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SOLUTION of (2-14.m):

First notice that

$$a_{\mathcal{M}}(v_h, v_h) = \sum_{K \in \mathcal{M}} \int_K |\mathbf{grad} v_h(\mathbf{x})|^2 d\mathbf{x} \geq 0.$$

Let us suppose that there exists a  $v_h \in \mathcal{CR}_0(\mathcal{M}) \setminus \{\mathbf{0}\}$  such that  $a_{\mathcal{M}}(v_h, v_h) = 0$ .

Then

$$\mathbf{grad} v_h(\mathbf{x}) = \mathbf{0}, \quad \forall K \in \mathcal{M},$$

and this implies  $v_h$  is constant  $\forall K \in \mathcal{M}$ . Since

$$v_h(\mathbf{m}) = 0, \quad \forall \mathbf{m} \in \mathcal{N} \cap \partial\Omega,$$

this means that  $v_h \equiv 0$  for all  $K \in \mathcal{M}$ , which poses a contradiction as we assumed  $v_h \in \mathcal{CR}_0(\mathcal{M}) \setminus \{\mathbf{0}\}$ . Therefore, we conclude the desired result.

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