SOLUTION of (2-14.i):

The local shape functions $b_K^1, b_K^2, b_K^3$ for $\mathcal{CR}(\mathcal{M})$ on a triangle $K$ are *affine linear*,

$$\forall v_h \in \mathcal{CR}(\mathcal{M}),\ \forall K \in \mathcal{M}: \quad v_h|_K \in \mathcal{P}_1(\mathbb{R}^2)\ ,$$

and, thus, can be written as a linear combination of the barycentric coordinate functions $\lambda_i$ of $K$.

We elaborate the derivation for $b_K^1$; after permutation of indices the considerations also apply to $b_K^2$ and $b_K^3$. We set

$$b_K^1 = \xi_1\lambda_1 + \xi_2\lambda_2 + \xi_3\lambda_3\ ,$$

with yet unknown coefficients $\xi_1, \xi_2, \xi_3 \in \mathbb{R}$. From (2.14.1) we obtain three equations:

$$\begin{aligned}
1 = b_K^1(\boldsymbol{m}_K^1) &= \xi_1\lambda_1(\boldsymbol{m}_K^1) + \xi_2\lambda_2(\boldsymbol{m}_K^1) + \xi_3\lambda_3(\boldsymbol{m}_K^1) = \tfrac{1}{2}\xi_1 + \tfrac{1}{2}\xi_2\ ,\\
0 = b_K^1(\boldsymbol{m}_K^2) &= \xi_1\lambda_1(\boldsymbol{m}_K^2) + \xi_2\lambda_2(\boldsymbol{m}_K^2) + \xi_3\lambda_3(\boldsymbol{m}_K^2) = \tfrac{1}{2}\xi_2 + \tfrac{1}{2}\xi_3\ ,\\
0 = b_K^1(\boldsymbol{m}_K^3) &= \xi_1\lambda_1(\boldsymbol{m}_K^3) + \xi_2\lambda_2(\boldsymbol{m}_K^3) + \xi_3\lambda_3(\boldsymbol{m}_K^3) = \tfrac{1}{2}\xi_1 + \tfrac{1}{2}\xi_3\ .
\end{aligned}$$

We have obtained a $3 \times 3$ linear system of equations for $\xi_1, \xi_2, \xi_3$:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\ ,$$

for which we find the solution

$$\xi_1 = 1\ , \quad \xi_2 = 1\ , \quad \xi_3 = -1 \quad \blacktriangleright \quad b_K^1 = \lambda_1 + \lambda_2 - \lambda_3\ .$$

This formula can be simplified based on the fact that $\lambda_1 + \lambda_2 + \lambda_3 \equiv 1$, which gives us the general formula

$$b_K^j = 1 - 2\lambda_{\mathrm{opp}(j)}\ , \quad j = 1, 2, 3\ . \tag{2.14.3}$$