1

ETH Lecture 401-0674-00L Numerical Methods for (Partial) Differential Equations

Numerical Methods for (Partial) Differential Equations

Prof. R. Hiptmair, SAM, ETH Zurich

Spring Term 2023

Useful Links and access information:

Course directory page:
https://www.vorlesungen.ethz.ch/Vorlesungsverzeichnis/lerneinheit.view?lerneinheitId=167947&semkez=2023S&ansicht=LEHRVERANSTALTUNGEN&lang=en
Moodle page: https://moodle-app2.let.ethz.ch/course/view.php?id=19025
Course notes: https://www.sam.math.ethz.ch/~grsam/NUMPDEFL/NUMPDE.pdf
(chapter 0 of the course notes gives you a general introduction)
Homework problem collection:
https://people.math.ethz.ch/~grsam/NUMPDEFL/HOMEWORK/NPDEFL_Problems.pdf
Course code repository: https://github.com/erickschulz/NPDECODES
Course repository: https://people.math.ethz.ch/~grsam/NUMPDEFL/
Course polybox folder: https://polybox.ethz.ch/index.php/s/xfmTwW3FTZDSlnX
PW: NPDE23 (contains course videos and tablet notes)

Welcome and Introduction

0.4 Topic

Better: Numerical methods for the simulation of continuum models with local interactions

continuum $\triangleq$ unknowns are functions / fields, e.g. $u = u(x)$, $x \in \mathbb{R}^d$
$\in$ ∞-dimensional function spaces

model $\triangleq$ $\underbrace{\text{mathematical description}}_{\text{Here: PDE-based}}$ of some aspects of reality

$\Rightarrow$ PDE embedded in physical context

$\rightarrow$ Not "just an equation" $F(u, Du) = 0$
some (partial) derivatives

$\rightarrow$ encodes structural properties

$\downarrow$

Should be respected / exploited by a numerical method ("Structure-preserving")

0.4.3 Example (Fluid mechanics)

(Stationary, incompressible) Navier-Stokes equations:

$$\begin{aligned} \underset{\substack{\uparrow\\ \text{dissipation}}}{-\nu\Delta\mathbf{u}} + D\mathbf{u}\cdot\mathbf{u} + \mathbf{grad}\, p &= \mathbf{f} \quad \text{in } \Omega\,, \\ \operatorname{div}\mathbf{u} &= 0 \quad \text{in } \Omega\,, \quad \longrightarrow \text{incompressibility} \\ \mathbf{u} &= 0 \quad \text{on } \partial\Omega\,. \end{aligned} \tag{0.4.3.1}$$

$\nu \;\hat{=}\;$ dynamic viscosity
$\mathbf{f} : \Omega \mapsto \mathbb{R}^3 \;\hat{=}\;$ given external force field
$\mathbf{u} : \Omega \mapsto \mathbb{R}^3 \;\hat{=}\;$ velocity field (unknown)
$p : \Omega \mapsto \mathbb{R} \;\hat{=}\;$ pressure (unknown)

0.3 Prerequisites

In (0.4.3.1): differential operators grad, div, Δ
$\longrightarrow$ advanced (vector) calculus
$\longrightarrow$ multi-dimensional integration/differentiation

Of course: Basic numerical methods (quadrature, numerical LA, numerical integration, ...)
and C++

Contents and Outline

Model classes covered in this course (2023 edition)

I. Equilibrium models

Elastic membranes, electrostatics, stationary heat conduction

II. Evolution models

Heat equation, wave propagation, Schrödinger equation

III. Transport models

③

0.1.2. Teaching Model

→ Flipped classroom format

A flipped-classroom course

This course will follow the **flipped-classroom** paradigm:

Learning by **self-study** guided by

instruction videos tablet notes	lecture notes	interactive Q&A sessions	homework tutorial classes

assigned every week (Moodle page)

Also supplementary

Submit questions through Discord channels

Define course contents

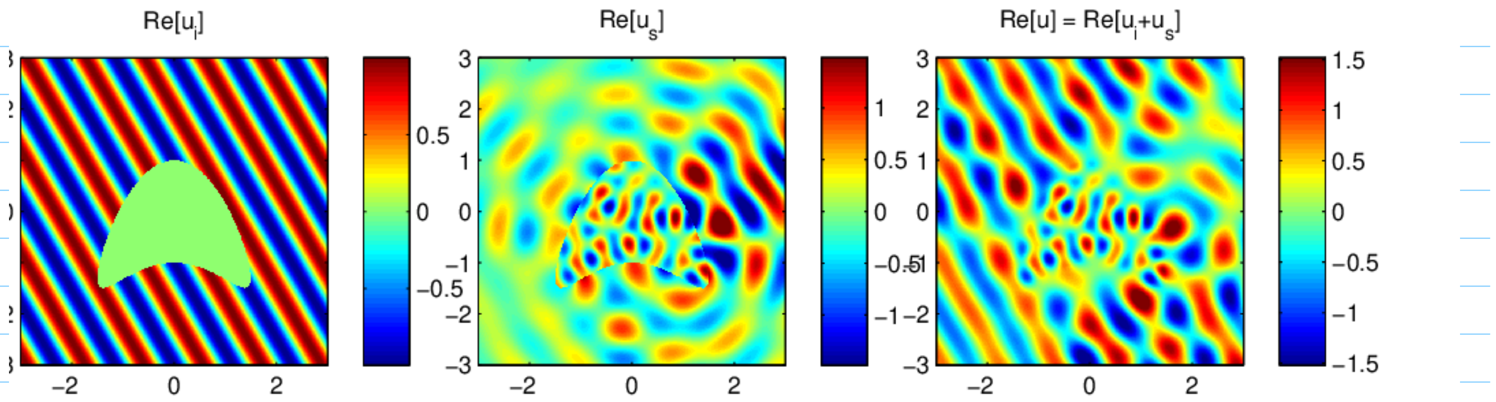
3-5 h/week

7-10 h/week

Flipped classroom advice:

- Reserve time slots
- Stable setting / routine
- Take breaks
- Group study

Much success with NumPDE !



Left: incident wave; middle: scattered field, right: total pressure p (real parts). Computation with method of particular solutions, using the software MPSPACK,