
SOLUTION of (2-14.m):

First notice that

$$a_{\mathcal{M}}(v_h, v_h) = \sum_{K \in \mathcal{M}} \int_K |\mathbf{grad} v_h(x)|^2 dx \geq 0.$$

Let us suppose that there exists a $v_h \in \mathcal{CR}_0(\mathcal{M}) \setminus \{0\}$ such that $a_{\mathcal{M}}(v_h, v_h) = 0$.

Then

$$\mathbf{grad} v_h(x) = 0, \quad \forall K \in \mathcal{M},$$

and this implies v_h is constant $\forall K \in \mathcal{M}$. Since

$$v_h(m) = 0, \quad \forall m \in \mathcal{N} \cap \partial\Omega,$$

this means that $v_h \equiv 0$ for all $K \in \mathcal{M}$, which poses a contradiction as we assumed $v_h \in \mathcal{CR}_0(\mathcal{M}) \setminus \{0\}$. Therefore, we conclude the desired result.
