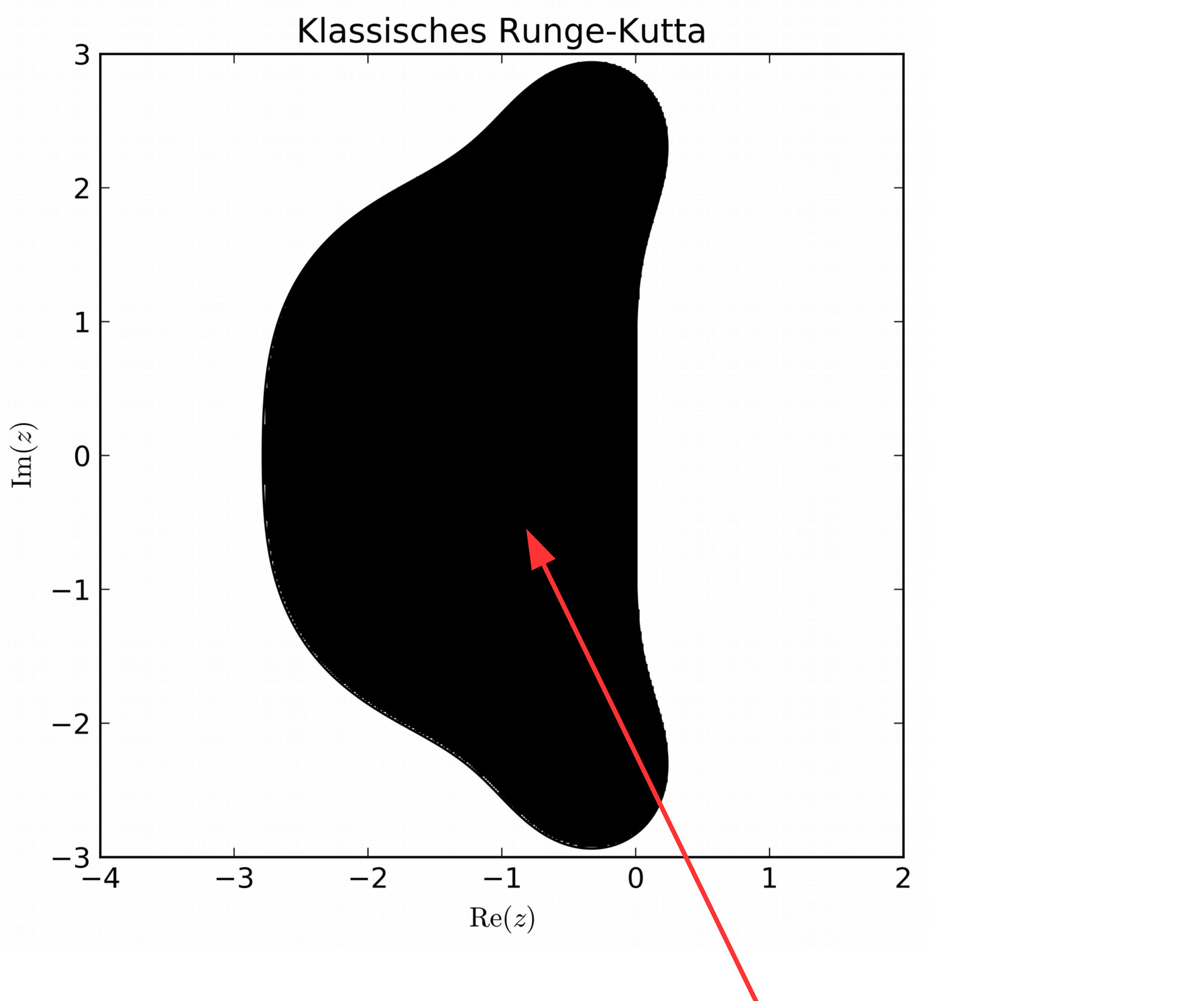

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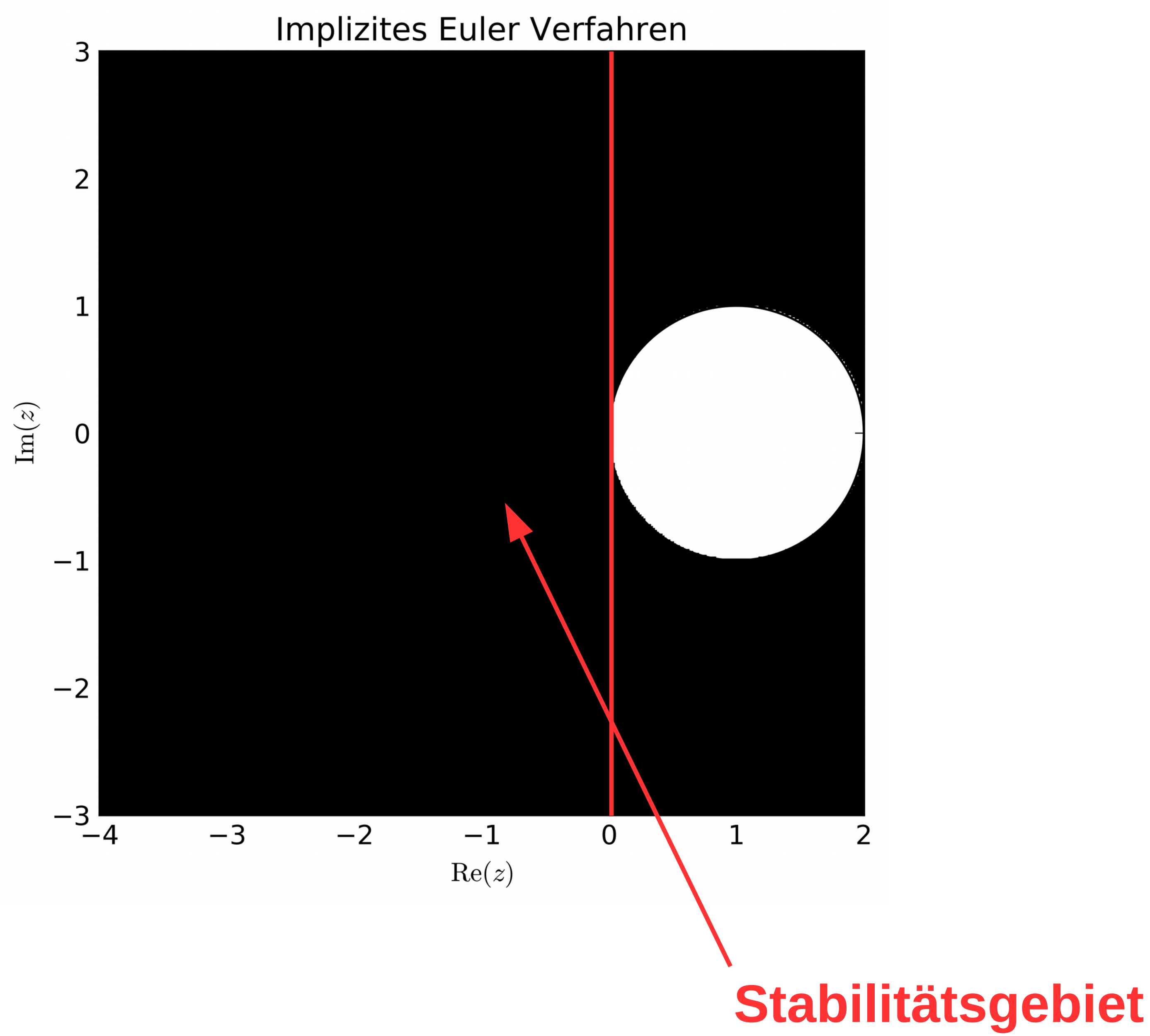

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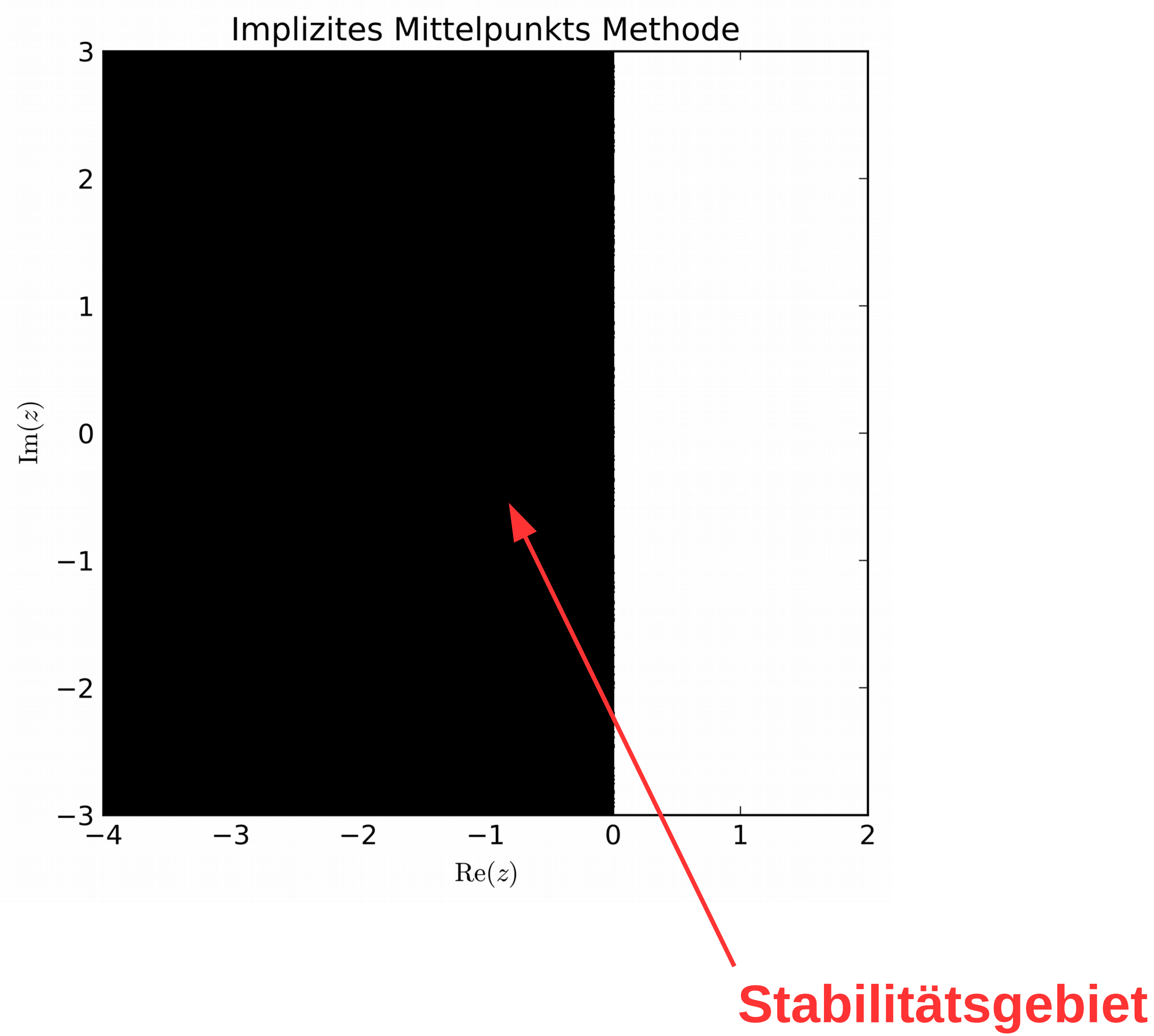


Steifes lineares AWP

$$\dot{\mathbf{y}}(t) = A\mathbf{y}(t)$$

$$\mathbf{y} = \begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{pmatrix} \qquad A = \begin{pmatrix} -\frac{1}{2} & -\frac{869}{10} & \frac{1521}{5} \\ 0 & -\frac{227}{2} & \frac{591}{2} \\ 0 & \frac{591}{2} & -\frac{1803}{2} \end{pmatrix}$$

$$\mathbf{y}_0 = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} \qquad 0 \leq t \leq 2$$

Steifes lineares AWP

D.h. „von Links“

$$\dot{\mathbf{y}}(t) = A\mathbf{y}(t)$$

$$= PDP^{-1}\mathbf{y}(t) \quad \Big| \times P^{-1}$$

Diagonal!

$$\mathrm{P}^{-1}\dot{\mathbf{y}}(t) = D\underbrace{P^{-1}\mathbf{y}(t)}_{\mathbf{z}(t)}$$

$$\dot{\mathbf{z}}(t) = D\mathbf{z}(t)$$

ENTKOPPELT!!!

Steifes lineares AWP

Durch rechnen (z.B. mit einem CAS!):

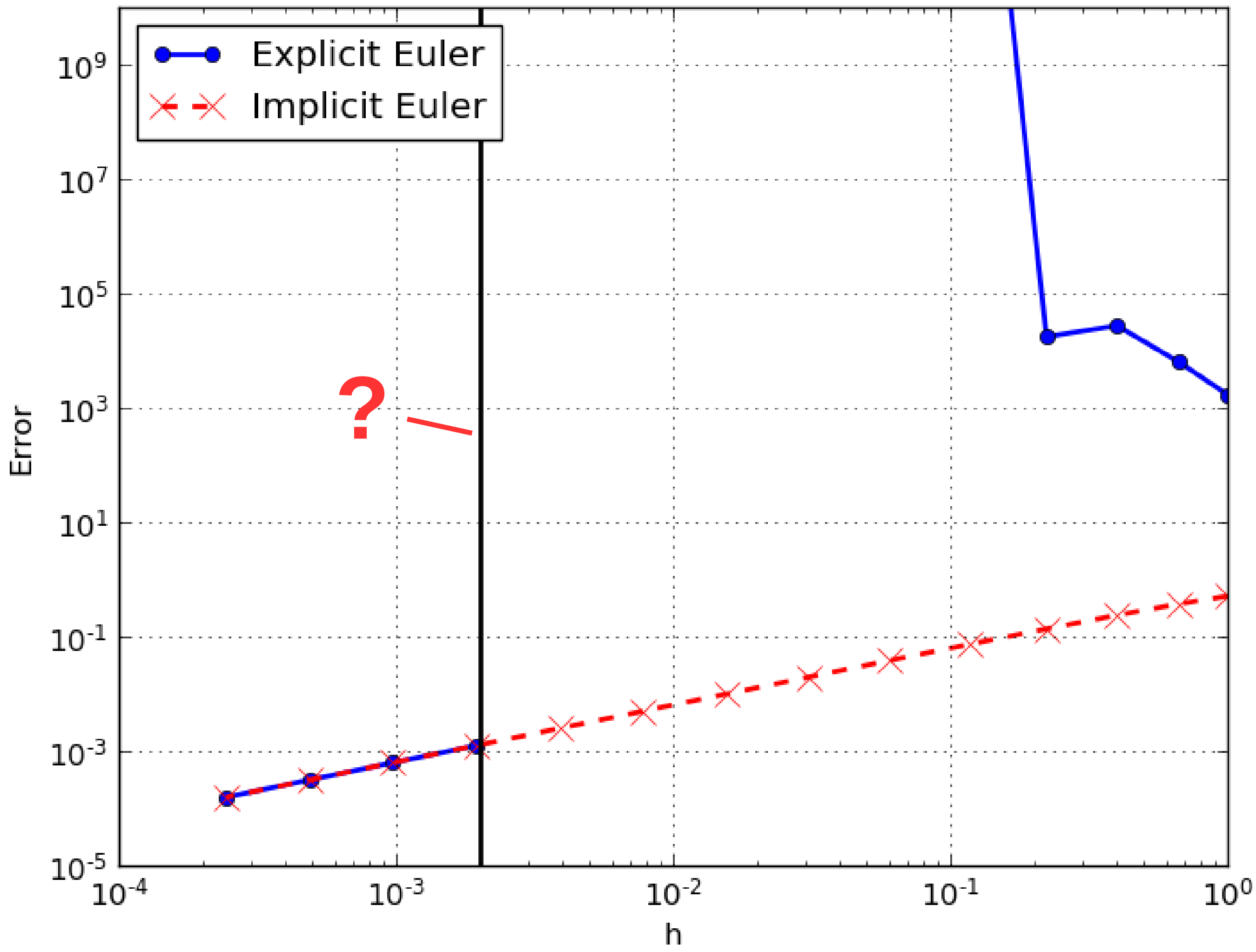
$$D = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -1000 \end{pmatrix}$$

$$P = \begin{pmatrix} 15 & -12 & 1 \\ 0 & 12 & 1 \\ 0 & 4 & -3 \end{pmatrix} \qquad P^{-1} = \begin{pmatrix} \frac{1}{15} & \frac{4}{75} & \frac{1}{25} \\ 0 & \frac{3}{40} & \frac{1}{40} \\ 0 & \frac{1}{10} & -\frac{3}{10} \end{pmatrix}$$

Durch einsetzen der AW:

$$\mathbf{y}(t) = \begin{pmatrix} 7e^{-\frac{1}{2}t} - 6e^{-15t} \\ 6e^{-15t} \\ 2e^{-15t} \end{pmatrix}$$

Steifes lineares AWP



Steifes nichtlineares AWP

$$\begin{aligned}
\dot{y}_A &= -0.1 y_A + 100 y_B y_C \\
\dot{y}_B &= +0.1 y_A - 100 y_B y_C - 500 y_B^2 \\
\dot{y}_C &= +500 y_B^2 - 0.5 y_C
\end{aligned}$$

$$y_A(0) = 0.5 \; y_B(0) = 0.5 \; y_C(0) = 0.5$$

$$0 \leq t \leq 1$$

Steifes nichtlineares AWP

$$\begin{aligned}
\dot{y}_A &= -0.1 y_A + 100 y_B y_C \\
\dot{y}_B &= +0.1 y_A - 100 y_B y_C - 500 y_B^2 \\
\dot{y}_C &= +500 y_B^2 - 0.5 y_C
\end{aligned}$$

NICHTLINEAR!!

$$\mathbf{y} = \begin{pmatrix} y_A \\ y_B \\ y_C \end{pmatrix} \qquad \mathbf{f}(x, \mathbf{y}) = \begin{pmatrix} -0.1 y_A + 100 y_B y_C \\ +0.1 y_A - 100 y_B y_C - 500 y_B^2 \\ +500 y_B^2 - 0.5 y_C \end{pmatrix}$$

$$y_A(0) = 0.5 \; y_B(0) = 0.5 \; y_C(0) = 0.5 \qquad \mathbf{y}_0 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}$$

Steifes nichtlineares AWP

Steif?

Linearisieren wir die rechte Seite der DGL:

$$\mathbf{f}(t,\mathbf{y}) = \begin{pmatrix} -0.1y_A + 100y_B y_C \\ +0.1y_A - 100y_B y_C - 500y_B^2 \\ +500y_B^2 - 0.5y_C \end{pmatrix}$$

$$\mathbf{f}(t,\mathbf{y}) \approx \mathbf{f}(t_1,\mathbf{y}_1) + \underbrace{\frac{\partial \mathbf{f}}{\partial t}(t_1,\mathbf{y}_1)}_{0} + \underbrace{\frac{\partial \mathbf{f}}{\partial \mathbf{y}}(t_1,\mathbf{y}_1)}$$

$$J(t,\mathbf{y}) = \frac{\partial \mathbf{f}}{\partial \mathbf{y}}(t,\mathbf{y}) = \begin{pmatrix} -0.1 & 100y_C & 100y_B \\ +0.1 & -1000y_B - 100y_C & -100y_B \\ 0 & 1000y_B & -0.5 \end{pmatrix}$$

Steifes nichtlineares AWP

Lokal Steif?

Linearisieren wir um den AW: $t_0 = 0 \quad \mathbf{y}_0 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}$

$$J(t_0, \mathbf{y}_0) = \frac{\partial \mathbf{f}}{\partial \mathbf{y}}(t_0, \mathbf{y}_0) = \begin{pmatrix} -0.1 & 50 & 50 \\ +0.1 & -550 & -50 \\ 0 & 500 & -0.5 \end{pmatrix}$$

MATLAB: eig →

$$\lambda_1 \approx -5.00 \times 10^2$$
$$\lambda_2 \approx -9.87 \times 10^{-4}$$
$$\lambda_3 \approx -5.07 \times 10^1$$

$$S = \frac{\max_j |\operatorname{Re}(\lambda_j)|}{\min_j |\operatorname{Re}(\lambda_j)|} \approx 5.06 \times 10^5$$

JA!!!