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2  template <typename MESHFUNCTION>
3  Eigen::VectorXd quasiInterpolate(
4      const If::uscalfe::FeSpaceLagrangeO1<double> &fe_space,
5      MESHFUNCTION &&v_mf) {
6      // Get quadrature points and weights on the reference triangle for a
7      // quadrature rule that integrates quadratic polynomials exactly
8      If::quad::QuadRule quadrule =
9          If::quad::make_QuadRule(If::base::RefEl::kTria(), 2);
10     const int P = quadrule.NumPoints();
11     Eigen::MatrixXd zeta_hat = quadrule.Points();
12     Eigen::VectorXd w_hat = quadrule.Weights();
13     // Precomputation: Cache values of  $(\hat{\psi}^1, \hat{\psi}^2, \hat{\psi}^3)$ 
14     // at reference quadrature nodes. Use the formulas (3.14.4).
15     Eigen::Matrix<double, 3, 2> A;
16     A << -24.0, -24.0, 24.0, 0.0, 0.0, 24.0;
17     Eigen::Vector3d b{18.0, -6.0, -6.0};
18     auto psi_hat = [A, b](Eigen::Vector2d x) -> Eigen::Vector3d {
19         return A * x + b;
20     };
21     Eigen::MatrixXd psi_hat_values(3, P);
22     for (int l = 0; l < P; ++l) {
23         psi_hat_values.col(l) = psi_hat(zeta_hat.col(l));
24     }
25     // Retrieve the map  $p \mapsto (K_p, j)$ 
26     std::shared_ptr<const If::mesh::Mesh> mesh_p = fe_space.Mesh();
27     If::mesh::utils::CodimMeshDataSet<
28         std::pair<const If::mesh::Entity *, unsigned int>>
29         Kp_mesh_data_set = findKp(mesh_p);
30
31     const If::assemble::DofHandler &dofh{fe_space.LocGlobMap()};
32     const int N = dofh.NumDofs();
33     // Vector for returning the basis expansion coefficients of the
34     // quasi-interpolant
35     Eigen::VectorXd coefficients(N);
36     // Compute projection onto each basis function
37     for (If::assemble::gdof_idx_t i = 0; i < N; ++i) {
38         // Get  $p$ ,  $K_p$  and the local index  $j$  of  $p$  in  $K_p$ 
39         const If::mesh::Entity &point = dofh.Entity(i);
40         std::pair<const If::mesh::Entity *, unsigned int> Kp_localindex =
41             Kp_mesh_data_set(point);
42         const If::mesh::Entity *Kp = Kp_localindex.first;
43         unsigned int j = Kp_localindex.second;
44         // Evaluate  $v$  on quadrature points in  $K_p$ 
45         const std::vector<double> v_zeta = v_mf(*Kp, zeta_hat);
46         // Compute integral on current triangle  $K_p$ 
47         double sum = 0.0;
48         for (int l = 0; l < P; ++l) {
49             sum += w_hat(l) * v_zeta[l] * psi_hat_values(j, l);
50         }
51         coefficients(i) = sum;
52     }
53     return coefficients;
54 }

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