

C++ code 9.10.29: Code for `leapfrogMixedWave()` → [GitLab](#)

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2  template <typename RHOFUNCTION, typename FFUNCTION,
3          typename RECORDER = std::function<void(double, double)>>
4  std::pair<Eigen::VectorXd, Eigen::VectorXd> leapfrogMixedWave(
5      const std::shared_ptr<If::uscalfe::FeSpaceLagrangeO1<double>> &fe_space_V,
6      const If::assemble::UniformFEDofHandler &dofh_Q, RHOFUNCTION &&rho,
7      FFUNCTION &&f, double T, unsigned int nb_timesteps,
8      RECORDER &&rec = []<(double, double) {}>() {
9      // Size of timestep
10     double stepsize = T / nb_timesteps;
11     // Index mapper for linear Lagrangian FE space
12     const If::assemble::DofHandler &dofh_V{fe_space_V->LocGlobMap()};
13     // Dimension of finite element space
14     const If::uscalfe::size_type N_dofs_Q(dofh_Q.NumDofs());
15     const If::uscalfe::size_type N_dofs_V(dofh_V.NumDofs());
16
17     // Zero INITIAL CONDITIONS
18     Eigen::VectorXd mu_init = Eigen::VectorXd::Zero(N_dofs_V);
19     Eigen::VectorXd kappa_init = Eigen::VectorXd::Zero(N_dofs_Q);
20
21     // PRECOMPUTATIONS: essential for efficiency
22     std::cout << "Precomputing Galerkin Matrices :" << std::endl;
23     // Galerkin matrix  $\widetilde{M}_V$ 
24     Eigen::SparseMatrix<double> M_V = computeMV(fe_space_V, rho);
25     std::cout << "Creating SparseLU solver for M_V: ";
26     Eigen::SparseLU<Eigen::SparseMatrix<double>> solver_MV;
27     solver_MV.compute(M_V);
28     LF_VERIFY_MSG(solver_MV.info() == Eigen::Success, "LU decomposition failed");
29     std::cout << " Done" << std::endl;
30     // Galerkin matrix  $\widetilde{M}_Q$ 
31     Eigen::SparseMatrix<double> M_Q = computeMQ(dofh_Q);
32     std::cout << "Creating SparseLU solver for M_Q: ";
33     Eigen::SparseLU<Eigen::SparseMatrix<double>> solver_MQ;
34     solver_MQ.compute(M_Q);
35     LF_VERIFY_MSG(solver_MQ.info() == Eigen::Success, "LU decomposition failed");
36     std::cout << "Done" << std::endl;
37     // Galerkin matrix  $\widetilde{B}$ 
38     Eigen::SparseMatrix<double> B = computeB(dofh_V, dofh_Q);
39
40     // EVOLUTION
41     // Vectors for returning result
42     Eigen::VectorXd mu;
43     Eigen::VectorXd kappa;
44     std::cout << "Evolution using the leapfrog timestepping scheme." << std::endl;
45     Eigen::VectorXd mu_cur = mu_init;           //  $\vec{\mu}^{(j)}$ 
46     Eigen::VectorXd kappa_cur = kappa_init;      //  $\vec{\kappa}^{(j+\frac{1}{2})}$ 
47     Eigen::VectorXd mu_next;                    //  $\vec{\mu}^{(j+1)}$ 
48     Eigen::VectorXd kappa_next;                 //  $\vec{\kappa}^{(j+\frac{3}{2})}$ 
49     Eigen::VectorXd kappa_avg;
50     for (int j = 0; j < nb_timesteps; j++) {
51         // Right hand side vector  $\vec{\varphi}(\tau(j+\frac{1}{2}))$ 
52         const Eigen::VectorXd rhs{computeRHS(fe_space_V, f, stepsize * (j + 0.5))};
53         // Update of  $\vec{\mu}$ 
54         mu_next = solver_MV.solve(stepsize * (rhs + B.transpose() * kappa_cur) +
55                                   M_V * mu_cur);

```