

SOLUTION of (2-14.g):

Yes, as it satisfies the 3 conditions introduced in [Lecture → Section 2.5.3]:

Third main ingredient of FEM:

locally supported basis functions  
(see [Lecture → Section 2.2] for role of bases in Galerkin  
discretization)

Basis functions  $b_h^1, \dots, b_h^N$  for a finite element trial/test space  $V_{0,h}$  built on a mesh  $\mathcal{M}$  **must** satisfy:

- ( $\mathfrak{B}_1$ )  $\mathfrak{B}_h := \{b_h^1, \dots, b_h^N\}$  is basis of  $V_{0,h}$   $\Rightarrow$   $N = \dim V_{0,h}$ ,
- ( $\mathfrak{B}_2$ ) each  $b_h^i$  is **associated** with a single geometric entity (cell/edge/face/vertex) of  $\mathcal{M}$ ,
- ( $\mathfrak{B}_3$ )  $\text{supp}(b_h^i) = \bigcup\{\bar{K} : K \in \mathcal{M}, p \subset \bar{K}\}$ , if  $b_h^i$  associated with cell/edge/face/vertex  $p$ .

1. follows by the definition of  $\mathcal{CR}(\mathcal{M})$  and Sub-problem (2-14.b). These two combined imply that the set of functions defined in (2.14.1) are a basis for an  $N$ -dimensional finite element space  $\mathcal{CR}(\mathcal{M}) \subset L^2(\Omega)$ .
2. holds as each  $b_h^i$  is associated to a single edge.
3. As we saw in Sub-problem (2-14.f), if  $b_h^i$  is associated with the edge  $e$ , then  $\text{supp}(b_h^i) = \bigcup\{\bar{K} : K \in \mathcal{M}, e \subset \bar{K}\}$ . Moreover,  $\#\{K \in \mathcal{M} : K \subset \text{supp}(b_h^i)\} = 2$ .