

SOLUTION of (2-14.e):

In fact, the basis functions are discontinuous, see Fig. 12: $b_h^i(x_n) = -1$ if x_n is opposite of m_i on some triangle, but $b_h^i = 0$ on all other triangles touching x_n , thus we have “different values from different sides”:

$$\blacktriangleright \quad \mathcal{CR}(\mathcal{M}) \not\subset C^0(\Omega) \quad (\text{, if } \#\mathcal{M} > 1).$$

Since b_h^j is piecewise smooth, the assumptions of [Lecture → Thm. 1.3.4.23] are violated:

Theorem [Lecture → Thm. 1.3.4.23]. Compatibility conditions for piecewise smooth functions in $H^1(\Omega)$

Let Ω be partitioned into sub-domains Ω_1 and Ω_2 . A function u that is continuously differentiable in both sub-domains and continuous up to their boundary, belongs to $H^1(\Omega)$, if and only if u is continuous on Ω .