
SOLUTION of (6-5.b):

By the product rule and using the fact that \vec{Y} is a solution of the IVP we obtain

$$\begin{aligned}\frac{d}{dt}(\mathbf{Y}^\top(t)\mathbf{Y}(t)) &= \frac{d}{dt}(\mathbf{Y}^\top(t))\mathbf{Y}(t) + \mathbf{Y}^\top(t)\frac{d}{dt}(\mathbf{Y}(t)) \\ &= (-(\mathbf{Y}(t) - \mathbf{Y}^\top(t))\mathbf{Y}(t))^\top\mathbf{Y}(t) + \mathbf{Y}^\top(t)(-(\mathbf{Y}(t) - \mathbf{Y}^\top(t))\mathbf{Y}(t)) \\ &= -\mathbf{Y}^\top(t)\mathbf{Y}^\top(t)\mathbf{Y}(t) + \mathbf{Y}^\top\mathbf{Y}(t)\mathbf{Y}(t) \\ &\quad - \mathbf{Y}^\top\mathbf{Y}(t)\mathbf{Y}(t) + \mathbf{Y}^\top(t)\mathbf{Y}^\top(t)\mathbf{Y}(t) = 0.\end{aligned}$$

implying that the map is constant.
