

SOLUTION of (2-14.g):

Yes, as it satisfies the 3 conditions introduced in [Lecture → Section 2.5.3]:

Third main ingredient of FEM:

locally supported basis functions
(see [Lecture → Section 2.2] for role of bases in Galerkin discretization)

Basis functions b_h^1, \dots, b_h^N for a finite element trial/test space $V_{0,h}$ built on a mesh \mathcal{M} **must** satisfy:

- (\mathfrak{B}_1) $\mathfrak{B}_h := \{b_h^1, \dots, b_h^N\}$ is basis of $V_{0,h} \Rightarrow N = \dim V_{0,h}$,
- (\mathfrak{B}_2) each b_h^i is **associated** with a single geometric entity (cell/edge/face/vertex) of \mathcal{M} ,
- (\mathfrak{B}_3) $\text{supp}(b_h^i) = \bigcup \{\bar{K} : K \in \mathcal{M}, p \subset \bar{K}\}$, if b_h^i associated with cell/edge/face/vertex p .

1. follows by the definition of $\mathcal{CR}(\mathcal{M})$ and Sub-problem (2-14.b). These two combined imply that the set of functions defined in (2.14.1) are a basis for an N -dimensional finite element space $\mathcal{CR}(\mathcal{M}) \subset L^2(\Omega)$.
2. holds as each b_h^i is associated to a single edge.
3. As we saw in Sub-problem (2-14.f), if b_h^i is associated with the edge \mathbf{e} , then $\text{supp}(b_h^i) = \bigcup \{\bar{K} : K \in \mathcal{M}, \mathbf{e} \subset \bar{K}\}$. Moreover, $\#\{K \in \mathcal{M} : K \subset \text{supp}(b_h^i)\} = 2$.