SOLUTION of (2-14.k):

We write $\widehat{b}^\ell$, $\ell = 1,2,3$, for the local shape functions on the reference element. We can simply use the formula (2.14.3) found in Sub-problem (2-14.i) and that the barycentric coordinate functions on $\widehat{K}$ are given by

$$\lambda_1(\boldsymbol{x}) = 1 - x_1 - x_2 \quad , \quad \lambda_2(\boldsymbol{x}) = x_1 \quad , \quad \lambda_3(\boldsymbol{x}) = x_2 \,, \quad \boldsymbol{x} := \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} .$$

Thus, from (2.14.3), we find

$$\begin{aligned} \widehat{b}^1(\widehat{\boldsymbol{x}}) &= 1 - 2\lambda_3 = 1 - 2x_2 \,, \\ \widehat{b}^2(\widehat{\boldsymbol{x}} &= 1 - 2\lambda_1 = 2(x_1 + x_2) - 1 \,, \quad \widehat{\boldsymbol{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \widehat{K} \,, \\ \widehat{b}^3(\widehat{\boldsymbol{x}} &= 1 - 2\lambda_2 = 1 - 2x_1 \,, \end{aligned} \tag{2.14.4}$$

with gradients

$$\begin{aligned} \mathbf{grad}\, \widehat{b}^1 &= -2\,\mathbf{grad}\,\lambda_3 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \\ \mathbf{grad}\, \widehat{b}^2 &= -2\,\mathbf{grad}\,\lambda_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \\ \mathbf{grad}\, \widehat{b}^3 &= -2\,\mathbf{grad}\,\lambda_2 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} . \end{aligned} \tag{2.14.5}$$